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ON INTERSECTIONS AND COMPOSITES OF MINIMAL RING EXTENSIONS

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A ring extension $A \subset B$ is **minimal** if A and B are the only A-subalgebras of B. In this case, there exists a maximal ideal M of A, called the **crucial** ideal of the extension, such that $A_P = B_P$ for each prime ideal P of A different from M. Let $A \subset B$ and $A \subset C$ be two distinct minimal (ring) extensions such that the composite $D = BC$ exists. In an earlier work [1], the two last authors examined the extensions $B \subset D$ and $C \subset D$, and, in particular, under which conditions they are minimal. In this talk, we give some complements to this study and consider the dual situation : let $B \subset D$ and $C \subset D$ be distinct minimal extensions and set $A = B \cap C$. We obtain conditions in order that $A \subset B$ and/or $A \subset C$ are minimal extensions, with a special attention to the case of integral extensions. Moreover, combining properties of intersections and composites of minimal extensions, the following result is gotten: Given two distinct rings B and C such that the composite $D = BC$ exists, and setting $A = B \cap C$, the following conditions are equivalent :

(1) $A \subset B$ and $A \subset C$ are minimal extensions with distinct crucial ideals.

(2) $B \subset D$ and $C \subset D$ are minimal extensions with crucial ideals lying over distinct maximal ideals of A.

REFERENCES

[1] D.E. Dobbs, J. Shapiro, 241–275, Composites of minimal ring extensions, 2007, 9, JP J. Algebra.

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