Workshop on Commutative Rings

Abstracts

June 4 - 10, 2006

Cortona, Italy

For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the conference.

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THE CLASS GROUP OF AN INTEGRAL DOMAIN

David F. Anderson

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Let R be an integral domain. The class group of R is the abelian group $Cl(R) = T(R)/Prin(R)$, where $T(R)$ is the group of t-invertible t-ideals of R under t-multiplication and $Prin(R)$ is its subgroup of principal ideals. If R is a Krull (resp., Prufer) domain, then $Cl(R)$ is just the usual divisor (resp., ideal) class group of R. We will discuss properties of the class group and compute the class group for several classes of integral domains. Special emphasis will be on how results for Krull domains extend (or fail to extend) to arbitrary integral domains.

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A GENERAL THEORY OF PRÜFER DOMAINS

Ayman Badawi

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In this talk, we study a class of domains with the property that $r\%$ of the finitely generated ideals are invertible, $0.5 < r < 100$. Under certain condition, we show that the theory of this class of domains resemble that of Prüfer domains.

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A SOLUTION TO THE BAER SPLITTING PROBLEM

SILVANA BAZZONI

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A module B over a commutative domain R is called a Baer module in case $\mathrm{Ext}^1_R(B, T) = 0$ for every torsion R-module T. This definition goes back to 1936 when R. Baer posed the question of characterizing the class of all abelian (torsion-free) groups G such that any extension of G with a torsion group splits.

In 1962 Kaplansky considered the case of modules over commutative domains. He raised the question whether Baer modules are projective.

The answer to the original problem raised by Baer was only given in 1969, when Griffith proved that the only Baer groups are the free groups.

In 1990 Eklof, Fuchs and Shelah used a version of Shelah's Singular Compactness Theorem to prove a crucial reduction theorem. Namely, they showed that Baer modules are projective provided that countably generated Baer modules are proective. Since then, the only substantial progress concerning the Baer problem was made by Griffith in 2003, who showed that Baer modules over local noetherian regular domains are free.

We show that all Baer modules are projective. This solves the general problem raised by Kaplansky.

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FACTORIALS, POLYNOMIALS, AND COMBINATORICS

Jean-Luc Chabert

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In 1946, P. Erdös [1] stated an inequality connected with $n!$: in the decomposition of $n!$ as a product of prime powers, if the exponent of the prime p is strictly greater than the exponent of the prime q, then the corresponding power of p is greater than the power of q (although p is necessarily less than q).

Using the notion of integer-valued polynomials, factorials defined as elements of the ring $\mathbf{Z}$ may be generalized to every integral domain D. So we may ask for an analogous inequality in this more general setting. We are particularly interested in the case when D is the ring of integers of a number field. For instance, we may prove that, when prime numbers are replaced by prime ideals and powers of primes are replaced by powers of norm of prime ideals, the same inequalities hold for every ring of integers of a cyclotomic field with the exception of $\mathbf{Z}[j]$.

There is also a generalization of factorials to subsets of any domain and we are here particularly interested in subsets of $\mathbf{Z}$. For instance, we prove that the same inequalities hold for the factorials with respect to the subset $\mathbf{P}$ formed by the prime numbers.

Another interesting thing is to find combinatorial interpretations of these generalized factorials (see M. Bhargava [2]). We will give some and ask for others.

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PRÜFER $*$-MULTIPLICATION DOMAINS AND KRONECKER FUNCTION RINGS

Gyu Whan Chang

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Let D be an integrally closed domain, $*$ a star-operation on D, X an indeterminate over D, and $N_* = \{f \in D[X] | (A_f)^* = D\}$. For an *e.a.b* star-operation $*_1$ on D, let $Kr(D, *_1)$ be the Kronecker function ring of D with respect to $*_1$. In this talk we define an *a.b.* star-operation $*_l$ on D. Then we prove that D is a Prüfer $*$-multiplication domain if and only if $D[X]_{N_*} = Kr(D, *_l)$, if and only if $Kr(D, *_l)$ is a quotient ring of $D[X]$, if and only if $Kr(D, *_l)$ is a flat $D[X]$-module, if and only if each $*$-linked overring of D is a Prüfer $*$-multiplication domain. This is a generalization of the following well-known fact that if D is a v-domain, then D is a Prüfer v-multiplication domain if and only if $Kr(D, v) = D[X]_{N_v}$, if and only if $Kr(D, v)$ is a quotient ring of $D[X]$, if and only if $Kr(D, v)$ is a flat $D[X]$-module.

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FACTORING INTEGER-VALUED POLYNOMIALS: RECENT RESULTS AND OPEN PROBLEMS

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Over the past 20 years, the study of properties relating to non-unique factorizations into irreducible elements in integral domains and monoids has become an area of independent research. Many of the results in this area center on the study of factorization properties of objects which are Noetherian (or more generally v-Noetherian). This talk focuses on the factorization properties of the ring

$$\mathrm{Int}(S, D) = \{f(X) \in K[X] \mid f(s) \in D \text{ for all } s \in D\}$$

where D is an integral domain with quotient field K and $S \subseteq D$. In most classical cases, this ring is not Noetherian. There is no known example of a ring D and infinite subset S such that $\mathrm{Int}(S, D)$ has finite elasticity.

