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FINITELY VALUATIVE DOMAINS

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For a pair of rings $S \subseteq T$, an element $t \in T \backslash S$ is said to be within n steps (resp., within finitely many) steps of S if there is a saturated chain of rings of length m, $S = S_0 \subset S_1 \subset \cdots \subset S_m = S[t]$, for some integer $m \leq n$ (resp., for some integer m). An integral domain R is said to be n-valuative (resp., finitely valuative) if for each nonzero element u in its quotient field, at least one of u and u^{-1} is within n steps (resp., within finitely many steps) of R. Valuative domains, presented by Tom Lucas, are thus the 1-valuative domains, and this talk presents a generalization. The integral closure of a finitely valuative domain is a Prüfer domain. An integrally closed n-valuative domain is a Bézout domain with at most $2n + 1$ maximal ideals, the prime ideals contained in more than n prime ideals are totally ordered and at most n maximal ideals fail to contain every such prime.

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