Errata-Corrige-Addenda-Variationes to the book
"*Foundations of fluid mechanics*", by G. Gallavotti

Note:
l. 2 means line 2 (from top of page);
l. -3 means line 3 from bottom of page
action to be taken for correction is typed in bold face

• p. 8, l. -1:
$f(\underline{w}) = e^{\frac{-m\underline{w}^2}{2k_BT}}(\frac{m}{2\pi k_BT})^{3/2}$ **should be** $f(\underline{w}) = (\frac{m}{2\pi k_BT})^{\frac{3}{2}}e^{-\frac{m\underline{w}^2}{2k_BT}}$

• p. 8, l. -5: **delete** $\tau = \lambda/\bar{v}$

• p. 9, l. 4: **delete** recalling that

• p. 9, l. 5: $\frac{dv(z)}{dz}$) **should be**
$\frac{dv(z)}{dz}$, by $\overline{v}$ the mean velocity $\overline{v} = (3k_BT/m)^{1/2}$ and the free flight time by $\tau = \lambda/\overline{v}$)

• p. 9, l. 14: and hence that **should be**
so that, if the collision cross section is denoted σ, it is

• p. 9, l. 6 in problem 1.1.4: kT **should be** k_BT

• p. 9, l. 5 in problem 1.1.5: deduce that $\eta = \bar{\eta}$; **should be**
deduce that η is the quantity studied in problem [1.1.4];

• p. 12, l. -1: per unit$\Omega_\Omega\,\Omega_{\Omega_\Omega}$ time **delete spurious characters** $\Omega_\Omega\,\Omega_{\Omega_\Omega}$

• p. 13, l. 8 in probl. 1.1.16: production Γ **should be** production Γ (see problem 1.1.14)

• p. 17: **before Sec. 1.2.3 add the following paragraph**
However one should remark that if the fluid is enclosed in a container Ω the above equations might lead to results that might be physically unacceptable: for instance if the NS equations are subject to a boundary condition $\underline{u} = \underline{0}$ then the third equation shows that it will not be in general possible to fix the temperature at the boundary because the equation would imply that $\partial_t s > 0$ on the boundary and s (hence T) will increase with time and could not be held fixed. Hence in presence of thermoconduction the above equations will be an acceptable model only in special cases; see Sec. 5 for a

treatment of the problem in presence of boundaries. The incompressible NS equations will therefore be acceptable only if the internal generation of heat can be completely neglected either because "small" or because it is assumed to be removed by some mechanism which is not described by the equations themselves or which is implemented through special boundary conditions, see Sect. 1.5.

- p. 18, l. 3: $\sigma(T) = \int c\, dT/T$, **should be** $\sigma(T) = \int c\, dT/T$ for some function $c = c(T)$,

- p. 18, l. 4 after (1.2.8): **delete paragraph starting with** The decoupling **and replace it by**:
The interpretation problems mentioned for the Euler and NS equations are still present. Furthermore the decoupling is often illusory because the equations for s and $\underline{u}$ might become incompatible with the boundary conditions.

- p. 18, l. 9 after (1.2.8): In addition **should be** In conclusion

- p. 19, l. 1 before (1.2.12): Hence **should be** Hence the condition (1.2.10) becomes

- p. 20, l. 2/3: (1.2.11) which, therefore, adds **should be**
(1.2.11): hence the (1.2.10) adds

- p. 21, l. 2: conditions. **should be**
conditions, apart from the problems with boundary conditions that we have only mentioned in Sect. 1.2.2 and 1.2.3, and which will be studied in some detail in a simple case in Sect. 1.5.

- p. 23, l. -3: $r\, \Delta \underline{u}$ **should be** $\nu\, \Delta \underline{u}$

- p. 26, l. -8: lowest order in ε. **should be** lowest order in ε at least.

- p. 28, l. 5 in theorem 1: time τ_0 **should be** time τ_0, dimensionless by definition,

- p. 28, l. 5 after (1.3.11): $t < \tau_0 l \varepsilon\, v_{sound}$ **should be** $t < \tau_0 l/\varepsilon\, v_{sound}$

- p. 28, l. -11: time τ_0 **should be** time $\tau_0 l/\varepsilon\, v_{sound}$

- p. 29, eq. (1.1.13): **Eq. (1.1.13) should be**

$$\frac{\nu}{\varepsilon l v_{sound}} \equiv \nu_0 \ll 1 \qquad \text{independently of } \varepsilon \tag{1.3.13}$$

- p. 31, l. -11: gas is **should be** gas was

- p. 40, l. 2: is the value **should be** which is the value

- p. 40, l. 9: in prob. [1.4.7] kilometre **should be** kilometer

- p. 41: problem [1.4.10]

ocean of depth $h > 0$, large enough.
should be
ocean of depth $h > 0$, large enough, and of density negligible with respect to that of the planet.

- p. 42: problem 1.4.14 at the end:

per century.) **should be**
per century, obviously showing the inadequacy of the model, see [MD00] for a theory of tides and despinning.)

- p. 42: problem 1.4.15: **delete problem 1.4.15**

- p. 44, l. 6 after (1.5.1): will be constants that **should be** will have values that

- p. 45, l. 2: **add**

, aside from problems that might be generated by boundary conditions.

- p. 54, l. 2 after (1.6.5): : this is a form of the **should be** according to the

- p. 54, l. -10: consider vector fields **should be** consider divergenceless vector fields

- p. 55, l -12: **displace here the figure 1.6.6 from the next page**

- p. 57, l. 1 before (1.6.12): and a set φ of solutions **should be** and calling φ a solution

- p. 64, in formula (!): $\left(\frac{L}{2\pi}\right)^p$ **should be** $\big(\frac{L}{2\pi}\big)^p$

• p. 64/65: **delete problems 1.6.17,1.6.18,1.6.19 and replace by**
[1.6.17]: (*Approximating an irrotational field with a gradient*)
Let $\gamma_n(\underline{x}) = e^{(\underline{x}^2-n^{-2})^{-1}}c_n$ for $|\underline{x}|<1/n$ and let $\gamma_n = 0$ otherwise. The constant c_n is such that $\int\gamma_n \equiv 1$, ($c_n \propto n^3$). Let $\underline{f}\in L_2(\Omega)$ and $\operatorname{rot}\underline{f}=\underline{0}$ in the sense that for each $\underline{A}\in C_0^\infty(\Omega)$ it is $\int_\Omega \underline{f}\cdot\operatorname{rot}\underline{A}=\underline{0}$. Show, (with the notations of the problem [1.6.16] and if $*$ denotes the convolution product in the domain Q considered as a torus):

(i) $\int_Q \underline{f}^e\cdot\operatorname{rot}\underline{A}=\underline{0}$ for $\underline{A}\in C_0^\infty(\cup_{k=1}^8\Omega_k)$
(ii) if n is large enough: $\int_Q \gamma_n * \underline{f}^e\cdot\operatorname{rot}\underline{A}^e \equiv 8\int_\Omega \gamma_n*\underline{f}^e\cdot\operatorname{rot}\underline{A}^e\equiv\underline{0}$
(large enough means that $\frac1n$ is smaller than the distance between $\partial\Omega$ and the support of $\underline{A}$);
(iii) $\operatorname{rot}(\gamma_n*\underline{f}^e)=\underline{0}$, in $\Omega_k^{(n)}=\{\underline{x}|\underline{x}\in\Omega_k,\, d(\underline{x},\partial\Omega_k)>\frac1n\}$
(iv) $\lim_{n\to\infty}\gamma_n*\underline{f}=\underline{f}$, in $L_2(\Omega)$.

[1.6.18]: Within the context of problems [1.6.16],[1.6.17] suppose Ω convex and let $\varphi_n(\underline{x})$ be defined for $\underline{x}\in\Omega_k^n$ in the interior of $\cup_{k=1}^8\Omega_k^{(n)}$ by

$$\varphi_n(\underline{x})=\int_{\Omega_k^{(n)}}\frac{d\underline{y}}{|\Omega_k^{(n)}|}\int_0^1 ds\,(\underline{x}-\underline{y})\cdot(\gamma_n*\underline{f}^e)(\underline{y}+s(\underline{x}-\underline{y}))$$

Check that for $\underline{x}\in\cup_{k=1}^8\Omega_k^{(n)}$

$$\partial_j\varphi_n(\underline{x})=\gamma_n*f_j^e(\underline{x}),\qquad \int_{\Omega_k^{(n)}}\frac{d\underline{y}}{|\Omega_k^{(n)}|}|\varphi_n(\underline{x})|\le C\,||f||_{L_2(\Omega)}$$

(*Hint:* The first relation is simply due to $\operatorname{rot}\gamma_n*f^e=\underline{0}$ in $\cup_{k=1}^8\Omega_k^{(n)}$, *c.f.r.* problem [1.6.17]. The integral in the second can be bounded by

$$\begin{aligned}&L\sqrt3\int_0^1 ds\int_{\Omega_k^{(n)}}\frac{d\underline{x}d\underline{y}}{|\Omega_k^{(n)}|}|(\gamma_n*\underline{f}^e)(\underline{y}+s(\underline{x}-\underline{y}))|\le\\&\le\frac{L\sqrt3}{|\Omega_k^{(n)}|}|\int_0^1\frac{ds}{(1-s)^3}\int_{|\underline{x}-\underline{z}|<(1-s)L\sqrt3}d\underline{x}d\underline{z}|(\gamma_n*\underline{f}^e)(\underline{z})|\le\\&\le C_1L\int_{\Omega_k^{(n)}}d\underline{z}|(\gamma_n*\underline{f}^e)(\underline{z})|\le\\&\le C_2L^{5/2}||\gamma_n*\underline{f}^e||_{L_2(Q)}\le CL^{5/2}||\underline{f}||_{L_2(\Omega)}\end{aligned}$$

(after changing variables $\underline{y}\to\underline{z}=\underline{y}+(\underline{x}-\underline{y})s$) for suitable constants C_1,C_2,C.)

[1.6.19] (*Representing an irrotational field defined in a convex domain as a gradient*) Let $F_j^{(n)}\in L_2(Q)$ and $\varphi_n\in\cup_{k=1}^8\Omega_k^{(n)}$ be chosen, in the context of problems [1.3.17], [1.3.18], so that

$$\partial_jF_j^{(n)}=\gamma_n*f_j^e \text{ in } L_2(Q),\qquad ||\varphi_n||_{L_2(\Omega_k^{(n)})}\le C||f||_{L_2(Q)}$$

Check first that the bounds obtained in problem [1.6.18] imply that in $\Omega_k^{(n)}$ it is $F_j^{(n)}(\underline{x}) = \partial_j\varphi_n(\underline{x}) + c_k^n$ with the constant c_k^n bounded by $C||f||_{L_2(\Omega)}$. Then extend φ_n to Q by setting it equal to 0 outside $\cup_{k=1}^8 \Omega_k^{(n)}$ and let $\varphi = \lim \varphi_{n_q}$ be a weak limit in $L_2(\Omega)$ of φ_n on a subsequence $n_q \to \infty$. Show that φ admits generalized first derivatives and $\underline{\partial}\varphi \equiv \underline{f}$. (*Hint:* One remarks that if $b \in C_0^\infty(\Omega)$ it is $\int_\Omega \partial_j b\varphi = \lim_{n\to\infty} \int_\Omega \partial_j b\, \varphi_n = -\int_\Omega b\partial_j\varphi_n = -\int_\Omega bf_j$ so that $|\int \varphi\partial_j b| \le ||f||_{L_2(\Omega)}\, ||b||_{L_2(\Omega)}$.)

• p. 65, l-8: **replace text from** Continue **to** flux generated **by**
Continue the field in such a way that its flux lines exit the right face of the cylinder (where $x = L$ and the other two coordinates are y, z, say) and reenter from the left side (where $x = 0$) at the point corresponding to the same coordinates y, z: this is done by defining $\underline{u}$ outside the cylinder so that the flux thus generated

• line 2 after formula (1.7.2) line 2: flouid **should be** fluid

• formula (1.7.5) line 2: $\cdot(d\underline{\ell} + t\,\underset{\sim}{\partial}\underline{u}(\underline{\ell},0))\cdot d\underset{\sim}{\ell}$ **should be** $\cdot(d\underline{\ell} + t\,\underset{\sim}{\partial}\underline{u}(\underline{\ell},0)\cdot d\underset{\sim}{\ell})$

• p. 70, l. -7: An incompressible **should be** A smooth incompressible

• p. 71, l. 7: a velocity **should be** a *smooth* velocity

• p. 72, l. 13/14: as in the case **should be** as it is possible in the case

• p. 73, l. 7: which make **should be** which makes

• p. 73, l. 6 after (1.7.26): $\delta F/\partial f(x)$ **should be** $\delta F/\delta f(x)$

• p. 75, l. 1 before (1.7.31): **Delete text from** Hence **through l. 1 before (1.7.36) and replace by**
Hence, denoting $\dot\zeta = \partial_t\zeta, \zeta' = \partial_x\zeta$, we obtain the boundary condition $-\partial_x\zeta\, u_x + u_z = \partial_t\zeta$ in $(x, \zeta(x,t))$; the latter relation has to be coupled with the condition that the pressure be constant, *e.g.* $p = 0$, at the free boundary. If $\underline{n}(x) \stackrel{def}{=} (-\zeta'(x), 1)/(1 + \zeta'(x)^2)^{1/2}$ is the external normal to the set of the (x, z) with $z < \zeta(x)$, the two conditions can be written as

$$
\begin{aligned}
&(\underline{n}\cdot\underline{\partial}\,\varphi)(x,\zeta(x))=\frac{\partial_t\zeta(x)}{(1+\zeta'(x)^2)^{1/2}}\\
&\partial_t\varphi+g\zeta+\frac{1}{2}(\underline{\partial}\varphi)^2=0
\end{aligned}
\tag{1.7.31}
$$

respectively, *c.f.r.* (1.7.7). The relations show that the two functions $(\zeta(x),\dot\zeta(x))$ uniquely determine the velocity field as well as their values at time $t+dt$.

