# HODGE THEORY

SAN SERVOLO - VENICE - ITALY JUNE 19 - 24, 2006

Sponsors:

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- $\bullet$  G.N.S.A.G.A.
- Japan Society for Promotion of Science (JSPS) Core-to-Core Program "New developments of Arithmetic Geometry, Motive, Galois Theory and Their Practical Applications"

Scientific/Organizing Committee:

- L. Barbieri-Viale (Padova)
- B. Chiarellotto (Padova)
- H. Esnault (Essen)
- B. van Geemen (Milano)

Co-Organizers:

- David A. Ellwood (Research Director, Clay Mathematics Institute)
- Francesca P. Nisii (Institutional Affairs, Venice International University)

Schedule. The first talk will be at 9:30 a.m. so that if you are \*not\* lodging in the Island of San Servolo you can take the boat N. 20 leaving at 9:15 a.m. from San Zaccaria and reach the conference site in less than 10 minutes. The schedule is organized as follows: a coffee break between 11:45 and 12:15 and lunch at the mensa of the Campus between 13:15 and 14:00. Instructions for dinner are below.

Morning Lectures	9:30 - 10:30	10:45 - 11:45	12:15 - 13:15
Monday	Fontaine	Breuil	Rapoport
Tuesday	*Griffiths	*Saito	Viehweg
Wednesday	Kato	Illusie	*Kedlaya
Thursday	*Nori	Bondarko	Ayoub
Friday	Faltings	Jannsen	*Olsson
Saturday	Raskind	Tsuji	

The \* stands for Clay Math Lectures.

Afternoon Lectures	15:00 - 16:00	16:15 - 17:15
Monday	Otwinowska	Lewis
Tuesday	Weber	
Wednesday	(free)	
Thursday	Klingler	
Friday	Vologodsky	

Speakers, their affiliations & titles of their talks.

- J. Ayoub (CNRS, Paris, France): "Motives of rigid varieties. Applications to motivic nearby cycles and to the cancellation theorem for DM (curve)"
- M. V. Bondarko (St. Petersburg State University, Russia): "Differential graded realizations of motives; weight filtrations"
  - C. Breuil (CNRS & IHES, Paris, France): "*p*-adic Hodge theory and the *p*-adic Langlands program"
  - G. Faltings (MPI, Bonn, Germany): "Higgs-bundles and Galois-representations"
  - J.-M. Fontaine (Université de Paris-Sud, Orsay, France): "The special fiber of a crystalline representation"
    - \*P. Griffiths (IAS, Princeton, USA): "Extended normal functions and their singularities"
      - L. Illusie (Université de Paris-Sud, Orsay, France): "Logarithmic Hodge degeneration"
    - U. Jannsen (University of Regensburg, Germany): "Finiteness results for motivic cohomology"
      - K. Kato (Kyoto University, Japan): "SL(2)-orbit theorem for degeneration of mixed Hodge structure"
- \*K. S. Kedlaya (MIT, Boston, USA): "Convergence of solutions of *p*-adic differential modules, and higher-dimensional ramification theory"
  - B. Klingler (University of Chicago, USA): "On the André-Oort conjecture"
  - J. D. Lewis (University of Alberta, Edmonton, Canada): "The Abel-Jacobi map for higher Chow groups, II"
    - \*M. Nori (University of Chicago, USA): "Mixed Tate motives and hyperplane arrangements"
  - \*M. Olsson (University of Texas, Austin, USA): "Grothendieck's six operations for stacks"
- A. Otwinowska (Université de Paris-Sud, Orsay, France): "Motivic sheaves on curves"

M. Rapoport (Universität Bonn, Germany): "On non-archimedean period domains"

- W.Raskind (University of Southern California, Los Angeles, USA): "Totally degenerate reduction and the conjectures of Hodge and Tate"
  - \*M. Saito (RIMS, Kyoto, Japan): "Generalization of Griffiths' theorem on rational integrals and related topics"
  - T. Tsuji (Tokyo University, Japan): "On semi-stable smooth *p*-adic sheaves"
- E. Viehweg (Universität Duisburg-Essen, Germany): "Stability of Hodge bundles and the uniformization of certain Shimura varieties"
- V. Vologodsky (University of Chicago, USA): "Integrality of instanton numbers"
  - A. Weber (Warsaw University, Warszawa, Poland): "Topological properties of the weight filtration"

