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VALUATIVE DOMAINS

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An integral domain R with quotient field K is said to be valuative if for each nonzero element $u \in K$, at least one of the ring extensions $R \subseteq R[u]$ and $R \subseteq R[u^{-1}]$ has no proper intermediate rings. This generalizes the notion of a valuation domain. In fact, an integrally closed local domain is valuative if and only if it is a valuation domain. There do exist (both local and nonlocal) valuative domains that are not integrally closed. For example, if $F \subsetneq F[t]$ is a field extension of prime degree, then $R = F + XF[t][[X]]$ is a local valuative domain that is not integrally closed. A complete characterization of valuative domains will be given. One necessary condition for a domain to be valuative is that it have at most three maximal ideals, another is that the set of nonmaximal primes must be linearly ordered. Also, at most one maximal ideal (of a valuative domain) does not contain each nonmaximal prime. In the integrally closed case, R is valuative if and only if it is a Bézout domain with at most three maximal ideals such that at most one maximal ideal does not contain each nonmaximal prime ideal. Except for a special type of "bad" valuative domain, every overring of a "good" valuative domain is valuative. Moreover, the overrings of a good valuative domain can be described in a way that is similar to how one may describe the overrings of a semilocal Bézout domain. [Joint work with Paul-Jean Cahen and David Dobbs.]

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