The talk will focus on two recent papers ([1] and [2]) co-authored with Barbara McClain and William W. Smith. In [1] problems involving the elasticity of $\mathrm{Int}(S, D)$ when D is a UFD and $|S| = \infty$ are studied. Among other things, we show in the case where $D = \mathbb{Z}$ that 1) for every rational $q \neq 0$, there are infinite many irreducible integer-valued polynomial with leading coefficient q, 2) for every rational $q \geq 1$ there is an integer-valued polynomial with *elasticity* q. In the case where $|S| < \infty$, elementary arguments can be used to show that $\mathrm{Int}(S, D)$ is not *atomic*. We define in [2] the notion of *restricted elasticity*, which generalizes elasticity in a natural manner, and produce rings of integer valued polynomials with finite restricted elasticity.

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DOMAINS WITH PRESCRIBED FACTORIZATIONS

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Factorizations in integral domains and monoids have recently been the subject of much study. In one sense, monoid factorizations get to the true heart of the matter, since monoid factorizations only attempt to consider one operation (usually thought of as the analog of multiplication in an integral domain). It has been shown that there is more “freedom” available when one considers monoid factorizations. For example, although the monoid of nonzero, nonunits of any Noetherian valuation domain is (after reduction modulo units) isomorphic to the natural numbers, there is no integral domain having the monoid of nonzero, nonunits isomorphic to the monoid $\{0, 2, 3, 4, 5, \cdots\}$. In this talk, we will outline some recent work (joint with Brenda Mammenga) that shows that given any (reduced, cancellative) atomic monoid, M, we can find a (not necessarily atomic) domain, R, with the the property that submonoid of elements of R generated by irreducibles is isomorphic to M.

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ON THE AMALGAMATED DUPLICATION OF A CURVE SINGULARITY ALONG AN IDEAL

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This is a joint work in progress with R. Re.

Let R be a commutative ring and let I be a proper ideal of R. We compare the idealization $R \ltimes I$ (defined by Nagata as the R-module $R \oplus I$ endowed with the multiplication $(r,i)(s,j) = (rs, rj + si)$) and the amalgamated duplication of R by I:

$$R \bowtie I := \{(r, r+i) \mid r \in R,\ i \in I\} \subseteq R \times R.$$

We show that the idealization can be obtained as a deformation of the amalgamated duplication and we apply this result to algebroid curves; in particular, when I is a canonical ideal we construct a family of one-dimensional Gorenstein rings and we study when these rings are domains.

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A CLASSIFICATION OF THE MINIMAL RING EXTENSIONS OF CERTAIN COMMUTATIVE RINGS

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All rings considered are commutative with identity and all ring extensions are unital. Let R be a ring with total quotient ring T. The integral minimal ring extensions of R are catalogued via generator-and-relations. If T is von Neumann regular and no maximal ideal of R is a minimal prime ideal of R, the minimal ring extensions of R are classified, up to R-algebra isomorphism, as the minimal overrings (within T) of R and, for maximal ideals M of R, the idealizations $R(+)R/M$ and the direct products $R \times R/M$. If T is von Neumann regular, the minimal ring extensions of R in which R is integrally closed are characterized as certain overrings, up to R-algebra isomorphism, in terms of Kaplansky transforms and divided prime ideals, generalizing work of Ayache on integrally closed domains; no restriction on T is needed if R is quasilocal. One application generalizes a recently announced result of Picavet and Picavet-L'Hermitte on the minimal overrings of a local Noetherian ring. Examples are given to indicate sharpness of the results.

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FINITELY GENERATED-FRAGMENTED DOMAINS

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An integral domain D is called a fragmented domain (concept introduced by David Dobbs), if every proper principal ideal aD of D is contained in $\cap_{n\geq 1} b^n D$ for some proper principal ideal bD. In this talk we introduce a related concept. We say that an integral domain D is a finitely generated-fragmented domain (FGF domain), if every proper finitely generated ideal I of D is contained in $\cap_{n\geq 1} J^n$ for some proper finitely generated ideal J.

We prove the following results. A semi-quasi-local fragmented domain is an FGF domain. An FGF domain which is not a field is infinite-dimensional. A domain D is FGF iff it is locally FGF, provided for each maximal ideal M of D there exists a finitely generated ideal I such that M is the only maximal ideal containing I.

We construct a fragmented non-FGF domain D which has a finitely generated maximal ideal M such that D_M is a Noetherian domain. We show that not all FGF domains are fragmented by constructing an FGF domain satisfying the ascending chain condition for the principal ideals. (Joint work with Jim Coykendall.)

