

## Commutative Ring Theory Days 2010

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### EXTENDING LENGTH FUNCTIONS TO POLYNOMIAL RINGS VIA ALGEBRAIC ENTROPY

LUIGI SALCE

[Joint work with Simone Virili]. Length functions  $L$  for categories  $\text{Mod}(R)$  of modules over arbitrary rings  $R$ , taking values in the non-negative reals plus infinity, have been introduced by Northcott and Reufel in 1965. They found all length functions over valuation domains, and Vámos found in 1968 all length functions  $L$  over commutative Noetherian rings.

I will present a recent result stating that, if  $L$  is a discrete length function on  $\text{Mod}(R)$ , where  $R$  is an arbitrary ring, there is a unique discrete length function  $h_L$  on the subcategory of  $\text{Mod}(R[X])$  consisting of the locally  $L$ -finite modules, such that, for every  $R$ -module  $M$  of finite length:

- (i)  $h_L(M_f) = 0$  for every endomorphism  $f$  of  $M$  ( $M_f$  is the  $R[X]$ -module  $M$  with  $X$  acting on it via  $f$ ).
- (ii)  $h_L(M^{(\mathbb{N})}_{\beta_M}) = L(M)$  where  $\beta_M$  is the right Bernoulli shift on  $M^{(\mathbb{N})}$ .

The length function  $h_L$  coincides with the algebraic  $L$ -entropy  $\text{ent}_L$ , introduced by L. S. and P. Zanardo in 2009; that is, for every endomorphism  $g$  of an arbitrary  $R$ -module  $N$ ,  $h_L(N_g) = \text{ent}_L(g)$ . The crucial point in obtaining this result relies on the proof of the Addition Theorem for the  $L$ -entropy  $\text{ent}_L$ .

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