

Counting Star Operations on an Integral Domain  
A Series of Lectures by Evan Houston

An integral domain is said to be *divisorial* if each of its nonzero ideals is divisorial. Heinzer characterized integrally closed divisorial domains as  $h$ -local Prüfer domains each of whose maximal ideals is invertible, and Bass and (independently) Matlis showed that a local Noetherian domain  $(R, M)$  is divisorial if and only if  $R$  has Krull dimension 1 and is such that  $(R : M)$  is a 2-generated  $R$ -module.

It is easy to see that a domain is divisorial if and only if it admits only one star operation (namely the trivial star operation). In a series of lectures I will discuss domains which admit only finitely many star operations. In particular, I will extend both the Heinzer and Bass/Matlis characterizations to domains which admit at most two star operations and discuss some of the roadblocks to handling the general finite case. I will sketch proofs of the results, leaving many of the details as exercises.