## Abstracts.

Joseph Ayoub "Motives of rigid varieties. Applications to motivic nearby cycles and to the cancellation theorem for DM (curve)"

We construct a category Rig DM(k) out of the category of smooth rigid varieties over k following Voevodsky's construction of DM(k). We give a structure theorem for Rig DM(k) and use it to produce a new definition of limiting motives and to extend Voevodsky's cancellation theorem to the case where the base is a curve.

Mikhail V. Bondarko "Differential graded realizations of motives; weight filtrations"

We describe the Voevodsky category DM of motives in terms of Suslin complexes of smooth projective varieties. We give a description of any triangulated subcategory of DM. We describe 'truncation' functors  $t_N$ for N > 0.  $t_0$  generalizes the weight complex of Soule and Gillet; its target is the homotopy category of Chow motives; it calculates  $K_0(DM)$ .  $t_N$  give a weight filtration and a 'motivic descent spectral sequence' for 'standard' (and more general) realizations; these include motivic cohomology (!).

Christophe Breuil "p-adic Hodge theory and the p-adic Langlands program"

I will give a survey of some questions and results in the p-adic Langlands program that are related to p-adic Hodge theory.

Gerd Faltings "Higgs-bundles and Galois-representations"

I review the relation between Higgs-bundles and étale sheaves provided by the theory of almost étale extensions, and try to discuss generalisations (replacing Hodge-Tate by crystalline theory).

Jean-Marc Fontaine "The special fiber of a crystalline representation"

(Part of a joint work in progress with Uwe Jannsen) Using suitable generalizations of finite and flat commutative group schemes and of Barsotti-Tate groups, we try to give a meaning to the title.

Phillip Griffiths "Extended normal functions and their singularities"

There is a well-known reciprocal relationship between normal functions associated to Lefshetz pencils and Hodge classes. Due to the failure in higher codimension of Jacobi inversion this relationship has been of limited use in constructing algebraic cycles. Consideration of all high degree hypersurface sections leads to the concept of an extended normal function (ENF) associated to a Hodge class and of its singular set. The singular set is (i) non-empty iff the Hodge conjecture is true, and (ii) is a component of the pullback of a boundary component of partially compactified classifying space for polarized Hodge structures. In this talk we will discuss the dimension of the singular set and the relation between its fundamental class and the pullback of the fundamental class of the boundary component.

Luc Illusie "Logarithmic Hodge degeneration"

A classical theorem, due to Deligne (1969), asserts that if Y is a scheme of characteristic zero and  $f: X \to Y$  is a proper and smooth morphism, then the relative Hodge to de Rham spectral sequence of f degenerates at  $E_1$  and its initial term is locally free of finite type. I will discuss logarithmic generalizations of this result, using Kummer étale sites and Kato-Nakayama spaces. I will sketch possible further developments. This is joint work with K. Kato and C. Nakayama.

Uwe Jannsen "Finiteness results for motivic cohomology"

Joint work with Shuji Saito.

Kazuya Kato "SL(2)-orbit theorem for degeneration of mixed Hodge structure"

This is a joint work with C. Nakayama and S. Usui. Cattani, Kaplan, and Schmid established the SL(2)-orbit theorem in several variables on degeneration of polarized Hodge structure (Annals of Math., 1986). We generalize it to degeneration of mixed Hodge structure whose graded quotients for the weight filtration are polarized. A work in this direction was done also by Pearlstein.