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UNIQUE REPRESENTATION DOMAINS

S. El Baghdadi (in collaboration with S. Gabelli and M. Zafrullah)

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Call an ideal I in an integral domain R a packet if I is a t-invertible t-ideal with a unique minimal prime ideal. A Unique Representation Domain (URD, for short) is a domain in which each t-invertible t-ideal is expressible uniquely as a t-product of finitely many mutually t-comaximal packets. In this talk, we give characterizations of URD's and, as an application, we consider the case of Prüfer domains and Prüfer v-multiplication domains.

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NORMAL BASES IN RINGS OF CONTINUOUS FUNCTIONS, BASED ON THE Q_N-DIGITS PRINCIPLE

Sabine Evrard

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Let V be a discrete valuation domain, with quotient field K, maximal ideal M and residue field k. Assume that V is complete. In [2], starting with a basis of the k-vector space $L(V,k)$, K. Conrad constructs normal basis in rings of continuous functions from V to K using q-digit expansions, when q denotes the cardinality of the field k supposed to be finite.

Here, we try to extend this constructions to some compact subsets S of V, even if k is infinite. First, we define the q_n-digit expansion, where q_n denotes the number of classes of S mod M^n. Then, we consider a particular class of subsets which allow us to answer our problem : the Legendre subsets, where q_r divides q_{r+1} for all r. These sets have been studied by Y. Amice in [1]. Finally, we characterize all the subsets S such that the q_n-digit principle works: The class of "very well balanced sets" which contains the class of Legendre subsets.

We end with some examples of such subsets and of normal bases obtained by q_n-digits expansion. Such a construction is particularly useful to obtain normal bases formed by polynomial functions.

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RATIONAL FUNCTIONS AND PRESERVATION OF FACTORIALS IN NUMBER FIELD

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Let K be a number field with ring of integers O_K and let E be an infinite subset of O_K. Bhargava associates to E a sequence of generalized factorials. We show that if a rational function $\varphi \in K(X)$ maps the subset E onto a subset $\varphi(E)$ with the same factorial sequence, then φ is an homographic function. For that, we prove first that every rational function $\varphi \in K(X)$ such that $\varphi(x) \in O_K$ for infinitely many $x \in O_K$ has at most one pole.

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POLYNOMIALS ACTING ON MATRICES

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If D is a domain with quotient field K and A a D-algebra (such as $A = M_n(D)$ the ring of $n \times n$ matrices over D), we investigate the ring $\mathrm{Int}(A)$ of polynomials with coefficients in K that map every element of A to an element of A, as well as, if A is commutative, the ring $\mathrm{Int}^n(A)$ of polynomials in $K[x_1, ..., x_n]$ that map every n-tuple of elements of A to an element of A.

For Noetherian D we give a criterion for $\mathrm{Int}(A)$ to be non-trivial (i.e., not equal to $D[x]$). For D a Dedekind ring we determine the spectrum of $\mathrm{Int}(A)$ and $\mathrm{Int}^n(A)$. Under what conditions rings of this kind are integrally closed is still a mystery: conjectures, helpful hints, etc. welcome.

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HOW MANY IDEALS ARE THERE IN A RING?

Ralf Fröberg

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This is joint work with Valentina Barucci, Roma.

If I is an ideal in a (commutative) ring R and $l_R(R/I) = h$, we say that I has colength h. We are interested in the class of rings in which there is a finite number of ideals of each colength, and how this number grows as a function of h. In Artinian rings there are no ideals of colength h, if $h >> 0$, and we show that, in a Noetherian ring of dimension at least two, the number of ideals of colength h grows at least exponentially with h. The ideals of colength 1 are the maximal ideals, thus our rings are semilocal, and each localization at a maximal ideal has the same property. We identify a large natural class of one-dimensional local Noetherian rings, where we describe the generating function of the number of ideals of colength h. In particular we show that the generating is a rational function.

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CATENARY AND TAME DEGREE FROM THE POINT OF VIEW OF A PRESENTATION OF A MONOID

Pedro A. García-Sánchez

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We explain how to compute the catenary and tame degree from a presentation of a finitely generated cancellative monoid. For the catenary degree it is enough to know certain minimal presentations of the monoid, whilst for the tame degree the set of factorizations of every irreducible in the congruence generated by the presentation of the monoid is required. We illustrate the method with several examples: some numerical semigroups and block monoids, half factorial submonoids of $\mathbf{N}^2$ generated by three elements with arbitrary catenary degree...

This is a joint work with S. T. Chapman, D. Llena, V. Ponomarenko and J. C. Rosales.