In fact given $\zeta,\dot\zeta$ we can compute the velocity field $\underline{u}=\underline{\partial}\varphi$ by determining φ as the harmonic function satisfying $\Delta\varphi=0$ in $D=\{(x,z)|\,x\in[0,L],z\in(-\infty,\zeta(x))\}$ with boundary condition $\partial_n\varphi$ given by the first of (1.7.31): a Neumann problem; furthermore from the second relation we can compute $\partial_t\varphi(x,z)$ by solving a Dirichlet problem in D with boundary condition $(\partial_t\varphi)(x,\zeta(x))$. The latter field can be used to compute $\ddot\zeta(x)$ by differentiating the boundary condition rewritten in the form $\partial_t\zeta=-\partial_x\zeta\,u_x+u_z$.

We can now put the equations into Hamiltonian form. We expect that, the motion being "ideal", the equations of motion must follow from a naive aplication of the action principle. We shall restrict the analysis to motions which are horizontally periodic over a length L, for simplicity. Since we have seen that the motions are naturally described in the coordinates $\zeta(x),\dot\zeta(x)$ we consider the action functional of $\zeta,\dot\zeta$ as the difference betwen the kinetic energy of the fluid and its potential energy in the gravity field.

Hence the domain D in which we shall consider the fluid is $D=\{(x,z)|\,x\in[0,L],z\in(-\infty,\zeta(x))\}$. In D the velocity field will be $\underline{u}=\underline{\partial}\varphi$ and its time derivative will be $\underline{\partial}\psi$ with φ harmonic, solutions of

$$
\Delta\varphi=0\qquad\text{in } D,\qquad \underline{n}\cdot\underline{\partial}\varphi=\frac{\dot\zeta}{(1+\zeta'(x))^{1/2}}\qquad\text{in }\partial D
\tag{1.7.32}
$$

The potential energy will obviously be infinite, being $\int_0^L\int_{-\infty}^{\zeta(x)}\rho\,g\,z\,dxdz$. However what counts are the energy variations which are formally the same as those of $\rho\int_m^L dx\frac{1}{2}g\zeta(x)^2$ where m is any quantity less than the minimum of $\zeta(x)$.

The kinetic energy will be $\frac{\rho}{2}\int_0^L dx\int_{-\infty}^{\zeta(x)}dz\,|\underline{\partial}\varphi(x,z)|^2$. Thus the action of the system, *i.e.* the difference between kinetic and potential energy is, if $ds\stackrel{def}{=}(1+\zeta'(x)^2)^{1/2}dx$ and if we introduce the auxiliary function $\psi(x)\stackrel{def}{=}\varphi(x,\zeta(x))$,

$$
\begin{aligned}
\mathcal{L}=&\frac{\rho}{2}\int_0^L\int_{-\infty}^{\zeta(x)}(\underline{\partial}\varphi)^2\,dxdz-\frac{\rho}{2}\int_0^L g\zeta(x)^2dx=\\
=&\frac{\rho}{2}\int_0^L(\psi(x)\frac{\dot\zeta(x)}{(1+\zeta'(x))^{1/2}}\,ds-g\zeta(x)^2\,dx)
\end{aligned}
\tag{1.7.33}
$$

after integrating by parts the first integral, defining if $\psi(x)\stackrel{def}{=}\varphi(x,\zeta(x)$ and making use of the first relation (1.7.31), (1.7.32).

The linearity of (1.7.32) shows that ψ is linear in $\dot\zeta(x)/(1+\zeta'(x))^{1/2}$: *i.e.* $\psi=M\dot\zeta/(1+(\zeta')^2)^{1/2}$ for some linear operator M (which depends on $\zeta(x)$, *i.e.* on the shape of D).

In general if φ_1,φ_2 are two harmonic functions in D with respective "Dirichlet value" φ_1,φ_2 on the boundary, ∂D, and respective "Neumann values" on the same boundary $\sigma_1=\underline{n}\cdot\underline{\partial}\varphi_1,\sigma_2=\underline{n}\cdot\underline{\partial}\varphi_2$ then, (on the boundary) by definition, $\varphi_i=M\sigma_i$ for a suitable linear operator M and $\int_{\partial D}ds\,\big((M\sigma_1)\,\sigma_2-\sigma_1\,(M\sigma_2)\big)=\int_D(\varphi_1\Delta\varphi_2-\varphi_2\Delta\varphi_1)\,dxdz=0$ so that the operator M is *symmetric* on $L_2(ds,[0,L])$. Note that the operator M transforms, by definition and by (1.7.34), a Neumann boundary datum for a harmonic function into the corresponding Dirichlet datum. Hence $\mathcal{L}$ becomes

$$\mathcal{L}=\frac{\rho}{2}\int_0^L\big(M(\frac{\dot\zeta}{(1+\zeta')^{1/2}})\frac{\dot\zeta}{(1+\zeta')^{1/2}}\,ds-g\zeta^2\,dx\big)\qquad(1.7.34)$$

Therefore we can write the equations of motion in terms of the Hamiltonian corresponding to the Lagrangian $\mathcal{L}$. Note that $\delta\mathcal{L}/\delta\dot\zeta(x)=M(\dot\zeta(x)/(1+(\zeta')^2)^{1/2})$, where we denote by $\delta/\delta\varphi(x)$ a generic functional derivative and we recall that $(1+\zeta'(x)^2)^{-1/2}ds\equiv dx$: therefore we can define the variable $\pi(x)$ conjugate to $\zeta(x)$ as $\pi(x)\stackrel{def}{=}\rho M\dot\zeta/(1+(\zeta')^2)^{1/2}=\rho\psi(x)$ and the Hamiltonian in the conjugate variables π,ζ is

$$\mathcal{H}=\frac{1}{2}\int_0^L\Big(\frac{1}{\rho}\pi(x)G\pi(x)+\rho g\zeta(x)^2\Big)\,dx\qquad(1.7.35)$$

where if Γ^D is the operator that solves the Dirichlet problem in D we set $G(\zeta)\pi(x)=\rho\big(-\partial_x\zeta(x)\partial_x\big(\Gamma^D\pi\big)(x,z)+\partial_z\big(\Gamma^D\pi\big)(x,z)\big)_{z=\zeta(x)}\big)$.

The check that the equations of motion generated by (1.7.35) are correct, *i.e.* coincide with (1.7.31) proceeds as follows. The first Hamilton equation is just $\dot\zeta=\frac{1}{\rho}G\pi$ which coincides with the first of (1.7.31). More interesting is the computation of the functional derivative $-\delta\mathcal{H}/\delta\zeta(x)$. It is

• p. 77: l. 1/2 after (1.7.36): deduced from **should be** studied by remarking that by its definition $(\Gamma_D\pi)(x,z)|_{z=\zeta(x)}\equiv\pi(x)$ hence near $z=0$ we get

• p. 85, l. -9: significant **should be** most significant

• p. 87, l. 1: Since the level lines of $\sigma_\varepsilon(\xi)$ are, **should be** The level lines of $\sigma_\varepsilon(\xi)$ will be,

• p. 87, l. 2: the gradient **should be** and the gradient

• p. 87, l. 16: its divergence **should be** the divergence

• p. 87, l. 17: tends **should be** would tend

• p. 88: in formulae (2.1.17) and (2.1.19): $\sum_{h=1}^{k}$ **should be** $\sum_{h=0}^{k-1}$

• p. 89, l. 3 after (2.1.9): $\varepsilon \to 0$. **should be**
$\varepsilon \to 0$ (see the last term in the second equation).

• p. 89, l. -10: (σ_ε) **should be** $(\underline{\partial}\sigma_\varepsilon)$

• p. 93, l. 1 in item (i) of prop. 2: $\tau, b, \eta > 0$, **should be**
$\tau, b, \eta > 0$ (implying, among other things, that T_0 is analytic entire in x)

• p. 93, l. 1 in item (iii) of prop. 2: if **should be** unless

• p. 93, l. 2 in item (i) of prop. 2: the solution of **should be**
the "usual", well known, solution $T(x,t)$ of

• p. 93, l. 1-2 in item (iii) of prop. 2: **delete**
(implying, among other things, that T_0 is analytic entire in x)
to be inserted in item (i)

• p. 93, l. 2 in item (iii) of prop. 2: is in general **should be** is not, in general

• p. 93, l. 1 after (2.1.23): $\omega = 2\pi n$ **should be** $\omega = n$

• p. 93, l. -5: $\overline{T}$ **should be** T **two replacements**

• p. 93, l. -5 -4: the solution of (2.1.12). **should be**
the "usual" solution of (2.1.12) with Fourier transform given by the r.h.s. of (2.1.24).

• p. 93, l. -1: $[-a, a]$). **should be** $[-a, a]$), hence it cannot be analytic for $t > 0$ (while $T(x,t)$ is analytic).

• p. 94, l. -13: a well defined solution and keeping **should be** a solution which is well defined and keeps

• p. 94, l. -9: equal to 0 **should be** equal to 0 or to each other

• p. 94, l. -4: are likely to be solutions **should be** are, in some sense, solutions

• p. 94: **change the footnote 3 to** [3] I do not know a reference for detailed analysis of this property.

• p. 95, l. 2: $\pm a$). **should be** $\pm a$, to say the least).

• p. 95, l. 5 and l. -1: $[-\pi, \pi]$. **should be** $[-\pi, \pi]$ and for all times $t \geq 0$.

• p. 95, l. 10 in remark (5): Euler equations or equations can "hide" major existence and Navier Stokes equations. and lead
should be
Euler or Navier–Stokes equations and lead

• p. 95, l. -1: $[-\pi, \pi]$. **should be** $[-\pi, \pi]$ and $t \geq 0$.

• p. 96, l. 4: (see [1.1.5]) **should be** (see problem [1.1.5])

• p. 96, l. -1: labelled **should be** labeled

• p. 97, l. 2: **delete** Note that the second bound is better than the first if εn is large.

• p. 98, probl. 2.1.8: **replace problen 2.1.8 by**
2.1.8 Consider, in the case $d = 2$, a solenoidal nonconservative force $\underline{g} \in C^\infty(\Omega)$ and let $\gamma = \partial\Omega$ be such that $I = \int_\gamma \underline{g} \cdot d\underline{x} \neq 0$. Show that in this case (2.1.9) is not, in general, soluble if the initial datum is $\underline{u}_0 = \underline{0}$. Find an example. (*Hint:* Let Ω be a disk of radius r and let $\underline{g}(\underline{x}) \stackrel{def}{=} \underline{\omega} \wedge \underline{x}$ with $\underline{\omega} = \omega \underline{e}$ orthogonal to the disk. Then the uniqueness of the solutions for the Neumann problem implies that $p = const$ hence $\underline{\partial} p = \underline{g} = \underline{0}$ but $I = 2\pi r^2 \omega$).

