

# Regularity results for time-dependent quasi-variational inequalities with applications to dynamic traffic network

by **Annamaria Barbagallo** (Catania)

## Abstract

The aim of this paper is to consider linear and nonlinear time-dependent quasi-variational inequalities and to study under which assumptions the continuity of solutions with respect to the time can be ensured. In order to achieve our analytical results, the set convergence in the Mosco's sense plays an important role. We present the continuity results for solutions to time-dependent quasi-variational inequalities associated to a general class of convex sets, and then we apply these results to dynamic traffic equilibrium problems. In particular, we apply our results to show that equilibrium solutions to the time-dependent elastic problem, which arose whenever travel demands are dependent on the equilibrium distribution, are continuous.

# Incomplete similarity in continuum mechanics

by **Grigory I. Barenblatt** (Berkeley, CA – U.S.A.)

## Abstract

The general concept of incomplete similarity will be introduced and discussed. The connection of this concept with the dimensional analysis, intermediate asymptotics and renormalization group will be presented. The examples of scaling laws having the property of incomplete similarity will be outlined, in particular in the mathematical models of turbulent flows, fatigue of metals and fracture.

# Polyharmonic Dirichlet problems

by **Heinrich Begehr** (Berlin, Germany)

## Abstract

The Cauchy Pompeiu representation related to the poly-harmonic operator is altered in replacing the fundamental solution in the higher order Pompeiu operator by some proper poly-harmonic Green function. Depending on the choice of the Green function this leads to the solution of different kinds of Dirichlet problems. The higher the order of the differential operator the more Green functions and hence the more Dirichlet problems exist.

Here plane problems are investigated and complex notation is used. In order to be explicit mainly particular domains like the unit disc and a half plane are considered.

# On the representation of pseudoanalytic functions in the plane and in the space

by **Peter Berglez** (Graz, Austria)

## Abstract

For different problems in mathematical physics and mechanics pseudoanalytic functions are of practical use. For certain classes of such pseudoanalytic functions in two and three dimensions we give different representations. One of these methods uses particular differential operators acting on holomorphic and monogenic functions respectively. Furthermore formal powers are constructed with which such pseudoanalytic functions may be approximated.

**$\Gamma$ -convergence for strongly local  
Dirichlet forms in open sets  
with holes with homogeneous  
Neumann boundary conditions**

by **Marco Biroli** (Milano)

**Abstract**

We consider a Schrödinger type problem relative to a  $p$ -homogeneous Riemannian Dirichlet form. We define a suitable Kato class of measures and we prove that if the potential is a measure in the Kato class, then a Harnack inequality holds on suitable small balls (for distance relative to the form).

# Qualitative properties of viscosity solutions of nonlinear elliptic equations

by **Italo Capuzzo Dolcetta** (Roma)

## Abstract

Continuous solutions of fully nonlinear second order partial equations

$$F(x, u, Du, D^2u) = 0,$$

in an open domain  $\Omega \subseteq \mathbf{R}^n$ , have been extensively analyzed in recent years in the framework of the theory of viscosity solutions. For elliptic  $F$ , this weak notion of solution is a suitable generalization of that of harmonic function. The aim of the lecture is to describe extensions to this general setting of some classical results for solutions of linear elliptic equations such as the Hadamard Three Circles, the Liouville as well as the Alexandrov-Bakelman-Pucci and Phragmen-Lindelof theorems.

# Evolution Problems in Heat Conduction with Memory: Some Recent Results

by **Sandra Carillo** (Roma)

## Abstract

The temperature's evolution within a rigid linear heat conductor with memory is considered. An evolution problem which arises in studying the thermodynamical state of the material with memory is considered. Specifically, the time evolution of the temperature distribution within a rigid heat conductor with memory is investigated. The constitutive equations which characterize heat conduction with memory, according to a wide literature which takes its origins in Cattaneo's work [2] involve an integral term since the temperature's time derivative is connected to the heat flux gradient. In the framework of Fabrizio, Gentili and Reynolds' model[5], an evolution problem is studied. Specifically, some recent results[1, 3, 4] are presented concerning the integro-differential problem initial and boundary value problem which describes the temperature's evolution within the rigid heat conductor. Key tools, further to the application of Fourier transforms, turn out to be represented by suitable expressions of the minimum free energy as well as the application of semigroup theory.

## References

- [1] G. Amendola, S. Carillo, Thermal work and minimum free energy in a heat conductor with memory, *Quart. J. of Mech. and Appl. Math.* **57(3)**, (2004) 429–446.
- [2] C. Cattaneo, sulla conduzione del calore *Atti Sem. Mat. Fis. Università Modena* **3** (1948), 83–101.
- [3] S. Carillo, Some remarks on materials with memory: heat conduction and viscoelasticity, *J. Nonlinear Math. Phys.* **12**, (2005), suppl. 1, 163–178.
- [4] S. Carillo, M. Fabrizio, , *preprint*, (2006).
- [5] M. Fabrizio, G. Gentili, D.W.Reynolds On rigid heat conductors with memory *Int. J. Eng. Sci.* **36** (1998), 765–782.

# Asymptotic Analysis and Dougall's Bilateral ${}_2H_2$ -Series Identity

by **Wenchang Chu** (Lecce)

## Abstract

By means of the modified Abel lemma on summation by parts, a recurrence relation for Dougall's bilateral  ${}_2H_2$ -series is established with an extra natural number parameter. Then the steepest descent method allows us to compute the limit for  $m \rightarrow \infty$ , which leads us to a completely new and surprising proof of the celebrated bilateral  ${}_2H_2$ -series identity due to Dougall (1907). This exemplifies a new connection between hypergeometric series and classical asymptotic analysis.

