Commutative Ring Theory Days 2010

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ABOUT SUBSETS OF Z WITH SIMULTANEOUS ORDERINGS

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The natural sequence $\{n\}_{n\geq 0}$ has the following nice property: whatever the integer $m \in \mathbf{Z}$ the product $\prod_{k=0}^{n-1}(m-k)$ is divisible by the product $\prod_{k=0}^{n-1}(n-k)$, that is, $n!$ divides $m!/(m-n)!$.

Analogously, consider a subset S of $\mathbf{Z}$. A sequence $\{a_n\}_{n\geq 0}$ of elements of S is said to be a *simultaneous ordering* of S when:

$$\forall x \in S \quad \prod_{k=0}^{n-1}(x-a_k) \quad \text{is divisible by} \quad \prod_{k=0}^{n-1}(a_n-a_k).$$

There are two well known examples of subsets which admit simultaneous orderings: $\{n^2 \mid n \geq 0\}$ and $\{q^n \mid n \geq 0\}$ where q is any integer ≥ 2. We are interested in finding other natural examples.

We characterize here the integer-valued polynomials f of degree 2 such that either $\{f(n) \mid n \geq 0\}$ or $\{f(n) \mid n \in \mathbf{Z}\}$ admits a simultaneous ordering.

We prove also that, for any $f \in \mathbf{Z}[X]$ and any integer $x \in \mathbf{Z}$, the orbit of x under the action of the iterates of f, that is, $\{f^n(x) \mid n \geq 0\}$ always admits a simultaneous ordering.

This is a joint work with David ADAM (Université Française du Pacifique) and Youssef FARES (LAMFA, Université de Picardie).

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