• p. 98, l. -13: $\equiv 2($ **should be** $\equiv ($

• p. 100, l. 10: avoid the condition that **should be** exclude that

• p. 100-101: in formulae ((2.2.7),(2.2.8),(2.2.10),(2.2.11),(2.2.12):
$\prod$ **should be** Π **i.e. capital greek** π

• p. 101, l. -10:
"*ultraviolet*" components also called the "*short wave*" components **should be**
"*short wave*"components, also called the "*ultraviolet*" components,

• p. 101, in (2.2.13): $\sum_{m=1}^{n}$ **should be** $\sum_{m=0}^{n-1}$

• p. 102, l. 10: shoud **should be** should

• p. 107, l. 1: $-\underline{k}_1^4 \nu \omega_{\underline{k}_1}$ **should be** $-\underline{k}_1^2 \nu \,\overline{\omega}_{\underline{k}_1}$

• p. 107, l. 10: $\omega_{\underline{k}}$'s thought **should be** $\omega_{\underline{k}}$'s, thought

• p. 107, l. 11: gyroscope *must* **should be** gyroscope, *must*

• p. 110, l. 5: solution **should be** solution of NS

• p. 111, problem (2.2.3): **the first line should be the first line of the next prob. (2.2.4), while the first line of probl. (2.2.3) should be**
(*Weak solutions ambiguities*)

• p. 111, l. 6. in prob. (2.2.4): (cf. (2.2.15) by the definition of D) **should be**
(by the definition of D, cf. (2.2.15))

• p. 111, l. 6. in prob. (2.2.5): $d\underset{\approx}{\underline{x}}$ **should be** $d\underline{x}$
and also dx **should be** $d\underline{x}$

• p. 111, l. 9. in prob. (2.2.5): $\int_Q (\underset{\sim}{\partial}\, \underline{u})^2$ **should be** $\int_Q (\underset{\sim}{\partial}\, \underline{u})^2 / \int_Q \underline{u}^2$

• p. 111, l. 10. in prob. (2.2.5): $C_0^\infty(\Omega\text{mega})$ **should be** $C_0^\infty(\Omega)$

• p. 112, in formula (2.2.39): $D(\underline{u}_{n_i})$ **should be** $D(\underline{\tilde{u}}_{n_i})$

• p. 112, l. -5: $||u_n - u|| \xrightarrow[n\to\infty]{} 0$ and $D(u_n - u) \xrightarrow[n\to\infty]{} 0$ **should be** $||u_n - u_m|| \xrightarrow[n,m\to\infty]{} 0$ and $D(u_n - u_m) \xrightarrow[n,m\to\infty]{} 0$

• p. 114-115-116-117: **replace problems 2.2.21, 2.2.22, 2.2.23 2.2,24, 2.2.25, 2.2.26 by**
[2.2.19] (*Traces*) Let Ω be a domain with a bounded smooth manifold as boundary $\partial\Omega$: *i.e.* such that $\partial\Omega$ is covered by a finite number N of small surface elements each of which can be regarded as a graph over a disk δ_i tangent to $\partial\Omega$ at its center $\xi_i \in \partial\Omega$, so that the parametric equations of σ_i can be written $z = z_i(\xi)$, $\xi \in \delta_i$ and the points $\underline{x}$ of Ω close enough to σ_i can be parameterized by $\underline{x} = (\xi, z_i(\xi) + z)$ with $\xi \in \delta_i, z \ge 0$. Note that if Ω has the above properties also the homothetic domains $\rho\Omega$ with $\rho \ge 1$ have the same properties and the σ_i can be so chosen that $diam(\sigma_i) = cL$ if L is the diameter of Ω and N, c are the same for all domains $\rho\Omega$ with $\rho \ge 1$.
Let $f \in \mathcal{C}^\infty(\Omega)$ and define $\partial^{\underline{\alpha}} \equiv \frac{\partial^{|\underline{\alpha}|}}{\partial_1^{\alpha_1}\ldots\partial_d^{\alpha_d}}$ and

$$||f||^2_{W^n(\Omega)} = \sum_{j=0}^{n} L^{2j-d} \sum_{|\underline{\alpha}|=j} \int_\Omega |\partial^{\underline{\alpha}} f(\underline{x})|^2 \, d\underline{x}$$
$$||f||^2_{W^n(\sigma_i)} = ||f_{\delta_i}||_{W^n(\delta_i)}, \qquad ||f||^2_{W^n(\partial\Omega)} = \max_i ||f_{\delta_i}||_{W^n(\delta_i)}$$

where $f_{\delta_i}(\xi) = f\big(\underline{x}(\xi, z_i(\xi))\big)$. Check that problem [2.2.18] implies

$$||f||_{W^{n-1}(\partial(\Omega))} \le \Gamma\, ||f||_{W^n(\Omega)}, \qquad n \ge 1$$

and the constant Γ can be taken to be the same for all domains of the form $\rho\Omega$, $\rho \ge 1$

[2.2.20] (*A "trace" theorem*) Given a scalar function $f \in L_2(\Omega)$ suppose that it admits generalized derivative up to the order n included, with n even. Hence $(-\Delta)^{n/2}$ exists in a generalized sense, because $|\langle f, (-\Delta)^{n/2} g\rangle| \le C||g||_2$ for $g \in C_0^\infty(\Omega)$ and for a suitable C, *c.f.r.* §1.6, (1.6.19) and problem [2.2.1] above. Suppose $n > d/2$.
Show that f is continuous in every point in the interior of Ω, together with its first j derivatives if $n - d/2 > j \ge 0$. Find an analogous property for n odd.
(*Hint:* Let Q_ε be a cube entirely contained in Ω and let $\chi \in C_0^\infty(Q_\varepsilon)$. The function $\chi f \equiv f_\chi$ thought of as an element of $L_2(Q_\varepsilon)$ admits generalized derivatives of order $\le n$ and $(-\Delta)^{n/2} f_\chi$ exists in a generalized sense (this is clear if $n/2$ is an integer because $\chi \partial^p g = \sum_{j=0}^p \partial^j(\chi^{(j)}\, g)$ where $\chi^{(j)}$ are suitable functions in $C_0^\infty(Q_\varepsilon)$).
Hence thinking of $g \in C_0^\infty(Q_\varepsilon)$ as a periodic function in Q_ε we see that there is a constant C_{Q_ε} such that the relation $|\langle f_\chi, (-\Delta)^{n/2} g\rangle| \le C_{Q_\varepsilon} ||g||_{L_2(Q_\varepsilon)}$ holds for each $g \in C_0^\infty(Q_\varepsilon)$, and therefore it must hold for each periodic $C^\infty(Q_\varepsilon)$–function g. Then, if $\hat f_\chi(\underline{k})$ is the Fourier transform of f_χ as element of $L_2(Q_\varepsilon)$ (so that $\underline{k} = 2\pi\varepsilon^{-1}\underline{m}$ with $\underline{m}$ an integer components vector), we

get $\varepsilon^d \sum_{\underline{k}} |\hat f_\chi(\underline{k})|^2 |\underline{k}|^{2n} \le C^2_{Q_\varepsilon}$. Hence, setting $n = \frac{d}{2} + j + \eta$ with $1 > \eta > 0$, we see that the Fourier series of $\partial^j f_\chi$ is bounded above by the series

$$\begin{aligned}
&\sum_{\underline{k}} |\underline{k}|^j |\hat f_\chi(\underline{k})| \equiv \sum_{\underline{k}} |\underline{k}|^{j+\eta+d/2} |\hat f_\chi(\underline{k})| |\underline{k}|^{-\eta-d/2} \le \\
&\le \Big(\sum_{\underline{k}} |\underline{k}|^{2n} |\hat f_\chi(\underline{k})|^2\Big)^{1/2} \Big(\sum_{\underline{k}} |\underline{k}|^{-2\eta-d}\Big)^{1/2} \le \\
&\le C_{Q_\varepsilon} \varepsilon^{-d/2} (\frac{\varepsilon}{2\pi})^{\eta+d/2} (\sum_{\underline{m}} |\underline{m}|^{-d-2\eta}\Big)^{1/2} \equiv \Gamma_d \varepsilon^{n-j-d/2} C_{Q_\varepsilon}
\end{aligned}$$

hence f has j continuous derivatives in Q_ε. If n is odd one can say, *by definition* that $(-\Delta)^{n/2} f$ exists if it is $| < f, \partial^j g > | \le C ||g||_2$ for each derivative of order $j \le n$ and for each $g \in C^\infty_0(\Omega)$; then the discussion is entirely parallel to the one in the n even case.)

[2.2.21] (*An auxiliary trace theorem*) Let $W^n(\Omega)$ be the space of the functions $f \in L_2(\Omega)$ with generalized derivatives of order $\le n$ and define.
Show that the method proposed for the solution of [2.2.19] implies that, if $n > j + d/2$ and $d(\underline{x}, \partial\Omega)$ denote the distance of $\underline{x}$ from $\partial\Omega$, then

$$L^j |\partial^{\underline{\alpha}} f(\underline{x})| \le \, (\frac{d(\underline{x}, \partial\Omega)}{L})^{-j-d/2} \, \Gamma \, ||f||_{W^n(\Omega)}, \qquad |\underline{\alpha}| = j$$

and Γ can be chosen to be independent of Ω. (*Hint:* Let Q_1 be the unit cube. Let $\chi_1 \in C^\infty_0(Q_1)$ be a function identically equal to 1 in the vicinity of the center of Q_1. Let $\chi_\varepsilon(\underline{x}) = \chi_1(\underline{x}\varepsilon^{-1})$ and show that the constant C_{Q_ε} considered in the estimates of problem [2.2.19] can be taken equal to $\gamma L^{d/2} \varepsilon^{-n} ||f||_{W^n(\Omega)}$, with γ independent from Ω. Then choose $\varepsilon = d(\underline{x}, \partial\Omega)$.)

[2.2.22] (*Trace theorem*) Infer from problem [2.2.19], [2.2.21] that if $j < n - 1 - d/2$ there is a constant Γ such that $\partial^{\underline{\alpha}} f(\underline{x})$, $|\underline{\alpha}| = j$ with $\underline{x} \in \partial\Omega$ is continuous and

$$L^j |\partial^{\underline{\alpha}} f(\underline{x})| \le \, \Gamma \, ||f||_{W^n(\Omega)}, \qquad |\underline{\alpha}| = j$$

(*Hint:* a point $\underline{x} \in \partial\Omega$ will be in some σ_i, *c.f.r.* problem [2.2.19], and at a distance $O(1)$ from its boundary. Then apply problem [2.2.21].) With a little extra effort one can obtain the "same" result under the weaker condition $j < n - d/2$.

• p. 118, l. 1: *Dirichlet form*) **should be**
Dirichlet form: scalar case)

• p. 118, l. 2: $f_n \in X^0_{rot}(\Omega)$ **should be** $||f_n||_2 = 1$

• p. 118, l. 5: **delete the hint and replace it by** (*Hint:* For each $g \in C_0^\infty(\Omega)$ we get, setting $(f,g)_D \equiv \int_\Omega \underline{\partial} f \cdot \underline{\partial} g \, d\underline{\xi}$, that

$$|(f_n, g)_D - \lambda_0^2 \langle f_n, g\rangle| \xrightarrow[n\to\infty]{} 0 \Rightarrow |(f_0, (-\Delta - \lambda_0^2) g|) \,=\, 0$$

Hence $|(f_0, -\Delta g)| \le \lambda_0^2 ||g||_2$ and f_0 has a Laplacian in a generalized sense, by definition of generalized derivative, and $-\Delta f_0 = \lambda_0^2 f_0$.)

• p. 119, l. 1 in problem [2.2.33]: **should be**
(*Estimates on the large order eigenvalues and eigenfunctions of the scalar Dirichlet form*) scalar

• p. 119, l. 1 before problem [2.2.34]: **add**
Lower bounds on $\lambda_j(\Omega)$ can be obtained by the minimax principle by enclosing Ω in a cube $Q' \supset Q \supset \Omega$.

• p. 119, l. 1 in problem [2.2.34]: **should be**
(*estimates on the large order eigenvalues and eigenfunctions of the scalar Dirichlet form*)

• p. 122, l. -11: make not **should be** not make

• p. 124, l. -10: $\omega = \omega_1$ **should be** $\omega \stackrel{def}{=} \omega_1$

• p. 124, l. -11: **add**
The distance $\sqrt{\Delta}$ has to be > 0 as we shall exclude initial data in which a pair of vortices occupy the same point.

• p. 125, l. -19/-18: **replace the sentence by**
: the circulation at ∞ has indeed the value ω.

• p. 126, l. -6: $\log(p'^2 + q^2)$ **should be** $\log(p'^2 + q'^2)$

• p. 127, l. 2 before prob. 2.3.9: linear and **should be** linear, and

• p. 128, l. 7: $2\frac{(\underline{n}\cdot\underline{\xi})^2 L^2}{(|\underline{n}|^2 L^2)^2}$ **should be** $\frac{1}{2}\frac{(\underline{n}\cdot\underline{\xi})^2 L^2}{(|\underline{n}|^2 L^2)^2}$

• p. 130, l. 1 before eq. (2.4.8): the *Biot–Savart law* **replace by**
a formula often called *Biot–Savart formula* because it says that the velocity field of the vorticity field is the "*magnetic field*" of an electrric current of

intensity Γ circulating on the filament as computed from the Biot-Savart law (units aside, of course)

• p. 132, l. -2: Frenets' **should be** Frenet's

• p. 134, l. 13: is not obviously defined **should be** cannot be, *a priori*, condidered defined

• p. 136, l.2/3 of (2.4.27) : $(D|\alpha|)$ **should be** $D|\alpha|$

• p. 137, l. -13: $\underline{\omega}^l$ **should be** $\underline{\omega}^\lambda$

• p. 139, l. 2 after (2.4.33): $s \to \underline{\xi}(s) = (s, \eta s)$ **should be** $s \to \underline{\xi}(s) = (s, \eta s)$, $0 \le s \le q_n$,

• p. 139, l. 4 after (2.4.33): (*c.f.r.* [Ga83] **should be** (cf. problem [5.1.7] below)

• p. 139, l. 9/10 after (2.4.33):
where dh is the size **should be** where $d\ell$ is the size

• p. 139, in eq. (2.4.35): $l\underline{n}\, dh\, dl$ **should be** $l\,\underline{\nu}\, dh\, dl$

• p. 140, l. 6: then $\underline{u}$ results integrable. **should be** then the integral expressing the value of the field $\underline{u}$ is convergent.