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GAUSSIAN PROPERTIES OF TOTAL RINGS OF QUOTIENTS

Sarah Glaz

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Let R be a commutative ring, and let f be a polynomial with coefficients in R. The *content of* f, $c(f)$, denotes the ideal of R generated by the coefficients of f. A polynomial f is called Gaussian if $c(fg) = c(f)c(g)$ for any polynomial g with coefficients in R. A ring R is called a *Gaussian ring* if every polynomial with coefficients in R is a Gaussian polynomial. Gaussian polynomials and rings have been a subject of investigation since their definition by Tsang in 1965. In 1967 Robert Gilmer had shown that an integral domain R is Prüfer if and only if it is Gaussian. Prompted by this result I considered several Gaussian properties for rings with zero divisors, and the extent to which these properties coincide if the ring is not a domain. These properties and their general relation to each other can be summarized as follows: semihereditary rings $\rightarrow$ rings of weak global dimension less or equal to one ($w.dim\ R \leq 1$) $\rightarrow$ Arithmetical rings $\rightarrow$ Gaussian Rings $\rightarrow$ Prüfer rings. None of the arrows are reversible in general.

This talk revolves around recent joint work with Silvana Bazzoni regarding the impact of the Gaussian and related properties on the total ring of quotients of a ring. In particular, we derived conditions on the total ring of quotients that allow for the reversing of the arrows in all the above implications.

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IDEAL SEMIGROUPS OF NOETHERIAN DOMAINS AND PONIZOVSKI DECOMPOSITIONS

Franz Halter-Koch

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Let R be a noetherian integral domain with quotient field K and $L \supset K$ a finite extension field. By an R-lattice in L we mean a finitely generated R-submodule of L containing a basis of L over K. The set $\mathcal{F}_L(R)$ of all R-lattices in L is a multiplicative semigroup whose Ponizovski decomposition reflects the ideal theory of R-orders in L. In particular, if R is one-dimensional, then $\mathcal{F}_L(R)$ is a complete semigroup. If R is a Dedekind domain and $L = K(\alpha)$ for some $\alpha \in L$, then $\mathcal{F}_L(R)$ is a Clifford semigroup if and only if $[L : K] \leq 2$.

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CONTINUOUS HOMOMORPHISMS AND INJECTIVE MODULES

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Let S be an R-algebra, a be an ideal of S and M be an R-module. With respect to the a-adic topology on S and the discrete topology on M, the S-module of continuous homomorphisms $Hom_R^a(S, M)$ has been used to construct injective hulls for concrete realizing Grothendieck duality in Noetherian case. We would like to generalize the construction for Noetherian rings to arbitrary rings. We find topological and algebraic conditions under which the functor $Hom_R^a(S, -)$ preserves injective modules and essential extensions, respectively. Existence of a trace pins down our construction to injective hulls of cyclic modules. Our construction of injective modules applies to certain rings obtained by a $D + M$ construction.

This talk is based on joint work with Shou-Te Chang, National Chung Cheng University.

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ON BOUVIER CONJECTURE

Salah-Eddine Kabbaj

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A finite-dimensional integral domain R is said to be Jaffard if $\dim(R[X_1, ..., X_n]) = n + \dim(R)$ for all $n \geq 1$; equivalently, if $\dim(R) = \dim_v(R)$, where $\dim(R)$ denotes the (Krull) dimension of R and $\dim_v(R)$ its valuative dimension. As this notion does not carry over to localizations, R is said to be locally Jaffard if R_p is Jaffard for each prime ideal p of R (equiv., $S^{-1}R$ is Jaffard for each multiplicative subset S of R). The class of locally Jaffard domains contains most well-known classes of rings involved in dimension theory such as Noetherian domains, Prüfer domains, universally catenarian domains, and stablly strong S-domains.

It is an open problem to compute the dimension of polynomial rings over Krull domains in general. In this vein, Bouvier conjectured that "*finite-dimensional Krull (or more particularly factorial) domains need not be Jaffard*" [1,2]. Obviously, this conjecture makes sense beyond the Noetherian context. As the notion of Krull domain is stable under formation of rings of fractions and adjunction of indeterminates, it merely claims "*the existence of a Krull domain R and a multiplicative subset S such that* $1 + \dim(S^{-1}R) \lneqq \dim(S^{-1}R[X])$."

However, finite-dimensional non-Noetherian Krull domains are scarce in the literature and one needs to test them and **their localizations** as well for the Jaffard property. We show that these families of examples satisfy the **locally Jaffard** property. This reflects the difficulty of proving or disproving this conjecture.

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SOME QUESTIONS ABOUT POWER SERIES RINGS

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We will present answers to a couple of questions about power series rings.

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INTERSECTIONS OF MONOIDS IN COMMUTATIVE RINGS

Ulrich Krause

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The talk investigates for an integral domain R with quotient field K the intersections of multiplicative monoids M with $R^* \subseteq M \subseteq K$. The following issues will be addressed:

- Irredundant and unique intersections.
- R as an intersection of uniquely determined monoids.
- For the above a prime role is played by prime cones, i.e., monoids M between R^* and K such that M equals the set of fractions $\frac{x}{y}$ with $x, y \in R^*$ and y a unit of M.
- Often (e.g., generalized Krull domains, Prüfer domains) prime cones are determined geometrically by the "facets" of the multiplicative cone R^*.
- From intersections of prime cones a divisor theory can be constructed.