# Fading memory principle and quasi-static problem in viscoelasticity transitions

by **Mauro Fabrizio** (Bologna)

## Abstract

The Fichera problem for the integro-differential equation of viscoelasticity is connected with the study of the corresponding quasi-static problem. Fichera observed that this problem cannot be resolved, as in linear elasticity, without providing the datum of the initial history and so working on the time interval  $[0,8)$ . In this paper, following the Fichera point of view, we prove that it is convenient to work on new spaces more natural than the ones studied up to now. Therefore, we shall consider the topology connected with the minimum free energy, which provides the largest space  $H$  on which to take the initial history. In the meantime, the space of the solutions is built by means of the dual space of the set  $H$  defined by the histories. For such a Fichera problem, by means of these new spaces, we are able to prove existence and uniqueness for a wide family of initial data.

# Shear motion of a material with stretching threshold

by **Antonio Fasano** (Firenze)

## Abstract

The biological material making tendons and ligaments behaves in an elastic way for small deformations, but it becomes very slightly beyond some threshold stress. We extrapolate this kind of stress-strain relationship with a linear elastic law for stress below the threshold  $\tau_0$  and with complete undeformability for stress above threshold and we apply it to the study of the dynamics of a layer  $0 < x < L$ , in which the boundary  $x = 0$  is immobile and a shear stress  $\sigma$  dependent of time and taking values larger than  $\tau_0$  is applied at  $x = L$ . The time intervals in which  $\sigma > \tau_0$  are characterized by the presence of a fully stretched zone, separated by the still elastic region by a free boundary. From the mathematical point of view the problem is a free boundary problem for a hyperbolic equation with rather complicated conditions on the interface.

A general qualitative analysis is performed and the solution is constructed for some model problems, showing that in the most interesting cases the interface is supersonic.

Another question which is studied is the possibility of approximating the singular constitutive law employed by means of

- (i) a sequence of laws with a linear stress-strain relationship for  $\sigma > \tau_0$  whose slope tends to infinity
- (ii) a sequence of elastic potentials  $\Psi_n(\sigma) \in C^1(0, \tau_0)$ , all tending to infinity for  $\tau \rightarrow \tau_0$ .

It is shown that only the second approach can be successful.

## References

- [1] A. Farina, A. Fasano, L. Fusi, K.R. Rajagopal. Modelling elastic materials with stretching threshold, (to appear).

# Acoustic Modeling and Osteoporotic Evaluation of Bone

by **Robert P. Gilbert** (Newark, DE – U.S.A.)

## Abstract

In this talk we discuss the modeling of bone using the methods of homogenization. This can lead to Biot type equations or more generalized equations. We also discuss the use of the Biot model and consider its applicability to cancellous bone. One of the questions this talk addresses is whether the clinical experiments customarily performed can be used to determine the parameters of the Biot model. A parameter recovery algorithm which uses parallel processing is developed and tested.

# Boundary Integral Equations Recast as Pseudodifferential Equations

by George C. Hsiao (Newark, DE – U.S.A.) and  
Wolfgang L. Wendland (Stuttgart)

## Abstract

It is known that the treatment of boundary value problems based on variational principles often leads to corresponding boundary integral equations in weak formulations. Their mapping properties can then be derived from those of the associated partial differential equations. However, this approach is restricted only to those boundary value problems which can be formulated in terms of general variational principles based on Gårding's inequality. On the other hand, boundary integral equations can also be recast as special classes of pseudodifferential equations. In this paper, we are concerned with the latter approach by applying pseudodifferential operator theory to a class of elliptic boundary value problems. In particular, the boundary value problems for the Helmholtz equation of scattering theory and the Lamé equations of linear elasticity will serve as model problems for illustrating the basic ideas how one can apply the theory of pseudodifferential operators and their calculus to obtain basic solution properties for the boundary integral equations.

# Variational Inequalities, Smooth Continuation and Bifurcation

by **Milan Kucera** (Prague, Czech Republic)

## Abstract

Variational inequalities are in general nonsmooth problems. However, in some particular situations, it is possible to show that a variational inequality is equivalent in a certain sense to a smooth operator equation in a neighbourhood of a given solution or a bifurcation point. The standard approaches can be used for this equation and the results can be translated for the inequality. The goal of the lecture is to mention some joint results with L. Recke and J. Eisner where the existence of smooth (at least differentiable) local bifurcation branches emanating from simple eigenvalues and a smooth dependence on parameters for variational inequalities of a certain type is proved in this way. The direction of bifurcating branches and in the potential case also their stability is also described. The bifurcation diagrams are sometimes surprising in a certain sense.

# Sturm Methods, inequalities and zeros of some special functions

by **Andrea Laforgia** (Roma)

## Abstract

This is a survey of Sturmian methods for finding inequalities, monotonicity and concavity (convexity) properties of zeros of classical orthogonal polynomials and Bessel functions. The main tool used is the Sturm comparison theorem in a form due to Szegő. We also consider Turan-type inequalities for some classes of special functions. These results are established using a generalization of an integral Schwarz inequality.

# Parabolic transmission problems across irregular layers

by **Maria Rosaria Lancia** (Roma)

## Abstract

We describe some transmission problems across fractal layers for second order elliptic or parabolic operators. The transmission condition is of order two. The fractal layer is obtained as the limit of suitable polyhedral surfaces (pre-fractal layers). The key point to be analyzed in the study of these problems is the convergence of the solutions obtained with the pre-fractal layers when the geometry becomes fractal. The main difficulty is due to the jump of the geometric dimension which is two for all the pre-fractal approximations and between two and three for the limit layer.

