A Geometric Approach to Weierstrass Points

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I shall report on two joint papers with G. Sacchiero.

Let X be a smooth non-hyperelliptic curve of genus g. For any $P \in X$, let us consider the Weierstrass gap sequence of X at P

$$G_P = (r_0(P) + 1, r_1(P) + 1, \ldots, r_{g-1}(P) + 1)$$

where, for any i, $r_i(P)$ is the multiplicity intersections in P of the $(i-1)$-th osculating space to the canonical model X_K with X_K itself. A point $P \in X$ is said to be a Weierstrass point if $G_P \neq (1, 2, \ldots, g)$, that is if P is an inflectionary point for the canonical embedding. The complementary subset $H_P = \mathbf{N} \backslash G_P$ is called the Weierstrass semigroup of X at P.

By fixing the gonality of a curve and by considering the rational normal scroll which contains the canonical model, one can describe the Weierstrass points of a given curve. On the other hand, it is possible to deduce some intrinsic properties of a curve having an assigned Weierstrass point, as for instance the gonality, or the existence of a certain covering map over a curve of low genus.

Moreover, a description of a certain surface ruled in conics turns out to be useful in giving an answer to the question whether any subsemigroup of $\mathbf{N}$, with finite complementary subset, is the Weierstrass semigroup of an algebraic curve. A negative answer was given by Buchweitz in 1980 for genera $g \geq 16$. We improve this result by giving examples of subsemigroups of genus $g \geq 10$, which are not Weierstrass.