• p. 140, l. 3 in prob. 2.4.2: labelled **should be** labeled

• p. 142, l. 1 before Biblio.: exponent 1/2 **should be** any exponent $< 1/2$

• p. 142, in Biblio.: **shift the sentence**
The Gaussian process defined in [2.4.4] and discussed in problems following [2.4.4] has periodic sample paths: it differs therefore from the usual brownian motion. However the difference is quite trivial, see [IM65] p.21, problem 3.
to problem 2.4.4 before the hint and set it in italics

• p. 143: In Sec. 3.1 from beginning to problems excluded globally replace first G with V and *then* Φ with G

• p. 143, l. -1: cf. [3.1.0] **should be** cf. Problem [3.1.0]

• p. 143, l. -4: **modify text from** There **to end of proposition as**
There is $B_0 > 0$ *such that if* $\xi_0 \stackrel{def}{=} k_0\xi \le 1$, $V_c \stackrel{def}{=} \nu L^{-1}$, $T_c \stackrel{def}{=} L^2\nu^{-1}$ *and* $V_0 \stackrel{def}{=} B_0\,(V + G\,T_c)\,\xi_0^{-d-1}$ *then (3.1.2), admits a solution* $\underline{u}(\underline{x}, t)$ *analytic in* $\underline{x}$ *and* t *with*

$$|\underline{\gamma}_{\underline{k}}(t)| \le V_0 e^{-\xi_0|\underline{k}|/2k_0}, \qquad \text{for} \quad 0 \le t \le t_0 \stackrel{def}{=} T_c\big(\frac{V_c}{V_0}\big)^2 \tag{3.1.3}$$

and the solution is the unique one enjoying the above properties in the time interval $[0, t_0]$.

• p. 144, l. 14: $4 > \frac{3}{2} + 1$ **should be** $4 > \frac{3}{2} + 2$

• p. 144, l. 14: [2.2.20] **should be** Problem [2.2.22]

• p. 144, in eq. (3.1.4): $-\nu\int_\Omega \delta\underline{u}^2\,d\underline{x}$ **should be** $-\nu\int_\Omega(\underset{\sim}{\partial}\,\delta\underline{u})^2\,d\underline{x}$

• p. 145, l. 1: (3.1.6) **should be** (3.1.5)

• p. 145, l. 1 after (3.1.6): $\varphi_{\underline{k}}$ of **should be** $\Pi_{\underline{k}}\hat{\underline{g}}_{\underline{k}}$ of

• p. 145, in (3.1.7),(3.1.8),(3.1.8),(3.1.9),(3.1.10),(3.12),:
$e^{-\nu\,kk^2(t-\tau)}$ **should be** $e^{-\nu\,\underline{k}^2(t-\tau)}$ **(repeated many times)**

• p. 150, l. 6: n nodes **should be** m nodes

• p. 150, l. 7: $n+1$ final **should be** $m+1$ final

• p. 150, l. 7: $n-1$ internal **should be** $m-1$ internal

• p. 150, l. 1 in eq. (3.1.13): $n+1$ **should be** $m+1$

• p. 150, l. 2 before eq. (3.1.14): by enlarging **should be** enlarging to $[0, t]$

• p. 150, eq. (3.1.14): **All pairs** kk **should be** $\underline{k}$

• p. 150, in eq. (3.1.15): 2^{4n} **should be** 2^{4m}

• p. 151, l. 9:
$= \nu$ and $(16B\,\overline{V}_0\xi_0^{-d-1})^{-2}$ **should be** $= \nu\,(16B\,\overline{V}_0\xi_0^{-d-1})^{-2}$

• p. 151, l. 8 after (3.1.17): enough time. **should be**
enough time: this is the case not only for the "normal viscosity" (friction term $\nu\Delta\underline{u}$) but also for the *ipoviscous NS* with friction term $-\nu|\Delta|^{\alpha}$ with $1 > \alpha > \frac{1}{2}$ and (of course) for the *hyperviscous NS* with $\alpha > 1$.

• p. 151, l. 8 after (3.1.17): **add entries in the subject index pointing here with the following four items**
ipoviscous NS, hyperviscous NS, ipoviscosity, hyperviscosity

- p. 152, l. 4 in prob. (3.1.2): $\equiv \int \underset{\sim}{u} \cdot \underset{\sim}{\partial}\, \underline{w}^2\, d\underline{x}$ **should be** $\equiv \frac{1}{2} \int \underset{\sim}{u} \cdot \underset{\sim}{\partial}\, \underline{w}^2\, d\underline{x}$

- p. 152, l. 6 in prob. (3.1.3): [2.2.20] **should be** [2.2.22]

- p. 152, l. 3 before prob. 3.1.4:[2.2.20] **should be** [2.2.22]

- p. 153, l. 2: [3.1.2] **should be** [3.1.3]

- p. 154, l. 8. in prob 3.1.10: $p > 0$ **should be** $p > 2d$

- p. 154, l. -8: quantities. **should be** quantities: *rather it is estimated in the shorter interval* $[0, \tau_{h+p}(R)]$.

- p. 154, l. 5 in prob. 3.1.9: **add in the subject index the item** constructive algorithm **pointing at l. 5**

- p. 155, l. 2/4: **the sentence after** points out **in l. 2 should be** points out that *although by abstract arguments* we can show existence and even some *a priori* bound on $||\underline{u}||_{W_k}$, in terms of t and $\underline{u}_0$, *nevertheless a constructive estimate* for $\underline{u}^N$ is not known.

- p. 155, l. 8.: **delete** , as in [3.1.6],

- p. 155, l. 8.: (whose use is strongly not recommended to everybody) **should be** (whose use we strongly disrecommend)

- p. 155, l. -4: **delete** of $\underline{u}$ solving the Euler equations and

- p. 155, l. 4 in prob. 3.1.11: $\nu = 0$ **should be** $\nu = 0, \underline{g} = \underline{o}$

- p. 155, l. 4 in prob. 3.1.11: (2.3.3) **should be** (2.3.3) with $\nu, \gamma = \underline{0}$

- p. 156, l. -3: The second one, $\underline{n} = \underline{0}$ term, is obtained **should be** The $\underline{n} = \underline{0}$ contribution to the second estimate

- p. 157, prob. 3.1.16: **Replace the entire text of problem by** **[3.1.16]:** (*Continuous dependence of the flow lines from the vorticity field*) There exists C_0 such that, defining $M_0 = ||\omega||_0$ and $\delta \overset{def}{=} e^{-C_0\, M_0\, T}$ for $T > 0$, then the currents generated by the two fields $\underline{v}^\zeta, \underline{v}^{\zeta'}$ with $\zeta, \zeta' \in \mathcal{M}_0$ are such that for all $\underline{x} \in \Omega$ and $s, t \in [0, T]$

$$|U^{\zeta}_{s,t}(\underline{x}) - U^{\zeta'}_{s,t}(\underline{x})| \le C_0\, M_0\, T\, \left(||\zeta - \zeta'||_0 M_0^{-1}\right)^{\delta} L$$

Hence the $\mathcal{R}$ can be thought of as defined on the closure $\overline{\mathcal{M}}_0$ of $\mathcal{M}_0$ with respect to the metric of the uniform convergence and it can be extended to all continuous vector fields ζ, without any differentiability property, verifying the (2),(3),(4) of problem [3.1.11]. (*Hint:* Setting $\rho_s = L^{-1}(U^{\zeta}_{s,t}(\underline{x}) - U^{\zeta'}_{s,t}(\underline{x}))$, note that $\rho_t = \underline{0}$; furthermore from problem [3.1.15], by subtracting and adding $\underline{v}^{\zeta}(U^{\zeta'}_{s,t}(\underline{x}), s)$, there exist constants C_1, C_2, C_0:

$$\begin{aligned}|\partial_s \rho_s| =& |L^{-1}|\underline{v}^{\zeta}(U^{\zeta}_{s,t}(\underline{x}), s) - \underline{v}^{\zeta'}(U^{\zeta'}_{s,t}(\underline{x}), s)| \le C_1 |\rho_s| \log_+ |\rho_s|^{-1} + \\ &+ L^{-1}|\underline{v}^{\zeta}(U^{\zeta'}_{s,t}(\underline{x}), s) - \underline{v}^{\zeta'}(U^{\zeta'}_{s,t}(\underline{x}), s)| \le \\ &\le C_1 |\rho_s| \log_+ |\rho_s|^{-1} + C_2 ||\zeta - \zeta'||_0 \le \\ &\le C_0 \big(|\rho_s| \log_+ |\rho_s|^{-1} + ||\zeta - \zeta'||_0\big)\end{aligned}$$

Since $0 \le s \le T$, by integration it follows that $|\rho_s| \le R$ if R is

$$\int_0^R \frac{d\rho}{\rho \log_+ \rho^{-1} + ||\zeta - \zeta'||_0 M_0^{-1}} = C_0 M_0 T$$

which means that R can be taken $R \le K(M_0 T)(||\zeta - \zeta'|| M_0^{-1})^{\delta}$ with $K(M_0 T)$ a continuous increasing function and $\delta = \exp - M_0 C_0 T$).

- p. 157, l. 7 in prob. 3.1.16: uniform so convergence it can **should be** uniform convergence and it can

- p. 167, l. -1: (2.2.20) **should be** Problem [2.2.20]
- p. 167, l. -1: $\underset{\sim}{\partial}\Upsilon$ **should be** $\underset{\sim}{\partial}\underline{u}$
- p. 168, l. 4: min **should be** max
- p. 169, l. 1 after (3.2.31): $\underline{\gamma}^{\infty}(t)$ **should be** $\underline{\gamma}^{\infty}_{\underline{k}}(t)$
- p. 169, l. 4 after (3.2.31): $(\nu t)^{-1}$ **should be** $(k_0^2 + (\nu t)^{-1})$
- p. 172, l. 4 in remarks: [3.1.5] **should be** [3.1.6]

- p. 173, l. -1: $|\underline{k}_1|$ o $|\underline{k}_2|$ in the summation **should be** $|\underline{k}_1|$ or $|\underline{k}_2|$

- p. 175, l. -7: reularity **should be** regularity

- p. 176, l. -8: does not *only* consist **should be** not *only* consists

- p. 176, l. -6: and therefore **should be** so that

- p. 176, l. -5: Indeed it is *conceivable* **should be**
It also consists in the *possibility*

- p. 177, in formula (3.3.3): $+i \sum_{\underline{k}_1+\underline{k}_2+\underline{k}=\underline{0}}$ **should be** $-i \sum_{\underline{k}_1+\underline{k}_2=\underline{k}}$

- p. 178, formula (3.3.5): $\max_{\underline{y}\in\Omega}$ **should be** $\max_{\underline{y}\in\Omega}$

- p. 179, l. 9: disappear) **should be** disappear) with $|\underline{k}| \le N$

- p. 179, l. 1 after (3.3.8):
where $E(t) = L^3 \sum_{\underline{k}} |\underline{\gamma}_{\underline{k}}(t)|^2 = \int |\underline{u}(t)|^2 d\underline{x}$ **should be**
where $L^3||\underline{\gamma}^{\lambda,N}(t)||_2^2 = \int |\underline{u}^{\lambda,N}(t)|^2\, d\underline{x}$

- p. 179, l. -3: we see that **should be**
we see that there is $C_{\alpha+1} < \infty$ such that

- p. 180, l. 6: loss regularity **should be** loss of regularity

- p. 180, l. 10: (3.2.12) **should be** (3.2.12) and (3.2.13)

- p. 181, formula (3.3.14): $\frac{F_m}{(L^{-2}\nu\,(t-t_0)^{\frac{m}{2}})}$ **should be** $\frac{F_m}{(L^{-2}\nu\,(t-t_0))^{\frac{m}{2}}}$

- p. 182, l. 1 before (3.3.19): [3.1.13] **should be** [3.1.15]

- p. 182, formula (3.3.19): **should have** an open parenthesis $\Big\{$ after the first summation symbol and a closed parenthesis $\Big\}$ at end of next line

- p. 182, l. 1 before (3.3.20): images(from **should be** images (from

- p. 182, l. 1 before (3.3.21): $rt \le L^2$ **should be** $\nu t \le L^2$, (*i.e.* $t \le T_c$)

• p. 182: **eq. (3.3.22) should be**

$$|\partial^{\underline{\alpha}}_{\underline{x}}\Gamma(\underline{x},t)|\ ,\ |\partial^{\underline{\alpha}}_{\underline{x}}T_{ij}(\underline{x},t)| \le \begin{cases} C_{|\underline{\alpha}|}\,(\underline{x}^2+\nu t)^{-(3+|\underline{\alpha}|)/2} & \text{for } \nu t\le L^2\\ C_{|\underline{\alpha}|}L^{-(3+|\underline{\alpha}|)}\,(L^2/\nu t)^{|\underline{\alpha}|/2} & \text{for } \nu t> L^2\end{cases} \tag{3.3.22}$$

• p. 183, l. 1: **delete l.1 until the period**

• p. 183, l. 2: problems [3.3.6] and [3.3.9] **should be** problems [3.3.9] and [3.3.10]

• p. 183, l. 4: and are **should be** which are

• p. 183, l. 1 before (3.3.24): we get **should be** we get, if $|\underline{\alpha}|=m$

• p. 183, l. -4: However **should be** However, for $T<T_c$,

• p. 183, l. -1: **add remark**
Remark: the condition $t<T_c$ appears because the inequalities have been derived supposing $t-\tau<T_c$ (*i.e.* $\nu(t-\tau)<L^2$).

• p. 184, l. 8: (3.2.29) **should be** (3.3.26)

• p. 184, l. 12: setting $F=$ **should be** setting $F\,'=$

• p. 184, l. 14:
for some small F, which **should be** which, by $\frac{a+b}{c+d}\ge\min(\frac{a}{c},\frac{b}{d})$,

• p. 184, l. 14:
$x<F\min\big(V_c/V_1,\,V_c/(xW_0)\big)$. **should be** $x<F\,'\min\big(V_c/V_1,\,V_c/(xW_0)\big)$.