In case R is not an integral domain but contains zero divisors it helps to consider prime cones with respect to addition instead of multiplication.

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THE SPACE OF VALUATION OVERRINGS OF AN INTEGRAL DOMAIN

K. Alan Loper

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When a collection of valuation domains with a common quotient field are intersected to obtain an integral domain it seems natural to separate into the cases where the result is a Prüfer domain and the cases where it is not a Prüfer domain. In this talk we discuss some alternative ways of distinguishing the results of intersections of valuation domains. In particular, we focus on the domains which have a unique Kronecker function ring as a generalization of Prüfer domains. We call such a ring a vacant ring. We look at properties of collections of valuation rings which intersect to produce vacant rings or not.

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CONSTRUCTING CHAINS OF PRIMES IN THE POWER SERIES RING OVER AN ALMOST DEDEKIND DOMAIN

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For an integral domain D of dimension n, the dimension of the polynomial ring $D[x]$ is known to be bounded by $n+1$ and $2n+1$. While $n+1$ is a lower bound for the dimension of the power series ring $D[[x]]$, it often happens that $D[[x]]$ has infinite chains of primes. For example, such chains exist if D is either an almost Dedekind domain that is not Dedekind or a rank one nondiscrete valuation domain. We will concentrate on a particular type of almost Dedekind domain. We say that D is a special almost Dedekind domain if it has only countably many maximal ideals, exactly one of these is not invertible, the rest are principal and there is an element ρ that generates each maximal ideal locally. We are interested in constructing chains of primes in $D[[x]]$. Using our construction scheme we will show there are chains of primes similar to the set of ω_1 transfinite sequences of 0's and 1's ordered lexicographically. All of the primes we construct contract to the noninvertible maximal ideal. The method can be extended to certain other integral domains, including all almost Dedekind domains that are not Dedekind.

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GAUSSIAN POLYNOMIALS AND CONTENT IDEAL IN TRIVIAL RING EXTENSIONS

Chahrazade Bakkari and Najib Mahdou*

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In this work, we are mainly concerned with Gaussian rings and rings (with zerodivisors) in which Gaussian polynomials have locally principal contents. Precisely, we study the possible transfer of these properties for various trivial ring extension contexts. Our results generate new families of examples of Gaussians rings and rings (with zerodivisors) whose Gaussian polynomials have locally principal contents.

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SEMISTAR OPERATIONS OF FINITE CHARACTER ON INTEGRAL DOMAINS

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Star operations are known as a very important tool to characterize some well-known classes of integral domains such us Prüfer domains, Mori domains, Krull domains, $PvMD$'s etc by studying some algebraic properties reflected in the sets of their fractional ideals. In 1994, A. Okabe and R. Matsuda introduced the notion of semistar-operations. This concept extends the classical concept of star-operations, as developed in Gilmer's book "Multiplicative Ideal Theory", and hence the related classical theory of ideal systems based on the work of W. Krull, E. Noether, H. Prüfer, and P. Lorenzen. Since then, many investigations of semistar-operations have been done. In this talk, we deal with a discussion about the cardinality of the set $SFc(R)$ of all semistar operations of finite character on integral domain R and its relationship with the Krull dimension.

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M-EQUIVALENCE OF INFINITE SETS UNDER POLYNOMIAL MAPPINGS

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Let R be an integral domain, E an infinite subset of R and f a non-constant uni-variate polynomial with coefficients in R. If the m-adic closures of E and $f(E)$ coincide for all maximal ideals of R, then E is said to be M-equivalent to $f(E)$. When R is "arithmetic" in nature the M-equivalence of E and $f(E)$ is possible only when f is of degree one. As a corollary, this yields an answer to a question posed by Gilmer and Smith regarding polynomial equivalence of E and $f(E)$.

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REPRESENTATIONS OF INTEGRALLY CLOSED DOMAINS AS INTERSECTIONS OF VALUATION RINGS

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Let H be an integral domain, and denote by F its quotient field. An *overring* of H is a ring R such that $H \subseteq R \subseteq F$. A classical theorem of Krull states that H is integrally closed if and only if H is the intersection of its valuation overrings. Because valuation rings can intersect in complicated ways, it is difficult in general to specify which collections of valuation overrings can be chosen to represent H in an intersection. For example, while the class of integrally closed overrings of a one-dimensional Noetherian domain is easily described, no comprehensive description of the integrally closed overrings of a two-dimensional Noetherian domain has yet been obtained, and it is this case that we focus on mostly. We describe in particular for an integrally closed overring R of the two-dimensional Noetherian domain D (e. g. R = quotient field of D), the rings H such that $D \subseteq H \subseteq R$ and $H = (\bigcap_{V \in \Sigma} V) \cap R$, where Σ is a Noetherian subspace of the Zariski-Riemann space of all valuation overrings of D. Even the special case in which Σ is finite character, or even finite, is quite subtle, but can occur naturally when considering certain subrings of Noetherian overrings of D. We give both intrinsic (in terms of H) and extrinsic (in terms of the valuation overrings of H) descriptions of the overrings of Noetherian domains of Krull dimension ≤ 2 that arise in this way from Noetherian spaces, and we explicitly describe how some strong versions of uniqueness hold for these representations.

