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FACTORIALS IN SEVERAL VARIABLES

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There is no need to introduce the factorial sequence $\{n!\}_{n\geq 0}$, and its arithmetic properties are well known

- For each $k, l \in \mathbb{N}$, $k!l!$ divides $(k+l)!$.
- For any sequence $x_0, x_1, \ldots, x_n$ of integers, the product $\prod_{0\leq i<j\leq n}(x_j - x_i)$ is divisible by $1!\cdots n!$.

We can also give some properties connecting polynomials and factorials:

- For every polynomial $f \in \mathbb{Z}[X]$, with unitary content and of degree n, $d(f) = gcd\{f(k) \mid k \in \mathbb{Z}\}$ divides $n!$.
- Every integer-valued polynomial f (that is, $f(\mathbb{Z}) \subset \mathbb{Z}$), of degree n,

$$n!f(X) \in \mathbb{Z}[X].$$

Bhargava introduced the notion of factorial sequence of a subset S of a Dedekind domain D, which generalizes the usual notion of $n!$, since it has arithmetical properties similar to the classical factorials. He introduced the factorial sequence of a subset S, in a local way, thanks to the notion of v-orderings of S. On the other hand, such a sequence may be defined in a global way, thanks to the notion of integer-valued polynomial on S. In this talk, we define factorials in several variables using both, integer-valued polynomials with d indeterminates and v-orderings of subsets of D^d. We will see that these factorial sequences still generalize some arithmetical properties of the factorial sequence $n!$.

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