# Potential theory for a class of diffusion equations: a Gaussian bounds approach

by **Ermanno Lanconelli** (Bologna)

## Abstract

Let  $L$  be a linear second order partial differential operator in  $R^N \times R$  with non-negative characteristic form. Assume  $L$  has a fundamental solution bounded from above and from below by Gaussian kernels modeled on subriemannian doubling distances in  $R^N$ . Under these assumptions we show that  $L$  endows  $R^N \times R$  with a structure of  $\beta$ -harmonic space. This allows to study boundary value problems for  $L$  with a Perron-Wiener-Brelot method, and to obtain a pointwise regularity at the boundary in terms of Wiener series modeled on the Gaussian kernels.

# “Approximate Approximations” on nonuniform grids

by Flavia Lanzara (Roma)

## Abstract

The method of approximate quasi-interpolation called “Approximate Approximations” was proposed by Vladimir Maz’ya in [1] and [2]. The quasi-interpolants are linear combinations of scaled translates of a sufficiently smooth and rapidly decaying basis function  $\eta$  and depend on two positive parameters, the meshsize  $h$  and the scale parameter  $D$ . If  $\mathcal{F}\eta - 1$  has a zero of order  $N$  at the origin ( $\mathcal{F}\eta$  denotes the Fourier transform of  $\eta$ ) then the quasi-interpolant on uniformly distributed nodes gives an approximation of order  $\mathcal{O}(h^N)$  up to a *saturation error* which can be made arbitrarily small if  $D$  is chosen large enough (see, e.g., [3]).

Here we present a generalization of the method of approximate quasi-interpolation to functions given on a set of nodes close to a uniform, not necessarily rectangular, grid  $\Lambda_h$  of size  $h$ . We construct a quasi-interpolant with centers at the grid points of  $\Lambda_h$  for which high order approximation of smooth functions up to some prescribed accuracy is possible.

These results were obtained together with V. Maz’ya (The Ohio State University, Columbus) and G. Schmidt (Weierstress Institute for Applied Analysis and Stochastics, Berlin).

## References

- [1] V. Maz’ya, A new approximation method and its applications to the calculation of volume potentials. Boundary point method, in: *3. DFG-Kolloquium des DFG-Forschungsschwerpunktes “Randelementmethoden”*, 1991.
- [2] V. Maz’ya, Approximate Approximations, in: J.R. Whiteman ed., *The Mathematics of Finite Elements and Applications. Highlights 1993*, (Wiley & Sons Chichester 1994).
- [3] V. Maz’ya and G. Schmidt, On approximate approximation using Gaussian kernels, *IMA J. of Numer. Anal.* **16** (1996) 13–29.

# Regularity results for the gradient of solutions of nonlinear elliptic equations with $L^{1,\lambda}$ data

by **G. Rita Cirmi** and **Salvatore Leonardi** (Catania)

## Abstract

We study the regularity of the entropy solution of the following Dirichlet problem

$$\begin{cases} -\operatorname{div}(A(x, Du)) = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \quad (1)$$

where  $\Omega$  is an open bounded subset of  $\mathbf{R}^n$  ( $n \geq 3$ ),  $u \mapsto A(x, Du)$  is a strongly monotone operator mapping  $W_0^{1,p}(\Omega)$  into its dual  $W^{-1,p'}$  ( $p \in [2, n[, \frac{1}{p} + \frac{1}{p'} = 1$ ) and  $f$  belongs to the Morrey space  $L^{1,\lambda}(\Omega)$ ,  $0 < \lambda \leq n - p$ .

We establish the following regularity properties:

(i) If  $\lambda \in ]0, n - p[$  then, for all  $H \subset\subset \Omega$ , we have

$$u \in \mathcal{M}^{p_\lambda, \lambda}(H), \quad Du \in \mathcal{M}^{\bar{p}_\lambda, \lambda}(H)$$

where  $p_\lambda = \frac{(p-1)(n-\lambda)}{n-\lambda-p}$  and  $\bar{p}_\lambda = \frac{(p-1)(n-\lambda)}{n-\lambda-1}$ ;

(ii) if  $\lambda = n - p$  then, for any cube  $Q \subset\subset \Omega$ , we have

$$u \in \operatorname{BMO}(Q), \quad Du \in \mathcal{M}^{\beta, \lambda}(Q), \quad \forall \beta < p.$$

here  $\mathcal{M}^{\beta, \lambda}$  denotes a suitable weak Morrey space.

Our results improve, at least locally, the regularity on  $u$  and  $Du$  obtained in previous papers without increasing the integrability on the datum  $f$ .

# Numerical Solution of Boundary Value Problems: BS methods and application to spacecraft rendezvous problems

by **Francesca Mazzia** (Bari)

## Abstract

Boundary Value Problems arise in many applicative fields. In particular we will analyze the equations that arise in modelling spacecraft rendezvous on elliptic orbits, considering both the noncooperative and the cooperative cases.

The numerical schemes used are the BS-methods, a new class of collocation methods based on B-spline basis, introduced and analyzed in [2], [3]. This class of methods has revealed a good performance when applied to BVPs and differs from the classical collocation methods in that it provides a much more smooth continuous extension. It is particularly efficient when used in combination with the hybrid mesh selection strategy implemented in the code TOM (<http://www.dm.uniba.it/mazzia/bvp/readme.html>, [4], [1]), based upon the conditioning of the associated continuous problem [1]. Numerical results will be presented showing the potentiality of the used numerical schemes.