Most of the results obtained rely in crucial ways on the fact that the base domain D is a Noetherian domain of Krull dimension 2. However, in treating even this case one encounters techniques that are more familiar in the study of non-Noetherian rather than Noetherian commutative rings. For example, some of the key tools involve Kronecker function rings, Prüfer domains, pullbacks, endomorphism rings of ideals, domains of finite real character, and an extension of the topology for the Zariski-Riemann space of valuation overrings to integrally closed overrings. We discuss in detail how these non-Noetherian concepts can be applied in this Noetherian context.

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A NOTE ON T-SFT-RINGS

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We define a nonzero ideal A of an integral domain R to be a t-SFT-ideal if there exist a finitely generated ideal $B \subseteq A$ and a positive integer k such that $a^k \in B_v$ for each $a \in A_t$, and a domain R to be a t-SFT-ring if each nonzero ideal of R is a t-SFT-ideal. In this talk we present a number of basic properties and stability results for t-SFT-rings. We show that an integral domain R is a Krull domain if and only if R is a completely integrally closed t-SFT-ring; for an integrally closed domain R, R is a t-SFT-ring if and only if $R[X]$ is a t-SFT-ring; if R is a t-SFT-domain, then $t\text{-}\dim R[\![X]\!] \geq t\text{-}\dim R$. We also give an example of a t-SFT Prüfer v-multiplication domain R such that $t\text{-}\dim R[\![X]\!] > t\text{-}\dim R$.

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SEMISTAR INVERTIBILITY AND RELATED PROPERTIES

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The notion of semistar invertibility, which generalizes the notion of invertibility of ideals and of invertibility with respect to the v and t-operation to the context of semistar operations, has been introduced in order to define the class of Prüfer semistar multiplication domains.

The observation that this notion does not allow a good generalization of the notion of Dedekind and Krull domain led us to introduce a different possible definition of invertibility with respect to a semistar operation, called "quasi-semistar-invertibility".

In this talk we compare "semistar invertibility" and "quasi-semistar-invertibility" and the classes of domains defined using these two notions.

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ON ELEMENTARY DIVISOR RINGS

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In this talk we will describe properties of elementary divisor rings; especially we will discuss the problem whether a Bezout domain of finite Krull dimension is necessarily an elementary divisor ring.

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PRÜFER DOMAINS AND DIVISIBLE MODULES

Luigi Salce

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[partly joint work with Peter Vámos]

It is well known that Dedekind domains are characterized by the property that divisible modules are injective. We show the well behavior of Prüfer domains with respect to certain classes of divisible modules. In particular, we prove that:

- the integral domains such that all the quotients of the injective modules are finitely injective are exactly the almost maximal Prüfer domains;
- the (Prüfer) domains all whose (FP-injective) divisible modules are finitely injective are exactly the Matlis almost maximal Prüfer domains;
- over Prüfer domains which are either almost maximal, or h-local Matlis, finitely injective torsion modules correspond, under the Matlis equivalence, to the complete torsionfree locally pure-injective modules;
- over Matlis valuation domains there exist finitely injective modules which are not direct sums of injectives.

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THE PROJECTIVE LINE OVER ORDERS

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In this talk we give some results toward a characterization of the underlying partially ordered set of the projective line over an order D in an algebraic number field, denoted by $Proj(D[h, k])$. We are continuing an investigation begun by Meral Arnavut, Aihua Li and Sylvia Wiegand. We show that $Proj(D[h, k])$ and $Spec(D[x])$ are not isomorphic as partially ordered sets. This is a joint work with Meral Arnavut.

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ON THE DISTRIBUTION OF PRIME IDEALS IN HALF-FACTORIAL KRULL MONOIDS WITH CLASS GROUP $C_{P^K}^R$

Wolfgang A. Schmid

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An atomic monoid H is called half-factorial if $a_1 \dots a_r = b_1 \dots b_s$ with irreducible elements $a_i, b_i \in H$ implies $r = s$. In case H is a Krull monoid (e.g., the multiplicative monoid of a Dedekind or a Krull domain), it is well-known that whether H is half-factorial or not, just depends on its class group G and the distribution of prime ideals over the classes. In other words, a Krull monoid H with class group G is half-factorial if and only if the subset $G_0 \subset G$ of classes containing prime ideals is a half-factorial set.

In this talk, we focus on half-factorial Krull monoids whose class group is the direct sum of r copies of a cyclic group of prime power order, i.e., it is isomorphic to $C_{p^k}^r$ for a prime p and integers k and r. In particular, for these monoids we obtain a (sharp) upper bound, in terms of p, k and r, on the number of classes containing prime ideals.

