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PARAMETRIZATION OF INTEGRAL VALUES OF POLYNOMIALS

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A polynomial $f \in \mathbb{Q}[X]$ is called integer-valued if $f(\mathbb{Z}) \subset \mathbb{Z}$. We give a complete classification of those integer-valued polynomials $f(X)$ whose image over the integers can be parametrized by a multivariate polynomial with integer coefficients, that is, the existence of $g \in \mathbb{Z}[X_1, \ldots, X_m]$ such that $f(\mathbb{Z}) = g(\mathbb{Z}^m)$. The necessary and sufficient condition for $f(X)$ is that $f \in \mathbb{Z}[B(X)]$ for some polynomial $B(X) = sX(sX - r)/2$, where s and r are coprime odd integers and s is a prime power or it is equal to 1. In particular we obtain that 2 is the only prime factor dividing the common denominator D of the coefficients of $f(X)$ and there exists a rational $\beta = r/s$ such that $f(X) = f(\beta - X)$. Moreover if $f(\mathbb{Z})$ is likewise parametrizable, then this can be done by a polynomial in one or two variables.

We will give also some ideas towards a generalization of this classification to number fields K, showing that the prime factors dividing D are related to the order of the roots of unity contained in K.