• p. 184, l. 15: the line
The latter relation is equivalent to $x<F\min(1,(V_c/V_1),\sqrt{(V_c/W_0)}$.
should be deleted

• p. 184, l. 2 before eq. (3.3.30) last condition **should be** last condition and of the $t<T_c$ (cf. remark above)

• p. 184, l. following (3.3.30)
which has a **should be** for F small enough, which has a

• p. 185, l. -4: identity (hence making also (3.2.6) an identity almost everywhere in $t>t_0$).

should be
identity, as well as (3.2.6), almost everywhere in $t > t_0$

- p. 185, l. -7: Remark **should be** *remarks:* (1)

- p. 185, l. -3: **add remark 2:**

(2) If there is no force all Leray's solutions will become eventually smooth, see problem [3.3.4]. These are examples of several results of Leray on global existence (see also problem [3.3.5]). The conclusion is that we miss an existence and uniqueness theorem under *general* initial data.

- p. 186, l. 7: estimate (3.2.12) **should be** estimate (3.3.8)

- p. 187, in l. following formula (3.3.39): λ **should be** the sequence λ_j

- p. 188, l. 11: sinluarities **should be** singularities

- p. 188, l. 3 after (3.3.35): (3.2.8) **should be** (3.3.8)

- p. 188, l. -16: derivative having **should be** derivative *without* having

- p. 188, l. -10: $(\underline{x}, t) \in U_\lambda(\underline{x}_0, t_0)$ **should be** $(\underline{x}, t) \in U_\rho(\underline{x}_0, t_0)$

- p. 188, l. -4:

regularity. **should be** regularity (cf. Sec. (3.2) Proposition 5).

- p. 189, l. 1 before Proposition 6:

[3.3.10], [3.3.11] **should be** [3.3.10], [3.3.11], [3.3.12]

- p. 190, l. -13:

(cf. (3.2.11) **should be** (cf. Definition 2 in Sec. (3.2))

- p. 190, l. -8:

The strongest **should be** Even the strongest

- p. 191, first line in formula (3.3.50) **should become**

$T_0 = \bar{F} T_c\,(R^4 + R_g^2)^{-1} = \bar{F}\tilde{T}_c\,(\tilde{R}^4 + \tilde{R}_g^2)^{-1}$,

- p. 191, l. -7: corresponding and the velocity **should become** corresponding velocity

- p. 192, l. 6 in probl. 3.3.2: by (3.3.20) **should be** by (3.3.21)

• p. 192, (3.3.52): **should be**

$$\sup_{\underline{y}} |\partial_{\underline{y}} \underset{\sim}{T}(\underline{y}, t-\tau)| \int_\Omega d\underline{y} |\langle \underline{u}(\underline{y},\tau)\rangle_\lambda| \, |\underline{u}(\underline{y},\tau)| \le \frac{L^{-4}E(\tau)}{((t-\tau)/T_c)^{\frac12}} C_1 \qquad (3.3.52)$$

• p. 192, formula (3.3.55): $\frac{B_3}{((t-\tau)/T_c)^2}$ **should be** $\frac{B_3}{((t-\tau)/T_c)^{\frac12}}$

• p. 193, l. 2 after (3.3.56):
cf. the second of (3.3.8) **should be** as in (3.2.14) with $\underline{f} = \underline{0}$

• p. 193, l. 2 after (3.3.56): nk_0^2E **should be** $-\nu k_0^2 E$

• p. 193, l. 2 before problem 3.3.5: $/2\nu T_c\big)^{-2}$ **should be** $/2\nu T_c V_c^2\big)^{-2}$

• p. 193, l. 1 of problem 3.3.5:
that if **should be** that if, for p large enough,

• p. 193, l. 2 of problem 3.3.5:
$\left(\frac{E(0)^{1/2}}{V_c}\right)^{p-3}$ **should be** $\left(\frac{(L^{-3}E(0))^{1/2}}{V_c}\right)^{p-3}$

• p. 193, l. -2: then **should be** then, for $\underline{g} = \underline{0}$

• p. 194, l. 2: see (3.2.2), (3.2.3) **should be** see (3.1.2), (3.1.3)

• p. 194, problem 3.3.8: **close parenthesis**) **at the end**

• p. 195, l. 7,
well chosen well **should be** suitably chosen we use the bound (3.2.22) for Γ

• p. 196, l. 1 before problem 3.3.12: **add**
, furthermore the Hölder exponent $\frac12$ in Proposition 5 can be replaced by any η with $\eta < 1$.

• p. 196, l. 7 after (3.3.61): by integration by parts. **should be**
because of the periodicity of the $\underline{y}$ integration.

• p. 196, l. 1 before Bibliography: majorizations). **should be**
majorizations. In a similar way one can check that the derivative is continuous).)

• p. 197, l. 12 : $\le \overline{C}_\alpha M_\alpha$ **should be** $\le \overline{C}_\alpha M_\alpha L^\alpha$

• p. 198, l. 1 in definition 2: Given a set **should be** Given a Borel set

• p. 199, l. 2 after (3.4.9):
(if $\gamma = \frac12$ then this means $2A^2 < F$). **should be**
: if $\gamma = \frac12$ we take $A^2 = \frac12 F$, for instance.

• p. 200, l. 2 after equ. (3.4.13): Sec. 3.2, **should be** Eq. (3.3.8),

• p. 201, l. -1: in question **should be**
in question, see Proposition II in Sect. §3.3 and remark (2) following it,

• p. 202: (3.4.16) **should be**

$$\underline{x} \in S(\underline{x}, r) \qquad \text{and} \qquad t \in (\vartheta, \vartheta + T_{cr} \min(1, \frac{F}{R_r^4 + R_{gr}^2})) \tag{3.4.16}$$

• p. 202, l. 2: **delete l. 2, completely**

• p. 202, l. 22: the integral would have size $O(r^3)$). **should be**
R_r^2 would have size $O(r^4)$.

• p. 202, in formula (3.4.18): $\frac{\nu\, d\vartheta}{r_i^2} \int_{t-r_i^2/\nu}^t$ **should be** $\frac{\nu}{r_i^2} \int_{t-r_i^2/\nu}^t d\vartheta$

• p. 203, l. 10-11:
at distance $\le 6r_i$ from the central point of F_i covers. **should be**
such that $|\underline{x}' - \underline{x}_i| < 5r_i$ and $|t' - t_i - \nu^{-1}\frac12 r_i^2| < \nu^{-1}(5r_i)^2$ covers

• p. 203, l. 13-14: 18 **should be** 11

• p. 203, l. 14: 36 **should be** 11

• p. 203, l. -2: $5, 6, 18$ **should be** $5, 11$

• p. 203, first line of formula (3.4.22): **should be**

$$\limsup_{r\to 0} \frac{\nu}{r^2} \int_{t-r^2/2\nu}^{t+r^2/2\nu} R_r(\vartheta)^2 < \varepsilon, \qquad \text{or}$$

• p. 204, l.2 after (3.4.22): **delete** is (much) weaker than (3.4.17) and it

• p. 204, l. 1 of Remark: **delete** An interesting question is whether (3.4.17) itself is true. However a conjecture **and replace by** A conjecture

• p. 204, l. -5: $5F_i$ **should be** λF_i

• p. 205, in problem 3.4.2: **delete** then the result still holds if one replaces $5F_i$ with λF_i where λ is a suitable homothety factor (with respect to the center of F_i). Show that if $\alpha = 1, 2$ then $\lambda = 5$ is enough (and, in general, $\lambda = (4^2 + 2^{2(1+\alpha)/\alpha})^{1/2}$ is enough). **and replace by**
then the result of problem [3.4.1] still holds if one interprets λF_i as the parabolic cylinder obtained by applying to F_i a homothety, with respect to the center of F_i, of scale λ on the first k coordinates and λ^2 on the others. Check that if $\alpha = 2$ the value $\lambda = 5$ is sufficient.

• p. 204: l. 6 in problem 3.4.3: $1 < 23^{-\alpha}$ **should be** $1 < 2 \cdot 3^{-\alpha}$

• p. 206, l. 1 before (3.5.2): implies that **should be**
implies, see below, that

• p. 207, in formula (3.5.3): $1 \le \alpha \le \infty$ **should be** $1 \le \alpha \le \frac{3}{2}$

• p. 207, in formula (3.5.5): **smaller modulus signs in the l.h.s.**

• p. 208, the remark on l. 7 **should be** *Remark:* (H) are a trivial extension of the Schwartz-Hölder inequalities; while (S) and (P) (mainly in the cases, important in what follows, $q = 6$ and $\alpha = \frac{3}{2}$) and (CZ) are less elementary and we refer to the literature, footnote at p.43 [So63], [St93], and p. 213, 219 of [LL01].

• p. 209, last l. in formula (3.5.10): $|\underline{u}|^2 \underset{\sim}{u} \cdot \underset{\sim}{\partial} \varphi$ **should be** $\frac{1}{2} |\underline{u}|^2 \underset{\sim}{u} \cdot \underset{\sim}{\partial} \varphi$

• p. 210, last line of (3.5.11): **delete the l.h.s., i.e., should be just**

$$= \Delta_r(t_0) \times B_r(\underline{x}_0) \equiv Q_r \tag{3.5.11}$$

• p. 211, in formula (3.5.19): : $(\underline{x}, t) \in Q_{r/2}$ **should be** $(\underline{x}, t) \notin Q_{r/2}$

• p. 212, l. 2: is the “local **should be** is an average of the “local

• p. 212, l. 3: see (3.4.22) **should be** (3.4.14), (3.4.22)

• p. 213, l. 2 before (3.5.24): **delete** , *for any* $p > 0, 1 > \delta > 0$

• p. 213, l. 1 before (3.5.24):
(if $p > 0$ and $0 < \delta < 1$) **should be** ,if $p > 0$ and $0 < \delta < 1$,

• p. 215, l. 13 and p. 216: **replace the text following l. 12 in p. 215 through the entire p. 216 by the following**
The relation (3.5.26) can be "iterated" by using the expressions (3.5.25) for G_{n+p}, J_{n+p} and then the first of (3.5.25) to express $G_{n+p}^{1/5}$ in terms of A_{n+2p} with n replaced by $n+p$:

$$\begin{aligned}\alpha_n \;\le\; C\,(&2^{-p}\alpha_{n+2p} + 2^{3p}\delta_{n+2p}^{1/2}\alpha_{n+2p}^{1/2} + \\ &+ 2^{p/5}(\alpha_{n+2p}\kappa_{n+2p})^{1/2} + 2^{7p/5}\delta_{n+2p}\alpha_{n+2p}^{7/20}\kappa_{n+2p}^{1/2} + \\ &+ 2^{3p}\delta_{n+2p}\alpha_{n+2p})\end{aligned} \tag{3.5.27}$$

It is convenient to take advantage of the simple inequalities $(ab)^{\frac12} \le za + z^{-1}b$ and $a^x \le 1 + a$ for $a, b, z, x > 0$, $x \le 1$.
The (3.5.27) can be turned into a relation between α_n and $\alpha_{n+p}, \kappa_{n+p}$ by replacing p by $\frac12 p$. Furthermore, in the relation between α_n and $\alpha_{n+p}, \kappa_{n+p}$ obtained after the latter replacement, we choose $z = 2^{-p/5}$ to disentangle $2^{p/10}(\alpha_{n+p}\kappa_{n+p})^{1/2}$ we obtain recurrent (generous) estimates for α_n, κ_2

$$\begin{aligned}\alpha_n \;&\le C\,(2^{-p/10}\alpha_{n+p} + 2^{3p/10}\kappa_{n+p} + \xi_{n+p}^{\alpha})\\ \kappa_n \;&\le C\,(2^{-4p/5}\kappa_{n+p} + \xi_{n+p}^{\kappa})\\ \xi_{n+p}^{\alpha} &\stackrel{def}{=} 2^{3p}\delta_{n+p}(\alpha_{n+p} + \kappa_{n+p} + 1)\\ \xi_{n+p}^{\kappa} &\stackrel{def}{=} 2^{3p}\delta_{n+p}\alpha_{n+p}\end{aligned} \tag{3.5.28}$$

We fix p once and for all such that $2^{-p/10}C < \frac13$.
Then if $C2^{3p}\delta_n$ is small enough, *i.e.* if δ_n is small enough, say for $\delta_n < \overline{\delta}$ *for all* $|n| \ge \overline{n}$, the matrix $M = C\begin{pmatrix} 2^{-p/10} + 2^{3p}\delta_{n+p} & 2^{3p/10} + 2^{3p}\delta_{n+p}\\ 0 & 2^{-4p/5} + 2^{3p}\delta_{n+p}\end{pmatrix}$ will have the two eigenvalues $< \frac12$ and iteration of (3.5.27) will contract any ball in the plane α, κ to the ball of radius $2\,\overline{\delta}$.
If α_n, κ_n are bounded by a constant $\overline{\delta}$ for all $|n|$ large enough the (3.5.25) show that also g_n, j_n are going to be eventually bounded proportionally to $\overline{\delta}$.
Hence by imposing that δ is so small that r.h.s. of (3.5.28) is ρ we see that proposition IV holds.