Kiran Kedlaya "Convergence of solutions of p-adic differential modules, and higherdimensional ramification theory"

> It is well-known that there is a strong analogy between irregularity of holomorphic connections on a curve, and wild ramification of a lisse l-adic sheaf. It is perhaps less well-known that in the world of p-adic cohomology, this analogy becomes a theorem equating irregularity of a certain connection with ramification (arising from an overconvergent F-isocrystal on a curve in characteristic p) of a certain Galois representation of an equal characteristic p local field. I'll start by recalling how this works. I'll then turn to the question of how one might extend this to overconvergent F-isocrystals on higher dimensional varieties; one can use differential data to define a ramification filtration on the Galois side, which obeys some nice convexity properties as one varies the choice of a boundary divisor, but it is unclear whether it reconciles with existing notions of higher-dimensional ramification filtrations (e.g., that of Abbes-Saito).

Bruno Klingler "On the André-Oort conjecture"

This is joint work with A. Yafaev. Given a set X of special points on a Shimura variety S, the André-Oort conjecture predicts that any irreducible component of the Zariski-closure of X in S is a subvariety of Hodge type. We prove this conjecture under the generalized Riemann Hypothesis.

James D. Lewis "The Abel-Jacobi map for higher Chow groups, II"

Let X be a projective algebraic manifold, and  $Y \subset X$  a NCD. Further, let  $\mathbf{A} \subset \mathbf{R}$  be a subring such that  $\mathbf{A} \otimes \mathbf{Q}$  is a field. We describe the regulator map (on the level of complexes) into absolute Hodge cohomology:  $CH^r(X-Y, \mathbf{A}) \to H^{2r-m}_{\mathcal{H}}(X-Y, \mathbf{A}(r))$ , and the corresponding Abel-Jacobi map  $CH^r_{\text{hom}}(X-Y,m) \to Ext^1_{\mathbf{A}-MHS}(\mathbf{A}(0), H^{2r-m-1}(X-Y,\mathbf{A}(r)))$ . This is joint work with Matt Kerr.

Madhav Nori "Mixed Tate motives and hyperplane arrangements"

An Abelian category of MTM arising from hyperplane arrangements will be constructed. It will be compared with the construction of Beilinson, Goncharov, Schectman and Varchenko. The Ext groups in this category will be compared with the K-groups of the base field.

Martin Olsson "Grothendieck's six operations for stacks"

I will discuss recent joint work with Yves Laszlo generalizing Grothendieck's six operations to Artin stacks. In the development of these operations a number of interesting technical issues present themselves which I will discuss in the talk. For example, morphisms of algebraic stacks are usually not compactifiable and do not have bounded cohomology. Also for applications to geometric Langland's program it is important to consider stacks which are not quasi-compact. I will explain how we overcome these difficulties in both the cases of finite and adic coefficients.

Ania Otwinowska "Motivic sheaves on curves"

Joint work in progress with A.Beilinson and V.Vologodsky.

Michael Rapoport "On non-archimedean period domains"

Period domains over the complex numbers were introduced by Griffiths. They are open subsets of partial flag varieties which parametrize Hodge structures. Non-archimedean period domains are their analogues over p-adic fields and parametrize p-adic Hodge structures. The most famous period domain is the Drinfeld half space (the complement of all rational hyperplanes in projective space). In this talk I will survey recent results on the cohomology of these spaces.

Morihiko Saito "Generalization of Griffiths' theorem on rational integrals and related topics (joint work with A. Dimca)"

We discuss a generalization of Griffiths' theorem on rational integrals to the case of a singular hypersurface of projective space. This is by joint work with A. Dimca.

"Chow-Künneth decomposition for varieties with low cohomological level"

We show that a smooth projective variety admits a Chow-Künneth decomposition if the cohomology has level at most one except for the middle degree. This can be extended to the relative case in a weak sense if the morphism has only isolated singularities, the base space is 1-dimensional, and the generic fiber satisfies the above condition.

### Takeshi Tsuji "On semi-stable smooth p-adic sheaves"

We have several types of p-adic representations of the absolute Galois groups of p-adic local fields: Hodge-Tate, de Rham, crystalline and semi-stable representations, the latter three of which were defined by J. -M. Fontaine. Generalizations of the first three types to smooth p-adic sheaves on varieites over p-adic local fields were defined and studied by O. Hyodo, N. Tsuzuki, G. Faltings, O. Brinon, ... I will talk on a generalization of the last one: semi-stable representations.