## References

- [1] F. Mazzia, D. Trigiante, A Hybrid Mesh Selection Strategy Based on Conditioning for Boundary Value ODE Problems, *Numerical Algorithms*, 36 (2004), 169–187.
- [2] F. Mazzia, A. Sestini, D. Trigiante, *B-spline linear multistep methods and their continuous extensions*, (preprint).
- [3] F. Mazzia, A. Sestini, D. Trigiante, BS linear multistep methods on non-uniform meshes, *Applied Numerical Analysis and Computational Mathematics*, (in press).
- [4] F. Mazzia, I. Sgura, Numerical Approximation of Nonlinear BVP by means of BVMs, *Appl. Numer. Math.*, 42 (2002), 337–352.

# Spectra between Scylla and Charybdis

by **Umberto Mosco** (Worcester, MA – U.S.A.)

## Abstract

Certain highly non-Euclidean structures - like fractals - are usually constructed by variational asymptotic procedures of the kind familiar in homogenization theory. In this process, the geometry of the structure is wildly perturbed. However, as an elastic body, the structure itself is kept away from degenerating into a "stone" or a "powder": spectra survive perturbations between Scylla and Charibdis! In our talk we go back to some pioneering ideas of Professor Fichera about the perturbation of spectra and describe certain metric and analytical features of such non-Euclidean variational constructions.

**On the sets of boundedness of solutions  
for a class of degenerate nonlinear elliptic  
fourth-order equations with  $L^1$  data**

by **Francesco Nicolosi** (Catania)

**Abstract**

We consider a class of degenerate nonlinear elliptic fourth-order equations in divergence form. Coefficients of the equations satisfy a strengthened ellipticity condition involving two weighted functions.

In regard to the Dirichlet problem of equation of given class we established a result on existence of solutions bounded on some sets where the involved weight functions is regular-enough.

# Existence, Non-Existence and Regularity of Radial Ground States for $p$ -Laplacian Equations with Singular Weights

by **Patrizia Pucci** (Perugia)

## Abstract

By using the Mountain Pass Theorem and the constrained minimization method we prove existence of positive or compactly supported radial ground states for quasilinear singular elliptic equations with weights. The paper also includes the discussion of regularity and the validity of useful qualitative properties of the solutions. Finally, we establish a Pohozaev type identity from which we deduce some non-existence results.

# Regularity for a class of elliptic systems

by **Maria Alessandra Ragusa** (Catania)

## Abstract

I present some regularity results, obtained in cooperation with Prof. Atsushi Tachikawa, of solutions of elliptic systems having discontinuous coefficients. We extend some known results obtained by Campanato in the case of constant coefficients and some results obtained in the case of systems having continuous coefficients.

# Multi-variable Gould-Hopper and Laguerre polynomials

by **Paolo Emilio Ricci** (Roma)

## Abstract

The *monomiality principle* was introduced by G. Dattoli in order to derive properties of special polynomials starting from the corresponding ones of monomials.

For particular polynomials sets, corresponding to suitable generating functions, the relevant properties can be easily derived by using a simple and uniform method. The leading set in this field is given by the Hermite-Kampé de Fériet or Gould-Hopper polynomials, since many sets of multi-variable or multi-index polynomials have been constructed starting from this important polynomial family.

In this article we show a quite general technique for constructing the monomiality operators of many-variables polynomial sets, starting from the corresponding operators of a basic monomial (or quasi-monomial) set of functions. Assuming the basic set of monomials  $x^n$  or  $x^n/n!$ , the many-variables Hermite and Laguerre-type polynomials are derived and the relevant properties are easily obtained, showing connections with the Bell polynomials.

The Laguerre derivative  $D_L := Dx D$ , and its iterations, defined by

$$D_{nL} := Dx Dx \cdots Dx D$$

(containing  $(n + 1)$  ordinary derivatives), are used in order to construct families of higher order multi-variable Laguerre polynomials, corresponding to the multi-variable Gould-Hopper ones.

Finally, an application of the Laguerre derivative in the framework of population dynamics models is shown.

# Singular integrals and pseudo-differential operators

by **Luigi Rodino** (Torino)

## Abstract

We survey some results obtained in the past century about singular integral operators, along the lines leading to the modern theory of pseudo-differential operators. Namely, after recalling basic facts on one-dimensional Cauchy singular integrals, and related contributions of Fichera, we review the preliminary results of Tricomi concerning 2-dimensional singular integral operators. Moving then to the application of the Fourier transform, we pass to the works of Giraud, Miklin, Calderon, Zygmund. We then approach, through the contributions of Seeley and others, to the modern theory of the pseudo-differential operators, as proposed by Kohn, Nirenberg and Hormander (1965). Some recent contributions are then considered, with applications in different contexts: Harmonic Analysis, Partial Differential Equations, Quantum Physics, Signal Theory.

# A new boundary value problem

by **F.A. Costabile, A. Bruzio** and  
Anna Rosa Serpe (Cosenza)

## Abstract

We consider the boundary value problem:

$$\begin{cases} x^{(m)}(t) = f(t, \bar{x}(t)), & a \leq t \leq b, \quad m > 1 \\ x(a) = \beta_0, \quad x(b) = \beta_1 \\ \Delta x^{(k)} \equiv x^{(k)}(b) - x^{(k)}(a) = \alpha_k, & k = 1, \dots, m - 2 \end{cases}$$

where  $\bar{x}(t) = (x(t), x'(t), \dots, x^{(m-1)}(t))$ , and  $f$  is continuous at least in the interior of the domain of interest.