• p. 218, l -13: $\frac{C}{\rho^3}$ **in last l. of formula should be** $\frac{C\,r^{9/5}}{\rho^3}$

• p. 219, l. 3 in prob. 3.5.4: $|\Delta\varphi| + \partial_t\varphi \le \frac{C}{\rho^2}$ **should be** $|\Delta\varphi + \partial_t\varphi| \le \frac{C}{\rho^2}$

- p. 220, l. 2: $|\underset{\sim}{\partial}\underline{\varphi}|$ **should be** $|\underset{\sim}{\partial}\varphi|$

- p. 220, l. 4: $\underline{u}|^3$ **should be** $|\underline{u}|^3$

- p. 220, l. 13: $B_{r_n} = B_n, \Delta_{r_n} = \Delta_n.Q_{r_n} = Q_n$ **should be**
$B^0_{r_n} = B^0_n, \Delta^0_{r_n} = \Delta^0_n, Q^0_{r_n} = Q^0_n$

- p. 221, l. 2: $\tilde{f}(r_{n_0}^{-1}\underline{x}, r_{n_0}^{-1/2}t)$ **should be** $\tilde{\varphi}(r_{n_0}^{-1}\underline{x}, r_{n_0}^{-1/2}t)$

- p. 221, l. 5: ${G^0_{n+p}}^{2/3}$ **should be** ${G^0}^{2/3}_{n+p}$

- p. 222, l. 13: for the use $\underline{x}, t$ **should be** for $\underline{x}, t$

- p. 222, l. 11: $(\underline{\partial}\chi)$ **should be** $(\underline{\partial}\chi_{n_0})$

- p. 223, l. 2: $\sum_{k=n+1}^{n_0-1}$ **should be** $\sum_{k=n+1}^{n_0}$

- p. 223, l. 13: $\int_{\Delta^0_n{}^0}$ **should be** $\int_{\Delta^0_n}$

- p. 223, l. 15: $= r_n^5 {G^0_n}^{1/3} \sum_{k=n+1}^{n_0-1} \frac{A^0_m}{r^3_m}$ **should be** $\le r_n^5 {G^0_n}^{1/3} \sum_{k=n+1}^{n_0} \frac{A^0_m}{r^3_m}$

- p. 224, l. 3-4 in problem [3.5.10]:
the first two sums $\sum_{k=n+1}^{n_0-1}$ **should be** $\sum_{k=n+1}^{n_0}$

- p. 224, l. 4 in problem [3.5.10]:
$\sum_{k=n+1}^{n_0-1} 2^k T_k^{1/2} \sum_{p=k}^{n_0-1} 2^{-3p} T_p$ **should be** $\sum_{k=n+1}^{n_0-1} 2^k T_k^{1/2} \sum_{q=k}^{n_0} 2^{-3q} T_q$

- p. 225: l. 2: **text should be**
Assuming $\alpha = 1$ prove (P). (*Hint:* Change variables as $\underline{y} \to \underline{z} = \underline{y} + (\underline{x} - \underline{y})s$ so that for α integer

- p. 225, l. 4: $\frac{ds}{(1-s)^2}$ **should be** $\frac{ds}{(1-s)^3}$

• p. 225, l. 13: $\frac{ds}{(1-s)^{1-3/\alpha}}$ **should be** $\frac{ds}{(1-s)^{3-3/\alpha}}$

• p. 225, l. 14:
getting (P) and an explicit estimate of the constant C^P_α.
should be
getting (P) and an explicit estimate of the constant C^P_α only for $\alpha = 1$ and a hint that (P) should hold for $\alpha < \frac{3}{2}$ at least.

• p. 225, second l. in Problem [3.5.14]:
Use this to get (P) for each $1 \le \alpha < \infty$.
should be
Use this to get (P) for each $1 \le \alpha < \alpha_0$ if it holds for $\alpha = \alpha_0$.

• p. 225, in problem [3.5.14]: **delete the entire hint**

• p. 225, l. 8 in Problem [3.5.15]: l **should be** λ

• p. 227, l. -11: this is true **should be** (this is true

• p. 229, l. 19: is time independent. **should be** is time independent (and of course smooth).

• p. 230, l. 3 in Sec. 4.1.1: , the trivial "*thermostatic solution*", **should be** (*i.e.* the trivial "*thermostatic solution*")

• p. 232, l. -3: Supposing ϑ **should be** Supposing that ϑ

• p. 235: l. 1 before E.(4.1.23): spectral form **should be**
spectral form, see (3.2.6), (3.2.26),

• p. 235, in Eq. (4.1.23): $g_{\underline{k}}$ **should be** $\Pi_{\underline{k}} g_{\underline{k}}$

• p. 235, l.1 before Eq. (4.1.24): The velocity field **should be** The velocity and external force fields are, respectively,

• p. 235, Eq. (4.1.24): **should be**

$$\underline{u}(\underline{x}) = \sum_{\underline{k}\neq\underline{0}} \gamma_{\underline{k}} \frac{\underline{k}^\perp}{|\underline{k}|} e^{i\underline{k}\cdot\underline{x}}, \qquad \underline{g}(\underline{x}) = \sum_{\underline{k}\neq\underline{0}} g_{\underline{k}} \frac{\underline{k}^\perp}{|\underline{k}|} e^{i\underline{k}\cdot\underline{x}}, \tag{4.1.24}$$

• p. 238, l. 1 in probl. 4.1.9: the solutions **should be**
the time independent solutions

• p. 238, l. -3: for the fourth order equation the eigenvalues **should be** the fourth order equation for the eigenvalues

• p. 239, l. 2: no periodic **should be** no stable periodic

• p. 239, l. 2 in problem 4.1.12: the *shortest mode* $\underline{k}_0 = (0,1)$ is the sole **should be** the *shortest modes* $\underline{k}_0 = (0,\pm 1)$ are the sole

• p. 239: **add the following hint to problem 4.1.13**
(*Hint:* Yes (*Marchioro's theorem*). Write (as suggested by *Falkoff*) the NS equations in dimension 2 and show that if $\Delta \stackrel{def}{=} \sum_{|\underline{k}|>k_0} (\frac{\underline{k}^2}{k_0^2} - 1)|\gamma_{\underline{k}}|^2$ then $\frac12 \dot\Delta = \sum_{|\underline{k}|>k_0} \underline{k}^2 (\frac{\underline{k}^2}{k_0^2} - 1)|\gamma_{\underline{k}}|^2$, hence $\frac12\dot\Delta \le -\nu k_0^2 \Delta$ and $\Delta(t) \le e^{-2\nu k_0^2 t}\Delta(0)$, *etc.*; see also proposition 4 in Sec. 3.2)

• p. 239, l. 3 in Bibliography:
The last two problems admit a remarkable generalization:
should be
The last problem is remarkable:

• p. 240, l. -5: in Chap. 7. **should be** in Sect. 5.5.3 and in Chap. 7.

• p. 242, l. -17: Laplacian **should be** Laplace

• p. 242, l. -13: Poincarè **should be** Poincaré

• p. 243, l. 8: modelled **should be** modeled

• p. 244, l. -7: should **should be** could

• p. 246, l. 2: [4.1.4], [4.1.5] **should be** problems [4.1.4], [4.1.5], [4.1.13].

• p. 246, l. 6: **delete** of Sect. 4.1

• p. 249, l. 2 in Example 1: $f(x) = r + ax^2$ **should be** $f(x) = r - ax^2$

• p. 248, l. 22: modelled **should be** modeled

• p. 251, l. 1 of example 5: map **should be** *map*

• p. 252, l. 3 of example 6: One expects, therefore, in such systems **should be** Therefore, in such systems, one expects

• p. 253, l. 2: phenomena. **should be**
phenomena occurring in incipient turbulence.

• p. 254, l. 8-9: of “nongeneric” bifurcations with **should be**
of bifurcations “non generic” because of

• p. 256, l. 4: two small real part conjugate eigenvectors **should be**
two conjugate eigenvectors with small real part eigenvalues

• p. 257, l. 8: [4.2.10] **should be** problem [4.2.10]

• p. 257, l. 4 in prob. 4.2.6: $(-\gamma)^{-1}$ **should be** γ^{-1}

• p. 257, l. 2 before prob. 4.2.8: with C constant. **should be**
with C a suitable constant..

• p. 257, l. -7: polynomial **should be** polynomial in r

• p. 258, l. 3-4 in prob. 4.2.10: written as **should be**
written, if $r_c = 0$, as

• p. 262, l. 8: becomes confused; one says it “collides” with **should be**
merges (one says it “collides”) with

• p. 262, l. -6: should **should be** could

• p. 262, l. -5: took **should be** take

• p. 263, l. 22: see [4.3.6] **should be** see problem [4.3.6]

• p. 265, l. 13: of an eigenvalue **should be** by an eigenvalue

• p. 265, l. 22: is essentially **should be** might still be essentially

• p. 270, l -16: one real eigenvalue **should be**
one real or two conjugate eigenvalues

• p. 271, l. 12 and 14: in the correspondence of **should be** on
i.e. delete "in the correspondence", replace "of" by "on"

• p. 273, l. 9 in prob, 4.3.3: $g^n(2\pi\, k) = g(0) + 2\pi\, k$ **should be**
$g^n(2\pi\, k) = g^n(0) + 2\pi\, k$

• p. 286, l. 9: **add the following paragraph**
Note that there is a very good reason for the choice of the mode on which the forcing acts: the simplest choice, *i.e.* the mode with minimum length $\underline{k}_0 = (0, \pm 1), (\pm 1, 0)$, is not sufficient to create interesting phenomenology: see problem [4.1.13]. However any other choice of the mode seems sufficient to generate interesting phenomenologies. How much the results depend on which mode is actually chosen for the forcing has not been studied: however it seems that the choice does not affect "substantially" the phenomenology and it would be interesting to clarify this point. Also the restriction that the components $\gamma_{\underline{k}}$ are real or imaginary (rather than complex) is an important issue partially studied in the literature and deserving further investigation.

• p. 288, l. 1: $O(u)$ **should be** $O(\underline{u})$

• p. 288, l. after (5.12): are **should be** were

• p. 291, l. -16 and -22: in L_1 **should be** locally in L_1

• p. 291, l. -9: $A_T(p)$ **should be** $A(p)$

• p. 293, l. 13: $y^2(dx^2 + dy^2)$ **should be** $y^{-2}(dx^2 + dy^2)$

• p. 296, l. -3: holds. **should be** hold in L_1 norm.

• p. 297, l. 1: $e^{2\pi\, i\, \nu\, x}$ **should be** $\hat{f}_\nu\, e^{2\pi\, i\, \nu\, x}$

• p. 297, l. 2: $|\hat{f}_\nu$ **should be** $|\hat{f}_\nu|$

• p. 297, l. 5: $|| \ge \varepsilon > 0$ **should be** $||_{L_1} \ge \varepsilon > 0$

• p. 297, l. 3 in problem 5.1.7: R_p **should be** R_k

• p. 298, l. 2 in problem 5.1.12:
$\alpha_{j-1} < \alpha < \alpha_{j+1}$ **should be** $\alpha_{j-1} > \alpha > \alpha_{j+1}$

- p. 298, l. -2: a p and **should be** numbers p and

- p. 299, last l. in problem 5.1.17: $q <\leq q_n$ **should be** $q \leq q_n$

- p. 299, l. 3 in problem 5.1.20: Viceversa **should be** However

- p. 299, l. 4 in problem 5.1.20: only hold **should be** even hold

- p. 299, l. 4 in problem 5.1.20:
are uniformly **should be** are not uniformly

- p. 302, l. 4 in problem 5.1.30: it maps **should be** they map

- p. 302, l. before last in problem 5.1.31: $-R^2$ **should be** R^2

- p. 302, l. -7: $z' = z\Gamma$. The **should be** $z' = z\Gamma$, if the

- p. 303, l. 2: Poincarè **should be** Poincaré

- p. 303, l. 1 in problem 5.1.35: 5.1.31 **should be** 5.1.34

- p. 303, l. 3 in problem 5.1.35:
in the sense of 5.1.28 **should be** in the sense of 5.1.31

- p. 303, l. before last in problem 5.1.35:
Fig. 5.5.1 **should be** Fig. 5.1.2

- p. 303, **change figure Fig 5.1.1**

- p. 303, l. 2 in Problem 5.1.34
$j\pi/4$ **should be** $\pi/8 + j\pi/4$

- p. 303, l. 3 in Problem 5.1.34:
$\pi/8 + \pi/4$ **should be** $\pi/8 + \pi/2$

- p. 303, l. 1 in Problem 5.1.35: 5.1.31 **should be** 5.1.34

- p. 304, l. -2: Show that **should be** Consider

- p. 305, l. 3 in Problem 5.1.40: z **should be** z in Σ_8

- p. 306, l. 4 in Problem 5.1.42: or irreducible **should be** of irreducible

- p. 306, l. 3 in Problem 5.1.43:
described in 5.1.39 **should be** described in [5.1.41]

- p. 306, l. -4: negative **should be** negative curvature

- p. 308, l. 15: Poincarè **should be** Poincaré

- p. 309, l. -6 : is L_1 **should be** is locally L_1

- p. 313, l. 11: **add** If g is the identity it will be omitted.