Moreover, we briefly discuss applications of our results to the investigation of the asymptotics of counting functions of algebraic integers with certain factorization-properties.

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ONE-DIMENSIONAL LOCAL RINGS AND THEIR CANONICAL IDEALS

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It is known that a local, one-dimensional, analytically irreducible ring R always has a canonical module (which is in fact a fractional ideal). Knowing the existence though does not tell us which fractional ideal is a canonical ideal. A characterization has been given for the case when R is residually rational (i.e. when R and its integral closure $\overline{R}$, have the same residue field).

The mentioned characterization relies essentially only on the semigroup of values of the ring R. However, for solving the problem in general (i.e. when R is not necessarily residually rational), the tool of the semigroup of values cannot suffice.

Further results has been achieved for the case when R is a generalized semigroup ring (notion introduced in [1]).

In the talk I shall present new results achieved in a collaboration with V. Barucci, aiming to cover an even more general case. The idea, coming from [2], is to consider the sequence of fractional ideals $F(i) = \{x \in F | v(x) \geq i\}$ $(i = 0, 1, 2, ..)$ associated to a fractional ideal F of R.

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FLAT EPIMORPHISMS AND A GENERALIZED KAPLANSKY IDEAL TRANSFORM

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We generalize the notion of the Kaplansky ideal transform $\Omega(I)$ to an ideal I in an arbitrary commutative ring R by defining $\Omega(I)$ to be the localization of R with respect to a certain filter of ideals. This agrees with the usual notion when R is a domain. We show that if the total ring of quotients of R is von Neumann regular, then $\Omega(I)$ is the ring of global sections over the open set $D(I)$. Moreover, for such rings any finitely generated (as an algebra) flat overring is the Kaplansky transform of a finitely generated ideal. In addition, generalizing a result of M. Fontana [Kaplansky ideal transform: A Survey, Lecture notes in Pure and Applied Mathematics, **205**, Marcel Dekker, New York, 1999 271–263], for such rings we characterize when the open set $D(I)$ is an affine scheme in terms of the flatness of $\Omega(I)$.

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ON THE HILBERT FUNCTION OF SOME 1-DIMENSIONAL LOCAL RING

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Let (R,m) be a 1-dimensional Cohen-Macaulay local ring, G be the associated graded ring with respect to m. The Hilbert function $H_R(n)$, defined as the Hilbert function of G, gives good informations on the singularity of Spec(R) at the closed point m. It is known G, in general, doesn't inherit Cohen-Macaulayness: depth (G) can be less than depth(R) and even depth(G) can be zero when R is Gorenstein. When G is not Cohen-Macaulay, one has: length$(m^{i*}/m^i) \geq 1$ for some $i \geq 2$, where $m^{i*} = \bigcup_k (m^{i+k} : m^k)$ is an ideal defined by Ratliff and Rush in 1978. Moreover, if G is not Cohen-Macaulay, the Hilbert function may sometimes be decreasing. Several authors proved $H_R(n)$ is not decreasing for some classes of rings: in particular, for certain semigroup rings, and when the multiplicity $e = e(R)$ and the embedding codimension h of R are such that $h \leq 2$, or $h+1 \leq e \leq h+3$. On the other hand, one can find examples with $e \geq h+4, H_R(i+1) < H_R(i)$ where length$(m^{i*}/m^i) \geq 4$ for some i. For analytically irreducible rings, we show one can "control" the Hilbert function through the cardinality of suitable subsets of the value semigroup $v(R)$, and in particular for semigroup rings we prove: if length$(m^{i*}/m^i) \leq 3$ for each i, then $H_R(n)$ is not decreasing. As a consequence, we can also show other classes of rings whose Hilbert function is not decreasing.

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DOMAINS IN WHICH THE OPERATION W IS THE IDENTITY AND SOME RELATIONS WITH PRÜFER DOMAINS.

Francesca Tartarone

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This is a joint paper with Giampaolo Picozza. Let D be a domain with quotient field K. We recall that the w-operation of a fractional ideal I of D is defined as follows:

$$I^w = \{x \in K \mid xJ \subseteq I \text{ for some } J \quad \text{Glaz-Vasconcelos ideal}\}.$$

It is well-known that much work has been done in order to characterize domains in which the divisorial closure is the identity (look, for instance, at H. Bass 1962, W. Heinzer 1968, S. Bazzoni & L. Salce 1996, S. Bazzoni 2000). In this direction, in a quite recent literature, we can find interesting papers investigating domains in which $v = t$ (TV-domains by E.G. Houston & M. Zafrullah 1988), or $v = w$ (VW-domains, El Baghdadi & S. Gabelli 2005) or $w = d$ (DW-domains, A Mimouni 2005).