We prove the existence and uniqueness of the solution under certain conditions, and give an algorithm for its calculation; finally the case  $m = 4$  is examined in an engineering application (the beam equation).

## References

- [1] R.P. Agarwal and P.J.Y. Wong, Lidstone polynomials and boundary value problems, *Computers Math. Appl.*, **17** (1989), 1377–1421.
- [2] R.P. Agarwal and G. Akrivis, Boundary value problem occurring in plate deflection theory, *Computers Math. Appl.*, **8** (1982), 145–154.
- [3] F.A. Costabile, Expansions of real functions in Bernoulli polynomial and applications, *Conferences Seminars Mathematics University of Bari*, **273** (1999), 1–13.

# Discrete Mathematics and Numerical Methods

by **Donato Trigiante** (Firenze)

## Abstract

Discrete mathematics has been neglected for a long time. It has been put in the shade by the striking success of continuous mathematics in the last two centuries, mainly because of the success of continuous models in physics, but also, perhaps, because of the greater difficulty in dealing with. In this presentation, starting from some sentences of Fichera about discrete and continuous world, we shall present some considerations about discrete phenomena which arise when designing numerical methods for hamiltonian problems.

# Eigenvectors and fixed points of nonlinear operators

by **Giulio Trombetta** (Cosenza)

## Abstract

Let  $X$  be an infinite-dimensional Banach space, and let

$$B(X) = \{x \in X : \|x\| \leq 1\} \quad \text{and} \quad S(X) = \{x \in X : \|x\| = 1\}.$$

We recall that a *retraction*  $R : B(X) \rightarrow S(X)$  is a continuous mapping such that  $x = Rx$ , for all  $x \in S(X)$ . Let  $\psi$  be a measure of noncompactness on  $X$ . A continuous mapping  $A : \text{dom}(A) \subseteq X \rightarrow X$  is called  $\psi$ -Lipschitz with constant  $k$  (also  $k$ - $\psi$ -contractive) if  $\psi(AM) \leq k\psi(M)$  for all bounded subset  $M \subseteq \text{dom}(A)$ ;  $A$  is called  $\psi$ -condensing if  $\psi(AM) < \psi(M)$  for each bounded  $M \subseteq \text{dom}(A)$  which is not relatively compact.

We consider the following quantitative characteristic:

$$k_\psi = \inf\{k \geq 1 : \text{there is a } k\text{-}\psi\text{-contractive retraction } R : B(X) \rightarrow S(X)\}.$$

Let  $\Omega$  be a nonempty bounded open subset of  $X$ , and denote by  $\bar{\Omega}$  and  $\partial\Omega$  the closure and the boundary of  $\Omega$ , respectively. A well known theorem of D. J. Guo [2] states that if a completely continuous mapping  $A : \bar{\Omega} \rightarrow X$  satisfies: (i)  $\inf_{x \in \partial\Omega} \|Ax\| > 0$  and (ii)  $Ax \neq \lambda x$  for  $x \in \partial\Omega$  and  $0 < \lambda \leq 1$ , then the Leray-Schauder degree  $\text{deg}(I - A, \Omega, 0) = 0$ .

In this talk we present a generalization of Guo's Theorem to  $k$ - $\psi$ -contractions with  $k < 1$ , and  $\psi$ -condensing mappings, under a boundary condition which depends on the characteristic  $k_\psi$ , the condition being optimal when  $k_\psi = 1$ . We derive results on eigenvectors and fixed points of  $k$ - $\psi$ -contractions and  $\psi$ -condensing mappings.

## References

- [1] D. Caponetti, A. Trombetta and G. Trombetta, *An extension of Guo's theorem via  $k$ - $\psi$ -contractive retractions*, Nonlinear Anal. **64** (2006), 1897-1907.
- [2] D. J. Guo, *Eigenvalues and eigenvectors of nonlinear operators*, Chinese Ann. Math. **2** (1981), 65-80.

# Harnack inequality for harmonic functions relative to a nonlinear $p$ -homogeneous riemannian Dirichlet form

by Paola G. Vernole (Roma)

## Abstract

We consider an extension of the notion of strongly local (regular) Dirichlet forms to the nonlinear case, more precisely in the  $p$ -homogeneous case. We give our assumptions on the energy measure of the form, whose existence is assumed. Using the properties of the energy measure we are able to prove suitable "chain" and Leibniz rule. These properties are the starting point for an investigation of local regularity of the harmonics relative to the form and in particular for the proof (under suitable assumptions) of a Harnack type inequality for positive harmonic; as a consequence we obtain also the Holder continuity. From the point of view of partial differential equations, the theory includes the  $p$ -Laplacian and the subelliptic  $p$ -Laplacian. These results are the object of two papers in collaboration with Prof. Marco Biroli of "Politecnico di Milano".

# Fractal and Euclidean interaction in some transmission problems

by Maria Agostina Vivaldi (Roma)

## Abstract

In this talk some model examples of second order elliptic transmission problems with highly conductive layers will be described . Regularity and numerical results for solutions of transmission problems across fractal layers imbedded in Euclidean domains will be presented, in the aim of better understanding the analytical problems which arise when fractal and Euclidean structures mutually interact.

# Quantum Mechanics and Fourier Type Analysis

by **Shuji Watanabe** (Gunma University)

## Abstract

We deal with a kind of differential operator in a bounded or unbounded domain with singular variable coefficients. This kind of operator appears in many quantum-mechanical systems. On the basis of the Hankel transform we construct an integral transform associated with the operator. We define spaces of Sobolev type using the transform, and show an embedding theorem for each space. Our embedding theorem is a generalization of the Sobolev embedding theorem. We apply our results both to partial differential equations in bounded or unbounded domains with singular variable coefficients and to quantum mechanics.

