Prüfer-like Properties in Ring Constructions

A Series of Lectures by Sarah Glaz

Let R be a commutative ring, and let $f \in R[x]$ be a polynomial over R. Denote by $c(f)$– *the content of f*– the ideal of R generated by the coefficients of f. A ring is called a *Gaussian ring* if $c(fg) = c(f)c(g)$ for all polynomials $f, g \in R[x]$. Gaussian rings were defined by Tsang in 1965, and became an active topic of investigation due to their connection to Kaplansky's conjecture, which was solved between 1997 and 2005. The focus of these investigations lied in the comparison between the Gaussian property and several related ring theoretic and homological properties. Specifically the properties under consideration are:

1. R is a semihereditary ring.
2. w.dim $R \leq 1$.
3. R is an arithmetical ring.
4. R is a Gaussian ring.
5. R is a locally Prüfer ring.
6. R is a Prüfer ring.

In 1967 Gilmer showed that an integral domain R is a Prüfer domain iff it is a Gaussian domain. In fact, if R is an integral domain all six properties coincide. This is no longer true if the ring R contains zero-divisors. The various relationships between these Prüfer-like properties in rings with zero-divisors were investigated in the last five years by a number of authors, among others, Bakkari, Bazzoni, Boynton, Glaz, Kabbaj, Lucas, Mahdou, and Mouanis.

This series of lectures will discuss the six Prüfer-like properties mentioned above in rings with zero-divisors. The lectures will start with a brief outline of the history of the subject, and the results and examples which highlight the relationships between the properties. It will then proceed to focus on recent completed work or work in progress, by the speaker and others, involving ascent and descent of these properties in various ring constructions, such as pullback rings, group rings, and a number of special localizations of the polynomial ring in one variable over R. The talks will include some outlines of proofs, many examples, and a number of open questions. My workshop talk involving these Prüfer-like properties in group rings constitutes a continuation of this lecture series.

This series of lectures will be accessible to graduate students specializing in Commutative Algebra.