Eckart Viehweg "Stability of Hodge bundles and the uniformization of certain Shimura varieties"

Let Y be a non-singular projective manifold with an ample canonical sheaf, and let V be a rational variation of Hodge structures of weight one on Y with Higgs bundle  $E^{1,0} + E^{0,1}$ . If Y is a curve the Arakelov Inequality says that  $\mu(E^{1,0}) - \mu(E^{0,1})$  is smaller than or equal to the degree of the canonical sheaf. The equality implies that V is the tensor product of a unitary bundle and the rank two variation of Hodge structures given by a theta characteristic; moreover Y is the Shimura curve corresponding to V.

We will discuss similar inequalities for surfaces Y. Here the equality will imply that Y is either a compact Hilbert modular surface, or a ball quotient, and again V is the tensor product of a canonical uniformizing variation of Hodge structures with a unitary bundle. Most of the results extend to variation of Hodge structures over quasi projective manifolds with a "nice" compactification and partly to the higher dimensional case. (Joint work with Kang Zuo)

Andrzej Weber "Topological properties of the weight filtration"

We study the weight filtration in homology of complex algebraic varieties. We explain a relation of the weight filtration with intersection homology of Goresky-MacPherson. For a complete variety X we study the canonical map  $IH_k(X) \rightarrow H_k(X)$ . We show that the image of this map coincides with  $W^kH_k(X)$  - the bottom term of the weight filtration. By duality we deduce that the pure quotient  $H^k(X)/W_{k-1}H^k(X)$ injects into intersection cohomology. The above statements provide a precise information about the Poincare duality map for singular varieties. Moreover, since intesection (co)homology is a topological invariant we obtain a striking corollary:  $W_k H^k(X)$  and  $W^k H_k(X)$  are topological invariants as well. To obtain slightly more general results we introduce "image homology" for noncomplete varieties. The proof is an easy application of the decomposition theorem or of a possibility to construct an induced morphism of intersection sheaves. We illustrate our constructions by two classes of examples: singular hypersurfaces in manifolds or varieties with an action of a large algebraic group.

## Participants (registered on-line).

Abbes, Ahmed - CNRS, Université de Paris 13 Abrashkin, Victor - Durham University, U.K. Aidam, Jesse Kwesi - University of Cape Coast Amini, Massoud - Tarbiat Modares University André, Yves - ENS, Paris Andreatta, Fabrizio - Università di Padova Axelsson, Reynir - University of Iceland, Reykjavik Baldassarri, Francesco - Università di Padova Bannai, Kenichi - Nagoya University/ENS, Paris Behzad, Khazaie - ENS, Paris Bertapelle, Alessandra - Università di Padova Berthelot, Pierre - Université de Rennes 1 Braeuer, Jonas - Universität Muenster Brinon, Olivier - Université de Paris 13 Buckingham, Paul - University of Sheffield Calabri, Alberto - Università di Padova Caro, Daniel - University of Durham Carlson, James - Clay Mathematics Institute Caruso, Xavier - Université de Paris 13 Caviedes Castro, Alexander - National University of Colombia Celalettin, Kaya - Middle East Technical University, Ankara Chae, Hi-joon - Hongik University Chinellato, Daniele - Università di Padova Collino, Alberto - Università di Torino Colombo, Elisabetta - Università di Milano Dan-Cohen, Ishai - UC Berkeley De Jeu, Rob - University of Durham, U.K. Déglise, Frédéric - LAGA, Université de Paris 13 Dhillon, Ajneet - University of Western Ontario Eriksson, Dennis - Université de Paris 7 Fabre, Bruno - Université de Paris 6 Fasel, Jean - ETH Zürich Filali, Ouidad - Universität Muenster Fiorot, Luisa - Università di Padova Flicker, Yuval - The Ohio State University Fornasiero, Marianna - Università di Padova Galluzzi, Federica - Università di Torino Garuti, Marco A. - Università di Padova Girivaru, Ravindra V. - Washington University in St. Louis Gros, Michel - IRMAR Université de Rennes 1 Hackstein, Urs - Universität Munster Harrison - Goodshepherd College Herz, Gabriel - Universität Muenster Hirica, Iulia Elena - University of Bucharest Hoevenaars, Luuk - Utrecht University Horozov, Ivan Emilov - University of Durham

