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NUMERICAL CHARACTERIZATIONS OF SOME INTEGRAL DOMAINS

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Let R be an integral domain and let K be its quotient field. An intermediate ring in the ring extension $R \subseteq K$ is called an overring of R . Several classes of integral domains are defined by properties satisfied by their overrings. Recently domains with only finitely many overrings have been investigated by several authors. Such integral domains have been named FO domains by Gilmer. The author gave an algorithm for computing the number of overrings of FO integrally closed domain. The main purpose of this talk is to investigate whether the number of overrings affects the properties of R . Precisely we have the following problems:

Problem 1

Given an FO integral domain R , is it possible to provide an exact list of all overrings of R ?

Problem 2

Given an FO integral domain R . Does the number of overrings characterize the properties enjoyed by R ?

We answer Problem 1 by providing an algorithm that produces all different overrings that are intersections of localizations of an integral domain with a tree as spectrum. The same algorithm gives a complete list of all overrings of an integrally closed domain. We then address Problem 2 by establishing a characterization of integrally closed domains in terms of the number of their overrings. We also give similar characterizations of valuation domains, Dedekind domains, integral domains with spectrums free of Y -subgraphs, and integral domains of Krull dimension 1.

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