

BIRS Workshop 11w5086 - Advancing numerical methods for viscosity solutions and applications

Abstracts of talks

February 11, 2011

- **M. Akian**, "Max-Plus algebra in the numerical solution of Hamilton-Jacobi and Isaacs Equations"

We shall present several examples where max-plus algebra can help to solve Hamilton-Jacobi and Isaacs equations. First, the max-plus linearity of first order Hamilton-Jacobi equations allows one to construct approximations of their solutions by max-plus linear combinations, then use max-plus tools for their study. Second, monotone schemes discretizations of stationary Isaacs equations yield to fixed point or spectral equations of dynamic programming operators of zero-sum two player stochastic games. But the set of subsolutions of such equations is a max-plus convex set. Also, using Perron-Frobenius properties inspired by the max-plus linear special case, a policy iteration algorithm was introduced for degenerate (multi-chain) equations. We shall show how policy algorithms behave combined with multigrid methods, and how this combination asks max-plus questions.

- **J.-D. Benamou**, F. Collino, S. Marmorat, "Local High Frequency wave content Analysis"

Given multi frequency domain wave data, the proposed new algorithm gives a pointwise estimate of the the number of rays, their slowness vectors and corresponding wavefront curvature. With time domain wave data and assuming the source wavelet is given, the method also estimates the traveltime. We present numerical results on synthetic data that demonstrate both the effectiveness and the robustness of the new method. Comparisons with more classical Algorithms tends to show the superiority of the new method.

- Y. Cheng, **O. Bokanowski**, C.-W. Shu, "Discontinuous Galerkin Scheme for Front Propagation with Obstacle"

We consider front propagation problems in the presence of obstacles, modeled by $\min(u_t + \mathcal{H}(x, \nabla u), u - g(x)) = 0$, where u is a level set function and $g(x)$ is an obstacle function (Bokanowski, Forcadel

and Zidani, SICON 2010). We propose a direct discontinuous galerkin method for this Hamilton-Jacobi equation, following the lines of Cheng and Shu (JCP 2007). We give several numerical examples of front propagation to illustrate the efficiency of the method, and propose also a narrow band approach. In some special cases we are furthermore able to prove stability estimates for standard fully explicit RK DG schemes.

- **F. Camilli**, D.Schieborn, "Shortest paths and Hamilton-Jacobi equations on a network"

In this talk we present an extension of the theory of viscosity solutions to topological networks. We discuss uniqueness, existence and approximation results for Hamilton-Jacobi equations of eikonal type. A prominent question in graph theory is how to efficiently detect shortest paths connecting a given vertex with prescribed source vertices in a weighted graph. We study a similar problem when the running cost varies in a continuous way along the edges.

- **E. Carlini**, "A Generalized Fast Marching Method on Unstructured Grid"

Recently, a new Method called Generalized Fast Marching Method has been developed. It generalizes the Fast Marching Method, in the sense that it applies to evolutive eikonal equations with speed that can change sign in time. In this talk, I present an extension of the GFM Method to unstructured grids. The motivation for this work comes from several applications, where methods to track interfaces have to be coupled with other solvers, which are based on nite elements or nite volumes built on unstructured grids. We prove a general convergence result that is based on some properties for the local solvers so that convergence is guaranteed provided the local solver satisfies some sufficient conditions. Some numerical tests will be presented.

- S. Cacace, **E. Cristiani**, M. Falcone, "Two New Ordered Upwind Methods for Hamilton–Jacobi Equations"

In this talk we present two generalizations of the Fast Marching method introduced by J.A. Sethian in 1996 to solve numerically the eikonal equation. The new methods are based on a semi-Lagrangian discretization and are suitable for Hamilton–Jacobi equations modeling monotonically advancing fronts, including anisotropic front propagation problems, Hamilton-Jacobi-Bellman and Hamilton–Jacobi–Isaacs equations. The algorithms are compared with Fast Marching and Fast Sweeping methods.

- **N. Forcadel**, "Generalized Fast Marching Method and applications"

The Fast Marching Method have been proposed by Sethian in 1996 to solve very efficiently front propagation problem when the front evolves in its normal direction with a positive speed depending only on space. The goal of this presentation is to give a generalization of this algorithm when the normal velocity depends also on time and can change sign. We will prove that the proposed algorithm is convergent and that the complexity is essentially the same as in the classical case. Finally, we will present some applications in dislocations dynamics and image segmentation.

- F. Cagnetti, **D. Gomes**, H. Tran, "djoint methods for obstacle problems and weakly coupled systems of PDE"

In this talk I will present some new results for obstacle problems and weakly coupled systems of PDE. The adjoint method, recently introduced by L. C. Evans, is used to study obstacle problems, weakly coupled systems, cell problems for weakly coupled systems of Hamilton–Jacobi equations, and weakly coupled systems of obstacle type. In particular, new results about the speed of convergence of common approximation procedures are derived.

- T. Hunt, **A.J. Krener**, "New Patchy Solutions of Hamilton Jacobi Bellman Equations"

We present a new patchy method for solving HJB equations of nonlinear optimal control problems whose linear quadratic part is a stabilizable and detectable linear quadratic regulator. The new method begins as did the old by computing the Al’brecht solution in a patch around the origin. It differs in the way the rest of the patches are computed. On a standard example the new method is 5 times more accurate than the old, 25 times more accurate than Al’brecht and 50 times more accurate than LQR.

