# TULSF VII

## Algebraic Geometry in
## Trieste - Udine - Ljubljana - SISSA - Ferrara

## Trieste, Friday February 17 2012

# Programme

10:00 - 10:45 Pietro De Poi (Udine)
*On the degree of irrationality*

Coffee

11.05 - 11:50 Valentina Beorchia (Trieste)
*The slope conjecture for fibrations in four-gonal curves*

12:00 - 12:40 Fabio Tanturri (SISSA)
*Pfaffian representations of cubic surfaces*

Lunch

14:30 - 15:15 Klemen Šivic (Ljubljana)
*Varieties of triples of commuting matrices*

15.25 - 16:10 Alexander Tikhomirov (Yaroslav)
*Moduli space of mathematical instanton vector bundles on projective space*

16:20 - 17:00 Alex Massarenti (SISSA)
*The automorphisms group of* $\overline{M}_{g,n}$

**All lectures will be held in Aula C, Building C, San Giovanni Campus, Via Weiss 1**

Co-sponsored by PRIN project "Geometria delle varietà algebriche e dei loro spazi di moduli" (cofin 2008).

**Abstracts**

**Valentina Beorchia: The slope conjecture for fibrations in four-gonal curves**
Let $f : S \to B$ be a relatively minimal fibred surface over a smooth curve $B$ of genus $b$, with smooth general fiber $C_b$ of genus $g \geq 2$. We denote by $K_f = K_S - f^*K_B$ the relative canonical divisor and by $\chi_f$ the relative Euler characteristic. If $f$ is not isotrivial, then $\chi_f > 0$. We investigate how the properties of the general fiber affect the slope $s(f) = K_f^2/\chi_f$. Harris and Stankova-Frenkel conjectured a bound on the slope of fibrations with general fiber a general $n$-gonal curve. The Conjecture is known to be true if $n \in \left\{ 2, 3, \left\lfloor \frac{g}{2} + 1 \right\rfloor \right\}$.

We give a complete description of fibrations in semistable curves, with general fiber a four-gonal curve with simply ramified gonal cover, and we prove the conjecture for $n = 4$ in this case.

**Pietro De Poi: On the degree of irrationality**
It is well known since M. Noether that the gonality of a smooth complex plane curve of degree $d > 3$ is $d-1$, and all the maps of minimal degree are obtained as a projection from a point of the curve. Perhaps the most natural generalisation of the notion of gonality for varieties of arbitrary dimension $k$ is the so called degree of irrationality, i.e. the minimal degree for which there exists a dominant rational map to the $k$-dimensional projective space. In this seminar, we will report a joint work with F. Bastianelli and R. Cortini, and we will show how to extend - in term of the degree of irrationality - Noether's result about plane curves, to smooth hypersurfaces of arbitrary dimension. We will show that the degree of irrationality of surfaces and threefolds of sufficiently high degree is always $d - 1$ but some cases - in which is $d - 2$ - that will describe. In particular, for generic surfaces and threefolds, all the minimal degree maps are only the projections from a point.

**Alex Massarenti: The Automorphisms group of $\overline{M}_{g,n}$**
The moduli stack $\overline{\mathcal{M}_{g,n}}$ parametrizing Deligne-Mumford stable $n$-pointed genus $g$ curves and its coarse moduli space $\overline{M}_{g,n}$: the Deligne-Mumford compactification of the moduli space of $n$-pointed genus $g$ smooth curves from several decades are among the most studied objects in algebraic geometry, despite this many natural questions about their biregular and birational geometry remain unanswered. In particular we are interested in their automorphisms groups. The symmetric group on $n$ elements $S_n$ acts on $\overline{\mathcal{M}}_{g,n}$ and $\overline{M}_{g,n}$ by permuting the marked points. We will prove that the automorphisms groups of $\overline{\mathcal{M}}_{g,n}$ and $\overline{M}_{g,n}$ are isomorphic to the symmetric group $S_n$ for any $g, n$ such that $2g - 2 + n \geq 3$, and compute the remaining cases. In doing this we will give an explicit description of $\overline{M}_{1,2}$ as a weighted blow-up of the weighted projective plane $\mathbb{P}(1, 2, 3)$.

**Klemen Šivic: Varieties of triples of commuting matrices**
The set $C(d, n)$ of all $d$-tuples of commuting $n \times n$ matrices over an algebraically closed field $F$ of characteristic zero can be viewed as an affine variety in $F^{dn^2}$. It is well-known that the variety $C(2, n)$ is irreducible for each $n$ and that for $d \geq 4$ the variety $C(d, n)$ is irreducible if and only if $n \leq 3$. However, the question of irreducibility of varieties of commuting triples is still an open problem. It has been shown that $C(3, n)$ is irreducible for $n \leq 7$ and reducible for $n \geq 30$. Using simultaneous commutative approximation of triples of matrices

by triples of generic matrices (i.e. matrices having $n$ distinct eigenvalues) we prove that $C(3,n)$ is irreducible for $n \leq 10$. We study also a related problem of irreducibility of varieties of pairs of commuting matrices in the centralizers of given matrices. We show that this variety is irreducible if the given matrix is 3-regular (i.e. each its eigenspace is at most 3-dimensional), but on the other hand, there exists 5-regular matrix with reducible variety of commuting pairs in its centralizer.

**Fabio Tanturri: Pfaffian representations of cubic surfaces**
Given an integer $k$ and a homogeneous polynomial $F \in \mathbb{K}[x_0, x_1, \ldots, x_n]$, one may ask whether $F^k$ is the determinant of a matrix with linear forms as entries. This problem is related to the existence of arithmetically Cohen-Macaulay sheaves of rank $k$ on the hypersurface $\mathrm{V}(F) \subset \mathbb{P}^n$. A relevant class of matrices with determinant $F^2$ are pfaffian representations, that is, skew-symmetric matrices of linear forms whose pfaffian is $cF$, for some $c \in \mathbb{K}$. We are interested in the case of a cubic surface $\mathbb{S}$ in $\mathbb{P}^3$. The existence of a pfaffian representation in the smooth case is equivalent to the existence of five points on $\mathbb{S}$ satisfying some properties. We describe an algorithm which requires a point on $\mathbb{S}$ with one additional property and ensures a pfaffian representation of $\mathbb{S}$. This enables us to state some results about the existence of pfaffian representations whose entries are linear forms in $\mathbb{K}'[x_0, x_1, x_2, x_3]$, $\mathbb{K}'$ being an algebraic extension of small degree of $\mathbb{K}$.

**Alexander Tikhomirov: Moduli space of mathematical instanton vector bundles on projective space**
We discuss the problem of irreducibility of the moduli space $I_n$ of rank - 2 mathematical instanton vector bundles with Chern classes $c_1 = 0$, $c_2 = n$ on the projective space $\mathbb{P}^3$. We give a sketch of the proof of the irreducibility of $I_n$ for an arbitrary positive integer $n$.