Program of the proposed course

HOMOLOGICAL ALGEBRA

Spring 2010, 24 hours

Irena Swanson

The main subject of the course is Homological Algebra, with applications to commutative algebra. I will start with projective and injective resolutions, Ext and Tor. I will be emphasizing the functorial properties, specifically with regards to the Hom and tensor functors, and I will also be emphasizing the bearings these constructions have on the ring and modules. In particular, I will present the information that the resolutions and Ext and Tor carry about dimension, depth, and singularities of a ring. Later in the course I will devote some time to the actual symbolic computation of these constructions: I will discuss the theoretical background of Gröbner bases that allows these computations, and I will be presenting actual computations by one of the symbolic computer algebra programs CoCoA, Macaulay 2, or Singular.

Some more specific topics will include: the Koszul and Taylor complexes and the Buchsbaum-Eisenbud criterion of exactness when discussing projective resolutions; and regular and Cohen–Macaulay and Gorenstein rings when discussing applications of homological algebra to ring theory. Depending on the interest of the audience, I could expand any of the topics above, or I could also add the implications of homological algebra on radicals and primary decompositions (work of Eisenbud, Huneke, and Vasconcelos), or on integral closure (work of Burch and of Corso, Huneke, Katz, and Vasconcelos).

There are several books that cover parts of the proposed material: *An Introduction to Homological Algebra* by C. A. Weibel, *An Introduction to Homological Algebra* by J. J. Rotman, *Commutative Algebra with a View toward Algebraic Geometry* by D. Eisenbud, and various books on Gröbner bases. Some papers (as listed above) may also be necessary. I plan to have the framework of the lectures typed up by the beginning of the course, and make them available to the audience via the web. It will depend on the audience what proofs and what background have to be explained in greater detail, and I plan to keep updating the web notes and finish them by the end of the course.