DEGENERATIONS OF SURFACES AND RELATED PROBLEMS

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I will report on joint work in collaboration with Alberto Calabri (Bologna), Ciro Ciliberto (Roma "Tor Vergata") and Rick Miranda (Colorado State).

The main subject of my talk is related to flat degenerations of surfaces, whose general fibre is a smooth projective surface and whose central fibre is a reduced, connected surface. A fundamental role is played by the case of degenerations which are embedded in projective spaces and such that the central fibre is a *union of planes*. Indeed, this particular case extends the curve case, where one considers *stick curves* (i.e. union of lines) as tools to study several problems concerning for example moduli spaces of curves or families of curves in projective spaces (e.g., the *Zeuthen problem*).

The interest in the subject has been stimulated by a series of papers by Guido Zappa (1940–50's).

The approach is as follows: I consider a reduced, connected, projective surface X which is a union of smooth surfaces (in particular a union of planes) whose singularities are:

- in codimension one, double curves which are smooth and irreducible, along which two surfaces meet transversally;
- multiple points (called *Zappatic singularities*), which are locally analytically isomorphic to the vertex of a cone over suitable stick curves with arithmetic genus either zero or one and which are projectively normal in the projective spaces they span.

I first present how to study some combinatorial properties of X, then I consider the case in which X is the central fibre of a (embedded) flat degeneration $\mathcal{X} \to \Delta$, where Δ is the complex unit disk. In this case, one deduces some properties of the general fibre $\mathcal{X}_t$, $t \neq 0$, of the degeneration from the aforementioned properties of the central fibre $\mathcal{X}_0 = X$.

The main results include formulas on the basic invariants of smoothable surfaces which, among other things, extend some results of topological nature of Clemens-Schmid. On the other hand, they are also used to prove a *Miyaoka-Yau type inequality* which improves some results of Zappa.

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