- p. 313, l. -11: with the map $S:$ **should be** with the map $S':$

- p. 313, l.-10: close **should be** similar

- p. 314, l. 21: this dependence **should be** such dependence

- p. 315, l. 4: the image of Q **should be** which is the image of Q

- p. 315, l. 5: map F, by simply setting **should be** map F: one simply sets

- p. 315, l.10: will not be. **should be** will not be absolutely continuous.

- p. 316, l. 3 in problem 5.2.1: $\tilde{S}$ of $[0,1]$, **should be** $\tilde{S}$, of $[0,1]$,

- p. 319, l. 16: written **should be** realized

- p. 320, l. 2: which to **should be** which with

- p. 320, l. 5 in definition 1: a singular **should be** a singularity

- p. 321, l. 2: not singular **should be** not a singularity point

• p. 323, l. 6: **delete** (see also the concluding remarks)

• p. 324, remark (ii): not constant **should be** positive and not constant

• p. 327, l. -10: say **should be** says

• p. 329, l. -18: $a\lambda_2 + b\lambda_1/(\lambda_1 + \lambda_2)$ **should be** $(a\lambda_2 + b\lambda_1)/(\lambda_1 + \lambda_2)$

• p. 329, l. -17: $a\lambda_1 + b\lambda_2/(\lambda_1 + \lambda_2)$ **should be** $(a\lambda_1 + b\lambda_2)/(\lambda_1 + \lambda_2)$

• p. 329, l. 2 in problem [3.3.6]: Q_O, Q_P **should be** Q_{O_0}, Q_{P_0}

• p. 329, l. -6, and l. -4: Q_O **should be** Q_{O_0}

• p. 329, l. 3 in problem [5.3.8]:
attaching a different label to each trajectory of an
should be
labeling each trajectory by one of its points in an

• p. 332, l. 1:
which we suppose has **should be** and we suppose that S has

• p. 332, l. 4:
two independent transversal **should be** two transversal

• p. 332, l. 16:
Remark: These systems ... **should be**:
Remarks: (i) The conditions (5.4.1) imply S-*covariance* of the decomposition $T_x = R^s(x) \oplus R^i(x)$, *i.e.* $R^\alpha(Sx) = \partial_x S(x) R^\alpha(x)$ for $\alpha = u, s$.
(ii) These systems ...

• p. 333, l. 12:
definition 1 of §5.2. **should be**
definition 1 of §5.2, illustrated in Fig. (5.2.1).

• p. 333, l. 18: orbit. The latter **should be** orbit: the latter

• p. 333, l. 5 in remark (iii):
every $x \in M$ tends to **should be** every $x \in M$ evolves towards

• p. 333,, l. 3 in remark (iv):
is *topologically mixing*, if **should be** is *topologically mixing on C*, if

• p. 334, l. 2 in definition 4:
and **should be** and, calling Ω the nonwandering set,

• p. 334, l. 3 in definition 4:
Denote by **should be** Denote, for $x, x' \in \Omega$, by

• p. 335, l. 3: satisfies **should be** satisfying

• p. 335, l. 17: has led **should be** have led

• p. 336, l. 3: derivatives **should be** derivative

• p. 337, l. 4: (i) **should be** (1)

• p. 337, l. 3 of definition 6:
definition 3, Sec. 5.3), **should be** definition 1, Sec. 5.3),

• p. 338, l. 22: definition 3, (c), **should be** definition 5, (b),

• p. 342, l. 12: $IS = S^{-1}I$. **should be** $IS = S^{-1}I$, cf. §7.1.

• p. 342, l. 21: cf. Sect. (6.1) **should be** cf. Sect. (7.1)

• p. 342, l. -19: the planes $\tilde{V}_j$ are
should be the planes $\tilde{V}^{(i)} = W_1 \oplus W_2 \oplus \cdots W_j$ are

• p. 342, l. -18: $\overline{\tilde{V}}_j^{(i)}$ **should be** $\overline{\tilde{V}}^{(i)}$

• p. 342, l. -15:
with $\tilde{V}_j(y)$ being the tangent plane to $\overline{\tilde{V}}_j$
should be
with $\tilde{V}^{(i)}(y)$ being the tangent plane to $\overline{\tilde{V}}^{(i)}$

• p. 342, l. -15 and -11: for almost all y. **should be** for almost all x.

• p. 342, l. -14:

Likewise if $\lambda_j < 0$ and $\lambda_{j-1} > 0$ there is, **should be** Likewise there is,

- p. 342, l. -13: $\overline{V}_j^{(s)}$ **should be** $\overline{V}^{(s)}$

- p. 342, l. -10: the plane $V_j(y)$ **should be**
the plane $V^{(s)}(y) = W_{j+1} \oplus W_2 \oplus \cdots W_n$

- p. 342, l. -2: are in a finite **should be** are *not* in a finite

- p. 343, l. -15: finte **should be** finite

- p. 344, l.8 before problem 5.4.7: e **should be** be

- p. 344, l. 2 in problem 5.4.7:
and **should be** and μ–almost everywhere it is

- p. 345, l. 13: $f_n(S^{-n}_x)$ **should be** $f_n(S^{-n}x)$

- p. 346, l. -1: $||(Au)^2||$, **should be** $||A\,u||^2$,

- p. 347, l. 8: $r' = 1$ and $r = 2$)of **should be** $r = 1$ and $r' = 2$) of

- p. 354, l. 23-24:
and $W_x^{\delta,i}$ where at each
should be
and $W_x^{\delta,i}$. Such surfaces

- p. 354, l. 28: $d(S_x^{-n}, S_y^{-n})$ **should be** $d(S^{-n}x, S^{-n}y)$

- p. 354, l. 31: then **should be** the

- p. 357, l. 3: more and nore **should be** more and more

- p. 358, in eq. (5.5.5): $f(\sigma)$ **should be** $f(\xi, \sigma)$

- p. 358, l. 2 after (5.5.5):
the surface element on $W_\xi^{\delta,i}$ and
should be
the surface element on $W_\xi^{\delta,i}$, a point $y \in \Delta$ located on the surface element

$d\sigma \in W_{\xi}^{\delta,i}$ is denoted (ξ, σ) and

- p. 360, l -7: requisite, *i.e.* **should be** requisites, *e.g.*

- p. 360, l -7: is also **should be** can also

- p. 365, l. 3 in problem 5.5.2: rotates, as a double pendulum, **should be** rotates as "double",

- p. 365, l. 1 in problem 5.5.3: write **should be** write (symbolically)

- p. 366, last l. in 5.5.5: if the (*) of [5.5.4] hold, **delete**

- p. 366, l. 1 in problem [5.5.6]:
If (M, S) **should be** If a differentiable dynamical system (M, S)

- p. 366, l. 3 in problem [5.5.8]: $\sqrt{2}$ **should be** 2

- p. 367, l. 1 and 2: $\{-1, 1\}^{Z}$ **should be** $\{-1, 1\}^{Z_+}$

- p. 370, l. 4: an *ergodic* **should be** a μ-regular *ergodic*

- p. 370, l. 4: Sect. 5.3 Definition 5, **should be**
Sect. 5.4 Definition 5 and Sect. 5.3 Definition 5,

- p. 370, l. -7: **delete** (to the left)

- p. 371, l. 7: for every j, it **should be** for every j it

- p. 371, l. 15: **delete** defined above

- p. 371, l. 15: **delete** a dynamical system

- p. 371, l. -8: for every **should be** from every

- p. 372, l. 12: if every continuous function f **should be** if every pair of continuous functions f, g

• p. 372, formula (5.6.6): **should be**

$$\Omega_{f,g}(n) = \int_{\mathcal{S}} \mu_{\underline{\sigma}}(d\underline{\sigma}')\, f(\underline{\sigma}')g(\tau^n\underline{\sigma}') \xrightarrow[n\to\infty]{} \langle f\rangle_{\underline{\sigma}}\langle g\rangle_{\underline{\sigma}} \tag{5.6.6}$$

• p. 372, l. 1 after equation (5.6.6):
i.e. $(\mathcal{S},\tau,\mu_{\underline{\sigma}}$ is mixing. **should be**
where $\langle\cdot\rangle_{\underline{\sigma}}$ denotes integration with respect to $\mu_{\underline{\sigma}}$, *i.e.* $(\mathcal{S},\tau,\mu_{\underline{\sigma}})$ is mixing.

• p. 373, l. 1 before (5.6.9): of this string satisfies **should be** satisfies

• p. 373, l. -9: $C\tau\underline{\sigma}' \equiv \tau C\underline{\sigma}'$; **should be** $C\tau\underline{\sigma} \equiv \tau C\underline{\sigma}$;

• p. 378, last l. in problem 5.6.1: ergodic.) **should be** ergodic. See also problem [5.2.3].)

• p. 378, l. -2: S **should be** s

• p. 379, l. 6 in problem 5.6.5:
$\mathcal{F}^j_{\sigma_0,\ldots,\sigma_{k-1}} \subset S^j\partial P_{\sigma_j}$ **should be** $\mathcal{F}^j_{\sigma_0,\ldots,\sigma_{k-1}} \subset \cup_j S^j\partial P_{\sigma_j}$

• p. 379, l. 8 in prob. 5.6.5: $\cup_{j=0}^{k-1} S^j \cup_\sigma P_\sigma$. **should be** $\cup_{j=0}^{k-1} S^j \cup_\sigma \partial P_\sigma$.

• p. 379, l. 8 in prob. 5.6.5:
$\max_{x,\xi} |\partial S_x\xi|/|\xi|$ **should be** $\max_{x,\xi,\delta=\pm1} |\partial S_x^\delta\xi|/|\xi|$

• p. 380, l. -15: among last problems **should be** among the last few problems

• p. 383, l. 11: , via **should be** via

• p. 387, l. 16-17:
largest side of the rectangles, $\le\delta$, *divided* ...
should be
largest side δ of the rectangles *divided* ...

• p. 387, l. -9: after the amplification of
should be after amplification by

• p. 389, l. 4: the definition **should be** the following definition

• p. 389, l. -15: pavement and parallel **should be** pavement parallel

• p. 389, l. -6: **newline before** Analogously

• p. 390, , l. -4: However ... **should be displaced to l. -8 between** neighbours **and** Furthermore

• p. 391, l. -5: cf. eq. (5.6.13) **should be** ,cf. eq. (5.6.13),

• p. 395, l. -15: Let $\Lambda_{e,T(x)}, \Lambda_{s,T(x)}$ **should be** Let $\Lambda_{e,T}(x), \Lambda_{s,T}(x)$

• p. 396, before Problems: **add**
A detailed analysis of the proof of the above theorems and of the construction of the Markov pavements for two dimensional Anosov maps can be found in [GBG04]

• p. 396, l. -9: $j = 1, \ldots, n-1$ **should be** $j = 0, \ldots, n-1$

• p. 397, l. 5 in problem 5.7.5: and C **should be** and C, λ

• p. 397, formula in l. 4 before problem 5.7.6:
replace σ_1 **and** σ'_1 **by** σ_0 **and** σ'_0 **respectively** (six times total)

• p. 397, last l. in problem 5.7.5: **close parenthesis**) **at line end**

• p. 397, l. -12: and hence show that is **should be** and hence such is

• p. 397, l. -10: $h\nu_0$ **should be** $\nu \stackrel{def}{=} h\nu_0$

• p. 398, l. 1:
the maximum of f,
should be
the maximum of f and $\nu = h\,\nu_0$ is the measure defined in problem [5.7.8],

• p. 398, l. 1: $||\mathcal{L}^k f|| \le B^{-1} \ldots$ **should be** $||\mathcal{L}^k f|| \ge B^{-1} \ldots$

• p. 398, l. 6-7: Use [5.7.2] **should be**
Use the Hölder continuity of h implied by [5.7.7]

- p. 399, l. 6: $\lambda' < \lambda]$ **should be** $0 < \lambda' < \lambda$

- p. 399, l. 12: with $n_+\lambda_+ = n_-|\lambda_-|$ and $\lambda_\pm$ **should be**
with (within one unit) $n_+\lambda_+ = n_-|\lambda_-|$ if $\lambda_\pm$

- p. 399, l. 2 in problem 5.7.20: [5.7.9] **should be** [5.7.19]

- p. 399, l. 1 in problem 5.7.21:
[5.7.9], [5.7.10] **should be** problems [5.7.19], [5.7.20]

- p. 399, l. -10: Sec. 5.6.2 **should be** Sec. (5.5.2)

- p. 399, l. -2: $\mathcal{P} = (p_1, \ldots, P_\mathcal{N})$ **should be** $\mathcal{P} = (P_1, \ldots, P_\mathcal{N})$

- p. 399, l. -3: **Replace the last three lines and the first two of the next page by**
(3) to the conjecture of the Sec. (5.5) (*Hint:* Find the geometric interpretation in terms of Markov partitions of the relation between of the third expression in (5.7.7) and the area of the sets E_j in (5.7.5): it will appear that the quantities $Z^0_p(\sigma_{-p}, \ldots, \sigma_p)$ differ by a factor the can be bounded above and below by a p-independent constant from the area of $\cap_{i=-p}^p S^{-j} P_{\sigma_j}$. Since the analysis in problems [5.7.4] through [5.7.14] depends only on the Hölder continuity of $\lambda(\underline{\sigma})$ it can be applied to the two factors defining Z^0 to infer that $\cap_0^p S^{-j} P_{\sigma_j}$ have area bounded above and below by a p-independent constant by $e^{-\sum_{j=0}^p \lambda_e(S^j\underline{\sigma})}$ hence the sum $C = \sum_{\sigma_0,\ldots,\sigma_p} e^{-\sum_{j=0}^p \lambda_e(S^j\underline{\sigma})}$ is uniformly bounded as $p \to \infty$, because $\sum_{\sigma_0,\ldots,\sigma_p} \mu_0(\cap_{i=-p}^p S^{-j} P_{\sigma_j}) = 1$, so that by the first of (5.7.7):

- p. 400, l. 3: $\bigcap_{i=0}^{\infty}$ **should be** $\cap_{i=0}^p$ **twice**

- p. 401: **add, at the end**
See, for instance, [GBG04] for detailed analysis of all the problems in Chapter VI and for proofs of most statements left unproved here.