- **F. Li**, "Exactly divergence-free central DG methods for ideal MHD equations"

Ideal MHD system consists of a set of nonlinear hyperbolic conservation laws, with a divergence-free constraint on the magnetic field. Though such constraint holds for the exact solution as long as it does initially, neglecting this condition numerically may lead to numerical instability. In this talk, high order central DG methods will be presented for ideal MHD equations, with the numerical magnetic field being exactly divergence-free. In principle, the methods could be of any order of accuracy.

- **J.-C. Nave**, "On some high-order, optimally local schemes for interface problems"

I will present two schemes, one for the advection equation and the other for Poisson's equation with interface discontinuities. The particularity of these schemes is that they are local, but manage to achieve 4th order convergence (in the L^∞ norm). The basic idea hinges on the locality of Hermite basis, and that of the ghost fluid method. I will discuss some applications and current thoughts on various extensions.

- **A. Oberman**, "Numerical Methods for Geometric Elliptic Partial Differential Equations"

Geometric Partial Differential Equations (PDEs) are at the forefront of current research in mathematics, as evidenced by Perelman's use of these equations in his proof of the Poincaré Conjecture and Cedric Villanis Fields Medal in 2010 for his work on Optimal Transportation. They can be used to describe, manipulate and construct shapes based on intrinsic geometric properties such as curvatures, volumes, and geodesic lengths.

These equations are useful in modern applications (Image Registration, Computer Animation) which require geometric manipulation surfaces and volumes. Convergent numerical schemes are important in these applications in order to capture geometric features such as folds and corners, and avoid artificial singularities which arise from bad representations of the operators.

In general these equations are considered too difficult to solve, which is why linearized models or other approximations are commonly used. Progress has recently been made in building solvers for a class of Geometric PDEs. I'll discuss a few important geometric PDEs which can be solved using a numerical method called Wide Stencil finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, Mean Curvature, and others.

Focusing in on the Monge-Ampere equation, which is the seminal geometric PDE, I'll show how naive schemes can work well for smooth solutions, but break down in the singular case. Several groups of researchers have proposed numerical schemes which fail to converge, or converge only in the case of smooth solutions. I'll present a convergent solver which is fast: comparable to solving the Laplace equation a few times.

- **S. Serna**, "Hamilton-Jacobi equations with shocks arising from general Fokker-Planck equations: analysis and numerical approximation"

We consider a class of Hamilton-Jacobi equations whose solutions admit shocks. This class of equations arise as the convective part of a general Fokker-Planck equation ruled by a non-negative diffusion function that depends on the unknown and on the gradient of the unknown. We explore the main features of the solution of the Hamilton-Jacobi

equations and propose a suitable numerical scheme that approximates the solution in a consistent way with respect to the solution of the associated Fokker–Planck equation. We present a set of numerical results performed under different piecewise constant initial data with compact support for specific equations including the one a two dimensional relativistic Hamilton–Jacobi equation and the porous media Hamilton–Jacobi equation.

- **J. Tasso**, TBA
- W. Chen, Z. Clawson, and S. Kirov, R. Takei, **A. Vladimirovsky**, "Optimal Control with Budget Constraints and Resets"

Many realistic control problems involve multiple criteria for optimality and/or integral constraints on allowable controls.

This can be conveniently modeled by introducing a budget for each secondary criterion/constraint. An augmented HJB equation is then solved on an expanded state space, and its discontinuous viscosity solution yields the value function for the primary criterion/cost. This formulation was previously used by Kumar & Vladimirovsky to build a fast (non-iterative) method for problems in which the resources/budgets are monotone decreasing.

We currently address a more challenging case, where the resources can be instantaneously renewed (& budgets can be "reset") upon entering a pre-specified subset of the state space. This leads to a hybrid control problem with more subtle causal properties of the value function. We illustrate by finding (time-or-energy) optimal trajectories for a robot in a room with obstacles, constrained by the maximum contiguous time of visibility to a stationary enemy observer.

- S. Luo, **Y. Yu**, H. Zhao, "A new approximation for effective Hamiltonians for homogenization of a class of Hamilton-Jacobi equations"

I will talk about a new formulation to compute effective Hamiltonians for homogenization of a class of Hamilton-Jacobi equations. Our formulation utilizes an observation made by Barron and Jensen about viscosity supersolutions of Hamilton-Jacobi equations. The key idea is how to link the effective Hamiltonian to a suitable effective equation. The main advantage of our formulation is that only one auxiliary equation needs to be solved in order to compute the effective Hamiltonian $\bar{H}(p)$ for all p . Error estimates and numerical examples will be presented.

- **H. Zidani**, "Convergence result of a non-monotone scheme for HJB equations"

We study the convergence of a non-monotonous scheme for one-dimensional first order Hamilton-Jacobi-Bellman equation of the form

$$\begin{cases} v_t + \max_{\alpha} (f(x, \alpha)v_x) = 0 \\ v(0, x) = v_0(x). \end{cases}$$

The scheme is based on the anti-diffusive method UltraBee. We show for general discontinuous initial data a first-order convergence of the scheme, in L^1 -norm, towards the viscosity solution. The non-diffusive behavior of the scheme and its relevance will be illustrated on several numerical examples.