In our paper we further study DW-domains in the more general context of semistar operations. In this talk I will focus my attention on the transfer of the DW-property between a domain D and its integral closure D'. In particular I will give some results about the DW-property and the Prüferianity of D', also examining a question raised by W. Vasconcelos some years ago. Finally I will give a characterization of DW-domains that are Mori or Noetherian.

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STRUCTURE THEOREMS FOR SPECIAL GORENSTEIN ARTINIAN LOCAL RINGS.

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I will discuss a structure theorem for Artinian Gorenstein local rings with the property that the square of the maximal ideal can be generated by two elements. Applications will be given to the study of the Hilbert scheme parametrizing zero-dimensional arithmetically Gorenstein subschemes of degree d in the projective space of dimension $d-2$. The results which we discuss are part of a joint paper with J. Elias.

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CONSTRUCTIONS OF INDECOMPOSABLE MODULES — OLD AND NEW

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Let R be a one-dimensional Noetherian local ring with maximal ideal $\mathbf{m}$. Modules are always assumed to be finitely generated. If R is not a discrete valuation ring, a construction going back to Kronecker and Weierstrass shows that there are indecomposable R-modules requiring arbitrarily many generators. On the other hand, if R has finite representation type, then of course there are bounds on the ranks and on the number of generators of the indecomposable *torsion-free* modules. (In fact, there is a *universal* bound, conjecturally equal to 3, on the rank of any indecomposable torsion-free module over any one-dimensional ring of finite representation type.) It is reasonable to ask, for such rings, whether there exist indecomposable finitely generated *mixed* modules of arbitrarily large rank. We show that this is indeed the case, unless R happens to be a *Dedekind-like* ring (essentially, an (A_1)-singularity, e.g., $\mathbb{C}[[x,y]]/(xy)$ or $\mathbb{R}[[x,y]]/(x^2+y^2)$). (The universal bound on the ranks of the indecomposables is, conjecturally, equal to 2 for Dedekind-like rings.)

If some power of $\mathbf{m}$ needs three or more generators, a direct construction, based on the classical one mentioned above, suffices to build modules of large rank. In other cases, where each power of $\mathbf{m}$ is two-generated (for example, the (A_n)-singularities $\mathbb{C}[[x,y]]/(y^2-x^{n+1}), n \geq 2$, and certain non-Cohen-Macaulay rings), a more subtle approach, using homological methods, seems to be required. I will sketch both approaches in this talk. This is joint work with Wolfgang Hassler, Lee Klingler and Ryan Karr.

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EXTENSIONS OF DOMAINS WITH TRIVIAL GENERIC FIBER

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We consider injective local maps from a local domain R to a local domain S such that the generic fiber of the inclusion map $R \hookrightarrow S$ is trivial, that is $P \cap R \neq (0)$ for every nonzero prime ideal P of S. We present examples of injective local maps involving power series that have this property. We also give some results on the dimension of the larger domain S in case R is a power series ring over a field.

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NOETHERIANNESS IN RINGS OF INTEGER-VALUED POLYNOMIALS

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The ring $\mathrm{Int}(\mathbb{Z})$ of integer-valued polynomials on $\mathbb{Z}$ is a classical example of non-Noetherian domain.

More generally, for D an integrally closed Noetherian domain and E a subset of D, the ring $\mathrm{Int}(E, D)$ of integer-valued polynomials on E is well known to be non-Noetherian (unless it is trivial, i.e. $\mathrm{Int}(E, D) = D[X]$). This property is a consequence of the fact that the ring $\mathrm{Int}(E, D)$ "separates the points" modulo the height 1 primes p : for any $a, b \in E$, $a \neq b$ there exists a polynomial $f \in \mathrm{Int}(E, D)$ such that

$$f(a) \equiv 0 \mod p \quad \text{and} \quad f(b) \equiv 1 \mod p$$

We will study two classes of subrings of $\mathrm{Int}(E, D)$ which don't separate the points, and show that the subrings of one are Noetherian, whereas on the contrary the subrings of the other are not.

This talk is based on a joint work with Manjul Bhargava and Paul-Jean Cahen.

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FINITELY GENERATED MODULES OVER LOCAL ONE-DIMENSIONAL DOMAINS.

Paolo Zanardo

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Let R be a local one-dimensional integral domain, with maximal ideal $\mathfrak{M}$. We assume that R is not a valuation domain. We are interested in a class of finitely presented mixed R-modules. We call them R-modules of Warfield type, since the idea for constructing them goes back to R. B. Warfield. We remark that a slight difference in the construction makes our modules mixed and finitely presented, while Warfield's original ones were torsion and not necessarily finitely presented. Our R-modules of Warfield type have local endomorphism rings, and hence they are indecomposable. We examine the torsion part $t(M)$ of a Warfield type module M, investigating the natural property $t(M) \subset \mathfrak{M}M$. This property is related to b/a being integral over R, where a and b are elements of R which define M. We also investigate $M/t(M)$ and determine its minimum number of generators.

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