Jun, Byungheup - Korea Institute for Advanced Study Kaba, Mustafa Devrim - Middle East Technical University, Ankara Kanev, Vassil - Università di Palermo Karoubi, Max - Université de Paris 7 Kasprowitz, Ralf - Universität Muenster Khomenko, Oleksandr - MPI für Mathematik, Bonn Kihel, Omar - Brock University, Ontario, Canada Kloosterman, Remke - Universität Hannover Kobayashi, Shinichi - Nagoya University Kopei, Fabian - Universität Muenster Lekaus, Silke - Università di Roma 1/Padova Liu, Tong - Université de Paris-Sud 11 Lombardo, Giuseppe - Università di Torino Longhi, Ignazio - Università di Milano Longo, Matteo - MPI für Mathematik, Bonn Ludsteck, Thomas H. - Universität Stuttgart Martínez Ramírez, Cristina - MPI für Mathematik, Bonn Massold, Heinrich - ETH Zürich Messing, William - University of Minnesota Migliorini, Luca - Universitá di Bologna Miller, Andrea Elisabeth - Math. Institut der Univ. Heidelberg Noot-Huyghe, Christine - Université Louis Pasteur (Strasbourg-France) Orgogozo, Fabrice - École polytechnique (France) Orlik, Sascha - Universität Leipzig Otsubo, Noriyuki - Chiba University/Institut de Math de Jussieu Pandey, Yashonidhi - Université de Montpellier II Patk, Seong Suk - Seoul National University Pedrini, Claudio - Università di Genova Prykarpatska, Natalia - WMS AGH Krakow Prykarpatsky, Anatoliy - WMS AGH Krakow Quick, Gereon - Universität Muenster Rababah, Abedallah - Department of Mathematics Radu, Laza - Columbia University Scholbach, Jakob - Universität Leipzig, Germany Solomon, Noam - Ben-Gurion University Sreekantan, Ramesh - TIFR, Mumbai, India (currently MPI) Stellari, Paolo - Università di Milano Stoppa, Jacopo - Università di Pavia Szamuely, Tamas - Renyi Institute, Hungarian Academy of Sciences Tommasi, Orsola - Universität Mainz Usnich, Alexandr - Université de Paris 6 Wang, Shanwen - ALGANT - Università di Padova Weibel, Charles - Rutgers University Wiech, Stefan - Universität Muenster Zeytin, Ayberk - Middle East Technical University, Ankara

### Eating out in Venice

Venice is a food paradise and the local cuisine is strictly recommended. Of course you will find international restaurants, but as everybody knows, especially around Piazza San Marco, you will have to pay a lot for food (and accommodation) which certainly does not always mean good value. The question of which kind of pasta or pizza is probably more mind-boggling than the question of where to eat it. Venice is provided with remarkable restaurants named *osteria* and *trattoria* - simpler eating places, but mostly of a good quality local cuisine. Besides the well-known italian seafood plates, a lot of places serve dishes containing fish finished off with *polenta*, a corn-based basis of various meals. The following traditional seafood restaurants are suggested reservation by phone is highly recommended.

A quoted traditional place is Trattoria Ca' D'Oro Alla Vedova (Tel. 0415285324 closed on thursday and sunday morning) just off Strada Nuova at the Vaporetto Station Ca' D'Oro. Nearby, you find Vini da Gigio (Tel. 0415285140 closed on monday and tuesday) where an excellent choice of wines is provided. Moreover, right after Campo Santi Apostoli, there is the less traditional but very good restaurant Al Vagon (Tel. 0415237558 closed on tuesday). Otherwise, a little further in the Cannaregio on Fondamenta de la Sensa you find L'Anice Stellato (Tel. 041720744 closed on monday); using spices more familiar on far Eastern menus (*Anice Stellato* means "star anice" a major component of five-spice powder), the chef manages to bring the flavors together in a way that brings home the idea that you're sitting upon the western edge of a major eastern trade route. Located at the entrance to the old Ghetto (Ghetto Vecchio) you find Gam Gam (open Sunday-Thursday: 12:00 - 10:00pm and Friday: 12:00 - two hours before Shabbat) a good Kosher Italian restaurant!