# Boundary integral equations for almost incompressible elastic materials

by **George C. Hsiao** (Newark, DE – U.S.A.) and  
**Wolfgang L. Wendland** (Stuttgart, Germany)

## Abstract

Based on previous work by O. Steinbach, we reconsider the relation between the governing equations of linearized elasticity and the inelastic behavior under Bingham's law of the inelastic flow. Whereas for elasticity well-known boundary integral equations of the first and the second kind model the materials' behavior, the inelastic behavior is governed by the boundary integral equations for the Stokes problem. The latter turns out to be the degenerate formulation if the elastic behavior is formulated in terms of a perturbation of the inelastic behavior, i.e., if the Poisson ratio  $\nu$  tends to  $\frac{1}{2}$ . For the pure Dirichlet problem, this perturbation turns out to be singular since in general the degenerate problem has no solution except that the average normal displacement on the boundary vanishes. The pure traction problem, however, turns out to be a regular perturbation of the degenerate Stokes problem.

Stable variational formulations and corresponding boundary element Galerkin formulations are presented not only for the above mentioned two kinds of problems but also for mixed boundary value problems with partly given displacements and traction boundary conditions.

# Maximally singular functions in Besov spaces

by **Darko Zubrinic** (Zagreb, Croatia)

## Abstract

Assuming that  $0 < p < N$ ,  $p, q \in (1, \infty)$ , we construct a class of functions in the Besov space  $B_\alpha^{p,q}(R^N)$  such that the Hausdorff dimension of their singular set is equal to  $N - \alpha p$ . We show that these functions are maximally singular, that is, the Hausdorff dimension of singular set of any other Besov function in  $B_\alpha^{p,q}(R^N)$  is  $\leq N - \alpha p$ . Similar results are obtained for Lizorkin-Triebel spaces  $F_\alpha^{p,q}(R^N)$  and for the Hardy space  $H^1(R^N)$ . Some open problems will be listed related to the program of finding singular dimension of various function spaces, and of solutions of PDE-s. The above results will be published in Archiv der Mathematik, and they are a continuation of author's previous work on finding maximally singular functions in Bessel potential spaces, including Sobolev spaces.

## *List of Participants*

1. Prof. Dr. **Angelo M. ANILE** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: anile@dmi.unict.it
2. Prof. Dr. **Stanislav ANTONTSEV** – Departamento de Informatica-Matematica, Universidade da Beira Interior, Rua Marquês d’Avila e Bolama, 6201-001 Covilhã, (Portugal).  
e.mail: anton@ubi.pt
3. Dr. **Annamaria BARBAGALLO** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: barbagallo@dmi.unict.it
4. Prof. Dr. **Grigory I. BARENBLATT** – Department of Mathematics, University of California, Berkeley, CA 94720 (U.S.A.)  
e.mail: gibar@Math.Berkeley.EDU
5. Prof. Dr. **Lucilla BASSOTTI RIZZA** – Dipartimento di Matematica, Università degli Studi di Parma, Via Linati, 5, 43100 - Parma (Italia).
6. Prof. Dr. **Heinrich BEGEHR** – Freie University Berlin, I, Mathematisches Institut Arnimallee 3, D-14195 – Berlin.  
e.mail: bekehr@math.fu-berlin.de
7. Dr. **Irene BENEDETTI** – Dipartimento di Energetica “S.Stecco” Università di Firenze, Via S. Marta, 3, 50139 - Firenze (italia).  
e.mail: benedetti@math.unifi.it
8. Prof. Dr. Ing. **Peter BERGLEZ** – 5010 Institut für Analysis und Computational Number Theory (Math A), 8010 Graz, Steyrergasse 30/II, (Austria).  
e.mail: berglez@weyl.math.tu-graz.ac.at
9. Prof. Dr. **Marco BIROLI** – Dipartimento di Matematica del Politecnico di Milano, Piazza Leonardo da Vinci, 32, 20133 - Milano (Italia).  
e.mail: marco.biroli@polimi.it
10. Prof. Dr. **Salvatore BONAFEDE** – Dipartimento di Economia dei Sistemi Agro-Forestali, Università degli Studi di Palermo, Viale delle Scienze, 90128 - Palermo (Italia).  
e.mail: bonafede@dmi.unict.it

11. Prof. Dr. **Primo BRANDI** – Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, Via Vanvitelli, 1, 06123 - Perugia (Italia).  
e.mail: mateas@unipg.it
12. Prof. Dr. **Italo CAPUZZO DOLCETTA** – Dipartimento di Matematica - Istituto “G. Castelnuovo”, Università di Roma “La Sapienza”, P.le A. Moro, 2, 00185 - Roma (Italia).  
e.mail: capuzzo@mat.uniroma1.it
13. Prof. Dr. **Sandra CARILLO** – Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università degli Studi di Roma “La Sapienza”, Via A. Scarpa 14, 00161 - Roma, (Italia).  
e.mail: carillo@dmmm.uniroma1.it
14. Prof. Dr. **Caterina CASSISA** – Dipartimento di Matematica - Istituto “G. Castelnuovo”, Università di Roma “La Sapienza”, P.le A. Moro, 2, 00185 - Roma (Italia).  
e.mail: cassisa@mat.uniroma1.it
15. Prof. Dr. **Wenchang CHU** – Dipartimento di Matematica, Università degli Studi di Lecce, Via Provinciale Lecce-Arnesano, P.O. BOX 193, 73100 - Lecce (Italia).  
e.mail: chu.wenchang@unile.it
16. Prof. Dr. **Alberto CIALDEA** – Dipartimento di Matematica, Università degli Studi della Basilicata, Contrada Macchia Romana, 85100 - Potenza (Italia).  
e.mail: cialdea@unibas.it
17. Dr. **Paolo CIANCI** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: cianci@dmi.unict.it
18. Dr. **Rita CIRMI** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: cirmi@dmi.unict.it
19. Prof. Dr. **Maria Pia COLAUTTI** – Dipartimento di Scienze Matematiche, Università degli Studi di Trieste, Via Alfonso Valerio 12/1, 34100 - Trieste (Italia).
20. Dr. **Salvatore D’ASERO** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: dasero@dmi.unict.it
21. Prof. Dr. **Giuseppe DATTOLI** – Unità Tecnico Scientifica Tecnologie Fisiche Avanzate, ENEA – Centro Ricerche Frascati – C.P. 65, Via E. Fermi, 45, 00044 - Frascati (RM) (Italia).  
e.mail: dattoli@frascati.enea.it