On the other side of Canal Grande - actually you can cross the Canal Grande with the *traghetto* gondola service at eight different places, e.g., between S. Sofia (Ca'd'Oro) and Rialto market (Pescaria) - near the old market, and the bridge of Rialto, is beautifully located the Bancogiro Osteria da Andrea Campo San Giacometto (Tel. 0415232061 closed on sunday night and monday) with tables outside in the Erberia facing the Canal Grande. In the Rialto area you can skip the touristic places with the help of the classical venetian restaurant Alla Madonna (Tel. 0415223824 closed on wednesday) in Calle della Madonna. Going further in the direction of Campo S. Polo you find the Osteria Vivaldi just near Campo S. Polo in Calle della Moddoneta (Tel. 0415238185) and also the classical Trattoria da Ignazio in Calle Saoneri (Tel. 0415234852 closed on saturday).

From both Vaporetto Stations S. Zaccaria and Arsenale you get to the top quality Corte Sconta in Calle del Pestrin (Tel. 0415227024 closed on sunday and monday). Nearby, you can trust the well established Al Covo in Campiello della Pescaria (Tel. 0415223812 closed on wednesday and thursday). Another one of the same level is L'Osteria di Santa Marina in Campo Santa Marina (Tel. 0415285239 closed on sunday and monday morning). From Campo S. Maria Formosa you get to Osteria Mascaron in Calle Longa Santa Maria Formosa (Tel. 0415225995 closed on sunday). Further on the same path you get to Enoteca (Wine Bar) Mascareta still in Calle Longa Santa Maria Formosa (Tel. 0415230744 open all nights) where you can also drink good wine till late night. Another remarkable but less fancy place is Antica Trattoria Bandierette in Barbaria delle Tole (Tel. 0415220619) which is just a few steps forward.

You may have a very good pizza at Acqua Pazza (Tel. 0412770688 closed on monday) in Campo S. Angelo. This place is a *pizzeria* and a *ristorante mediterraneo* born to make a name of the quality of Campania's cooking and in particular of the old amalfitana's cooking: they claim this is possible by using products arrived directly from Amalfi!

In the private Island of San Clemente, just between San Marco and the Island of San Servolo, in the spaces of the old Camaldolese monastery, alongside the Renaissance church, stands Ca dei Frari (Tel. 0412445001 closed on sunday and monday) the exclusive gourmet restaurant of the San Clemente Palace. The exquisite delicacies of a refined cuisine dauteur, the lagoon spread out before ones eyes ... a dream to live with the eyes and savour with the palate, in the elegant privacy of its facilities at a quite high prize!

If you really like to visit the lagoon then you can reach the Litorale of Pellestrina far off the Lido in the direction of Chioggia. In Pellestrina there is a good restaurant da Celeste in Sestiere Vianelli 625/b (Tel. 041967043 closed on wednesday). You can get there by going to the Lido and then taking the bus N. 11.

Besides these restaurants, all around in Vencie, you can find the so-called *bacari* - typical Venetian taverns selling wine and snacks, and homey restaurants (very small!) line the streets. These are usually packed with local people drinking an *ombra* at dinner-time: *ombra*, literally translated, is shade, and in Venetian dialect refers to a glass of wine. Just put a little forth the effort to find out the best for yourself!

Finally, Venice is, of course, also famous for its sweets, cakes and ice-creams in all sorts and sizes which are a must if you are walking around: just look for the *pasticceria* most close to you! For example, Rosa Salva in Campo S. S. Giovanni & Paolo is truly outstanding for ice-creams.