22. Prof. Dr. **Rosalba DI VINCENZO** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: divincenzo@dmi.unict.it
23. Prof. Dr. **Mauro FABRIZIO** – Dipartimento di Matematica, Università degli Studi di Bologna, Piazza di Porta S. Donato, 5, 40126 - Bologna (Italia).  
e.mail: fabrizio@dm.unibo.it
24. Prof. Dr. **Antonio FASANO** – Dipartimento di Matematica “U. Dini”, Università degli Studi di Firenze, Viale Morgagni, 67/A, 50134 - Firenze (Italia).  
e.mail: fasano@math.unifi.it
25. Prof. Dr. **Bruno FIRMANI** – Dipartimento di Ingegneria Meccanica e Strutturale, Università degli Studi di Trento, Via Mesiano, 77, 38050 - Povo (TN) (Italia).  
e.mail: bruno.firmani@ing.unitn.it
26. Prof. Dr. **Bruna GERMANO** – Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università degli Studi di Roma “La Sapienza”, Via A. Scarpa 14, 00161 - Roma (Italia).  
e.mail: germano@dmmm.uniroma1.it
27. Prof. Dr. **Robert P. GILBERT** – University of Delaware, Department of Mathematical Sciences, Newark, Delaware 19716 (U.S.A.).  
e.mail: gilbert@math.udel.edu
28. Prof. Dr. **Sandro GRAFFI** – Dipartimento di Matematica, Università degli Studi di Bologna, Piazza di Porta S. Donato, 5, 40126 - Bologna (Italia).  
e.mail: graffi@dm.unibo.it
29. Prof. Dr. **George C. HSIAO** – University of Delaware, Department of Mathematical Sciences, Newark, Delaware 19716 (U.S.A.).  
e.mail: hsiao@math.udel.edu
30. Prof. Dr. **Milan KUCERA** – Mathematical Institute, Academy of Sciences, Zitna 25, Prague 1, (Czech Republic)  
e.mail: kucera@math.cas.cz
31. Prof. Dr. **Andrea LAFORGIA** – Dipartimento di Matematica, Università degli Studi Roma Tre, Largo S. Leonardo Murialdo, 1, 00146 - Roma (Italia).  
e.mail: laforgia@mat.uniroma3.it

32. Prof. Dr. **Maria Rosaria LANCIA** – Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università degli Studi di Roma “La Sapienza”, Via A. Scarpa 16, 00161 - Roma, (Italia).  
e.mail: [lancia@dmmm.uniroma1.it](mailto:lancia@dmmm.uniroma1.it)
33. Prof. Dr. **Ermanno LANCONELLI** – Dipartimento di Matematica, Università degli Studi di Bologna, Piazza di Porta S. Donato, 5, 40126 - Bologna (Italia).  
e.mail: [lanconel@dm.unibo.it](mailto:lanconel@dm.unibo.it)
34. Dr. **Flavia LANZARA** – Dipartimento di Matematica - Istituto “G. Castelnuovo”, Università di Roma “La Sapienza”, P.le A. Moro, 2, 00185 - Roma (Italia).  
e.mail: [lanzara@mat.uniroma1.it](mailto:lanzara@mat.uniroma1.it)
35. Prof. Dr. **Michel L. LAPIDUS** – University of California at Riverside, Department of Mathematics, Riverside, CA 92521 (U.S.A.).  
e.mail: [lapidus@ucr.edu](mailto:lapidus@ucr.edu)
36. Dr. **Salvatore LEONARDI** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: [leonardi@dmi.unict.it](mailto:leonardi@dmi.unict.it)
37. Prof. Dr. **Maria Laura LEUZZI** – Dipartimento di Matematica - Istituto “G. Castelnuovo”, Università di Roma “La Sapienza”, P.le A. Moro, 2, 00185 - Roma (Italia).
38. Prof. Dr. **Maria Renata MARTINELLI** – Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università degli Studi di Roma “La Sapienza”, Via A. Scarpa 14, 00161 - Roma (Italia).  
e.mail: [martinelli@dmmm.uniroma1.it](mailto:martinelli@dmmm.uniroma1.it)
39. Prof. Dr. **Michele MATZEU** – Dipartimento di Matematica, Università di Roma “Tor Vergata”, Via della Ricerca Scientifica, 00133 - Roma (Italia).  
e.mail: [matzeu@mat.uniroma2.it](mailto:matzeu@mat.uniroma2.it)
40. Prof. Dr. **Francesca MAZZIA** – Dipartimento di Matematica, Università degli Studi di Bari, Via Orabona, 4, 70125 - Bari (Italia).  
e.mail: [mazzia@dm.uniba.it](mailto:mazzia@dm.uniba.it)
41. Prof. Dr. **Umberto MOSCO** – Worcester Polytechnic Institute, Department of Mathematical Sciences, 100, Institute Road, Worcester, MA 01609-2280 (U.S.A.).  
e.mail: [mosco@WPI.EDU](mailto:mosco@WPI.EDU)

42. Prof. Dr. **M.K. Venkatesha MURTHY** – Dipartimento di Matematica “L. Tonelli”, Università degli Studi di Pisa, Via Filippo Buonarroti, 2, 56127 - Pisa (Italia).  
e.mail: murthy@dm.unipi.it
43. Prof. Dr. **Pierpaolo NATALINI** – Dipartimento di Matematica, Università degli Studi Roma Tre, Largo S. Leonardo Murialdo, 1, 00146 - Roma (Italia).  
e.mail: natalini@mat.uniroma3.it
44. Prof. Dr. **Francesco NICOLOSI** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: fnicolosi@dmi.unict.it
45. Prof. Dr. **Patrizia PUCCI** – Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, Via Vanvitelli, 1, 06123 - Perugia (Italia).  
e.mail: pucci@unipg.it
46. Dr. **Maria Alessandra RAGUSA** – Dipartimento di Matematica e Informatica, Università degli Studi di Catania, Viale A. Doria, 6, 95125 - Catania (Italia).  
e.mail: maragusa@dmi.unict.it
47. Prof. Dr. **Paolo Emilio RICCI** – Dipartimento di Matematica - Istituto “G. Castelnuovo”, Università di Roma “La Sapienza”, P.le A. Moro, 2, 00185 - Roma (Italia).  
e.mail: riccip@uniroma1.it
48. Prof. Dr. **Luigi RODINO** – Dipartimento di Matematica, Università degli Studi di Torino, Via Carlo Alberto, 10, 10123 - Torino (Italia).  
e.mail: luigi.rodino@unito.it
49. Prof. Dr. **Maria Cesarina SALVATORI** – Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, Via Vanvitelli, 1, 06123 - Perugia (Italia).  
e.mail: salva@dipmat.unipg.it
50. Prof. Dr. **Carlo SBORDONE** – Dipartimento di Matematica e Applicazioni, Via Cintia - Complesso Universitario di Monte S. Angelo, 80126 - Napoli (Italia).  
e.mail: sbordone@unina.it
51. Dr. **Anna Rosa SERPE** – Dipartimento di Matematica Università della Calabria - cubo 30B, Ponte Attrezzato P. Bucci, 87036 - Arcavacata di Rende (CS) (Italia).  
e.mail: annarosa.serpe@uncal.it

52. Prof. Dr. **Marinella TORTORICI** – Dipartimento di Matematica ed Applicazioni, Università degli Studi di Palermo, Via Archirafi, 34, 90123 Palermo (Italia).
53. Prof. Dr. **Donato TRIGIANTE** – Dipartimento di Energetica “S. Stecco”, Università degli Studi di Firenze, Viale S. Marta, 3, 50139 - Firenze (Italia).  
e.mail: [trigiant@unifi.it](mailto:trigiant@unifi.it)
54. Prof. Dr. **Giulio TROMBETTA** – Dipartimento di Matematica, Università degli Studi della Calabria, cubo 30B Ponte Attrezzato P. Bucci, 87036 - Arcavacata di Rende, Cosenza (Italia).  
e.mail: [trombetta@unical.it](mailto:trombetta@unical.it)
55. Prof. Dr. **Giorgio VERGARA CAFFARELLI** – Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università degli Studi di Roma “La Sapienza”, Via A. Scarpa 14, 00161 - Roma (Italia).  
e.mail: [vergara@dmmm.uniroma1.it](mailto:vergara@dmmm.uniroma1.it)
56. Prof. Dr. **Paola G. VERNOLE** – Dipartimento di Matematica - Istituto “G. Castelnuovo”, Università di Roma “La Sapienza”, P.le A. Moro, 2, 00185 - Roma (Italia).  
e.mail: [vernole@mat.uniroma1.it](mailto:vernole@mat.uniroma1.it)
57. Prof. Dr. **Maria Agostina VIVALDI** – Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università degli Studi di Roma “La Sapienza”, Via A. Scarpa 14, 00161 - Roma (Italia).  
e.mail: [vivaldi@dmmm.uniroma1.it](mailto:vivaldi@dmmm.uniroma1.it)
58. Prof. Dr. **Shuji WATANABE** – Department of Mathematics, Faculty of Engineering, Gunma University, 4-2 Aramaki-machi, Maebashi 371-8510, (Japan).  
e.mail: [watanabe@fs.aramaki.gunma-u.ac.jp](mailto:watanabe@fs.aramaki.gunma-u.ac.jp)
59. Prof. Dr. Ing. **Wolfgang L. WENDLAND** – Institut für Angewandte Analysis und numerische Simulation, Lehrstuhl für Angewandte Mathematik, Universität Stuttgart, Pfaffenwaldring, 57, D-70569 – Stuttgart, (Germany)  
e.mail: [Wolfgang.Wendland@mathematik.uni-stuttgart.de](mailto:Wolfgang.Wendland@mathematik.uni-stuttgart.de)
60. Prof. Dr. **Darko ZUBRINIC** – Faculty of Electrical Engineering and Computing, Department of Applied Mathematics, Unska 3, 10000 Zagreb (Croatia).  
e.mail: [darko.zubrinic@gmail.com](mailto:darko.zubrinic@gmail.com)