- p. 405, l. -8: (6.1.7) with Green's function Γ with the
should be (6.1.7), where Γ is the periodic heat equation Green's function, cf. (3.3.17))

- p. 406, l. 5: up to $t - t_0$) **should be** up to $t-t_0$ *excluded*)

• p. 406, last l. in (6.1.14): $e^{-\nu\underline{k}^2(t+\tau+2\Theta)}$ **should be** $e^{-\nu\underline{k}^2|t+\tau+2\Theta|}$

• p. 408, l. -3: wish or are willing to forget mathematically checking **should be** we wish to, or are willing to, forget the mathematical check

• p. 409, l. 8/9: that we shall suppose with zero divergence (without **should be**
(that we shall suppose with zero divergence without

• p. 409, l. 2 before (6.1.25): of the field theories **should be** of a field theory

• p. 409, l. -5: did not turn out **should be** did not yet turn out

• p. 410, l. 4: cf. [BG95]. **should be** for instance cf. [BG95].

• p. 411, l. -1: in this section **should be** in this section for $d=2$ fluids.

• p. 415, in eq. (6.2.3): **leave more space between the two summation symbols so that their subscripts become distinct**

• p. 421, l. 6: **insert new paragraph**
The energy and enstrophy dissipations per unit volume and time are $\varepsilon=\nu L^{-3}\int_\Omega|\underline{\partial}\wedge\underline{u}|^2d\underline{x}$ and $\sigma=\nu L^{-3}\int_\Omega|\Delta u|^2 d\underline{x}$: their dimensions are of a velocity cube over a length and of a velocity cube over a length cube (*i.e.* $[v^3/l]$ and $[v^3/l^3]$) respectively.

• p. 421, l. 7: This hypothesis **should be** The above hypothesis

• p. 421, l. 17: $k_\nu=(\sigma^{2/3}\nu^{-3})^{1/6}$ **should be** $k_\nu=(\sigma\nu^{-3})^{1/6}$

• p. 425, l. 19: turbulene **should be** turbulence

• p. 426, by further conjecturing and assuming properties **should be** subject to further assumptions

• p. 426, l. 1/2/3 after (6.2.16): **replace the sentence by**
where $R=v_L L\nu^{-1}$ is the Reynolds number defined in terms of the "velocity

on the scale L of the forcing" by setting $||\varepsilon||_{L_{1+d/2}} \stackrel{def}{=} v_L^3 L^{-1}$.
new paragraph at the period

• p. 426, in footnote: $||\varepsilon|_{L_{1+d/2}}$ **should be** $||\varepsilon||_{L_{1+d/2}}$

• p. 427, l. -6: *Hint:* **should be** *Hint:* (NS case)

• p. 430, l. -3: $||\underline{g}||_{L_2} L^2/n$ **should be** $||\underline{g}||_{L_2} L^2/\nu$

• p. 432, l. -3: cf. [Ma86] **should be** cf. problem [4.1.13] and [Ma86]

• p. 433, l. -2: seriously in a model **should be** seriously as a model

• p. 434, l. 3: first integral **should be** integral of motion

• p. 434, l. 3: only **should be** only poorly

• p. 438, l. 15: $-M'_{kk'}$ **should be** $-M'_{k'k}$

• p. 438, l. 1 before (6.3.22): condition as **should be**
condition, implicitly defiining F, as

• p. 439, in problem [6.3.1]: **add the paragraph**
(*Hint:* Check first that, as a consequence of the negative signs in the terms with n odd in (6.3.9) the argument used in problem [4.1.13] in the case of the two dimensional Navier Stokes equation does not apply to the present case.)

• p. 441, l. 2 after (7.1.1): **add two new paragraphs:**
Note that this definition is *more general* than the often used and more common definition which takes i to be the "*velocity reversal with unchanged positions*" map, which in the case of simple fluids becomes simply velocity reversal and which for clarity of exposition will be called here, perhaps be more appropriately, *velocity reversal* symmetry. It is clear that while Newton's equations are reversible in the latter sense the NS equations do not have such velocity reversal symmetry and for this reason they are called irreversible. In principle a system may admit time reversal symmetry in the sense of (7.1.1) even though it does not admit the special velocity reversal symmetry: we shall see some interesting examples below.

The negation of above notion of reversibility is not "irreversibility": *it is instead the property that a map i does not verify* (7.1.1). This is likely to generate misunderstandings as the word irreversibility usually refers to lack of velocity reversal symmetry in systems whose microscopic description is or should be velocity reversal symmetric

• p. 441, l. 2/3 after (7.1.1): reversible microscopic dynamics **should be** reversible microscopic dynamics, in the sense of velocity reversal or in the more general sense in (7.1.1),

• p. 441, l. 3/4 after (7.1.1): **delete** (as the NS equations manifestly are)

• p. 442, l. -16: their average **should be** some average

• p. 443, l. 14 microscopic reversible **should be** microscopic velocity reversal symmetric

• p. 449, l. 14: **replace text in parenthesis by**
(Think of a gas of charged particles in a toroidal container subjected to an axially directed weak, but not vanishingly small, electric field).

• p. 449, l. 3 before definition: §5.7 **should be** §5.5

• p. 450, l. -20: To conclude the discussion, wishing to exemplify, what said by the well **should be**
We conclude the discussion by discussing, to provide a concrete example, various aspects of the abiove analysis in the case of the well

• p. 450, l. -15: energy). **should be**
energy: we imagine the "thermostsat" to consist of a system of forces applied to the constituents of the gas).

• p. 451, l. 2 in problem [7.1.3]: velocity **should be** velocities

• p. 451, l. 2/3 in problem [7.1.3]: from the constraint holonomic **should be** by the holonomic constraint

• p. 451, l. 4 in problem [7.1.3]: a constraint anholonomic **should be** an anholonomic constraint

• p. 451/452: problem [7.1.4]: **Delete problems 7.1.4, 7.1.5, 7.1.6 and replace the three by the single one:**
[7.1.4]: Consider a system of N particles subjected to a conservative force

with potential energy V. Consider the system of points subject to the force $\underline{f}=-\underline{\partial}V$ and to a Gaussian constraint imposing that $T=const$ (where T is the kinetic energy $T=\sum_i \underline{p}_i^2/2m$). Verify that the equations of motion are (by [7.1.3]):

$$m\underline{\dot{x}}_i=\underline{p}_i,\ \underline{\dot{p}}_i=-\underline{\partial}_{\underline{q}_i}V-\alpha\,\underline{p}_i\stackrel{def}{=}\underline{F}_i,\qquad \alpha=-\frac{\sum_i\underline{\partial}_{\underline{q}_i}V\cdot\underline{p}_i}{\sum_i\underline{p}_i^2}$$

Show that for arbitrary choices of the function $r(T)$ the probability distribution on phase space with density: $\rho(\underline{p},\underline{q})=r(T)e^{-\beta V(\underline{q})}$ is invariant if, defined ϑ as $3N\,k_B\vartheta/2\stackrel{def}{=}T$: $\beta=3N-1/(3Nk_B\,\vartheta)$. (*Hint:* The continuity equation is indeed $\partial_t\rho+\sum_i\underline{\partial}_{\underline{p}_i}(\rho\underline{F}_i)+\sum_i\underline{\partial}_{\underline{q}_i}(\rho\,\underline{p}_i/m)=0$).

• p. 453, l. 16 after (7.2.1): and do not **should be** and can be chosen so that they do not

• p. 455, l. 19: **delete** the ED or

• p. 459, remark (3): **delete remark (3) and replace it by**
(3) If we drop the condition that the poles are smooth and just require that motion on them is topologically mixing then the resulting weaker notion is structurally stable: this is implied by the quoted theorem by Robbin, [BG97].

• p. 461, l. 11: **complete the paragraph by adding after** time evolution. **the sentence**
The unsatisfactory aspect of the above analysis is that in general a perturbation of a system verifying Axiom C will have all the properties of the unperturbed system with the possible exception that the poles might lose the property of being smooth surfaces and a better understanding of this is desirable.

• p. 461, l. 1 after beginning of Sec. 7.2.5: stability **should be** stability properties

• p. 461, l. 1 after beginning of Sec. 7.2.5: has been **should be** have been

• p. 461, l. -12: Sect. 7.2.3 **should be** Sect. 7.2.4

• p. 462, l. 18: cf. Sect. 5.4. **should be** cf. Definition 5 in Sect. 5.4.

• p. 472, l. 1 after Theorem 3: **add Remark:**
Remark: an equivalent way to write (7.3.27) is

$$\frac{\pi\big(F(S_t\cdot) = h(t),\, t \in W_T,\, p\big)}{\pi\big(F(S_t\cdot) = -h(-t),\, t \in W_T,\, -p\big)} = e^{-p\langle\lambda\rangle_+ t}$$

expressing the same relation as (7.3.27) in terms of joint probabilities rather than probabilities conditined to the event that in the time interval it is $\sigma_t = \langle\lambda\rangle_+ \, p\, T$

• p. 473, l. 10 after (7.3.30): **delete paragraph starting with** The above intermittency

• p. 473, l. -14: However, if we regard **should be** However one can regard

• p. 473, l. -7: fluctutation **should be** fluctuation

• p. 473, l. -2: conditions each of **should be** conditions, each of

• p. 474, l. 1: picture and to **should be** picture of the above situation and to

• p. 474, l. 7 after (4.3.3): tried **should be** try

• p. 476, l. 3 before (7.3.43): magnitude **should be** quantity

• p. 477, l. 10: based only **should be** solely based

• p. 479, l. 3: to suppose **should be** to suppose heuristically

• p. 479, l. -1: **delete**
, and $\Pi_{\underline{k}}$ is the orthogonal projection on the plane orthogonal to $\underline{k}$.

• p. 480, l. 3 after (7.4.8): total energy $\int_\Omega \underline{u}^2 d\underline{x}$. **should be**
total energy $\int_\Omega \underline{u}^2 d\underline{x}$, constant in the ED evolution

• p. 482, l. 1: $R = \varepsilon^{1/3} L^{4/3} \nu^{-1}$, **should be** $R = \eta^{1/3} L^{4/3} \nu^{-1}$,

• p. 483, l. 17: modelled **should be** modeled

• p. 484, l. 17: to the rigorously **should be** to be rigorously

• p. 487, l. -9: (7.4.8) **should be** (7.4.10)

• p. 490, l. 3: Sect. 6.2.4 **should be** Sect. 6.2.6

• p. 490, l. 8 in problem 7.4.1: **delete text from** Check **through the end of problem and replace, adding also a new problem, by:**
Check that, in toroidal geometry (*i.e.* periodic boundary conditions), imposing $\mathcal{E}_1(\underline{a})$ on divergenceless fields $\underline{u}$ with the constraint $\varphi \stackrel{def}{=} \int(\underline{u})^2\,d\underline{x}$, leads to the GED equations while imposing $\mathcal{E}_2$ leads to equations that look like the incompressible NS equations. In the latter case the equations obtained are not the GNS equations of §7.1, or (7.4.1), (7.4.2): they coincide with the second of (7.4.1) but the multiplier β is different. Compute β in the latter case. (*Hint:* β has to be such that energy rather than dissipation stays exactly constant in time.)

[7.4.2] Check that, in toroidal geometry, the GNS equations in (7.4.1), (7.4.2) can be obtained by applying the Gauss principle with effort $\mathcal{E}_1(\underline{a}) \stackrel{def}{=} \big((\underline{a}+\underline{\partial}p-\underline{f}),(\underline{a}+\underline{\partial}p-\underline{f})\big)$ and constraint $\varphi \stackrel{def}{=} \int(\underset{\sim}{\partial}\underline{u})^2 d\underline{x} = const$ on the divergenceless fields $\underline{u}$. (*Hint:* Note that $\varphi = \int \underline{u}\cdot\Delta\underline{u}\,d\underline{x}$.)

• **add Reference**
[LL01] **Lieb, E., Loss, M.**: *Analysis*, American Mathematical Society, second edition, Providence, 2001.

• **add Reference**
[GBG04] **Gallavotti, G., Bonetto, F., Gentile, G.**: *Aslpects of the ergodic, qualitative and statistical properties of motion*, Springer–Verlag, Berlin, 2004.

• Reference [LY73] **should be** : **186**, 481–488, 1973.

• **add Reference**
[MD00] Murray, C.D., Dermott, S.F.: *Solar system dynamics*, Cambridge University Press, 2000.

• **scale down figure fig. 7.1.1, postscript is attached and Figure 5.1.1 should be modified as follows, postscript is attached**

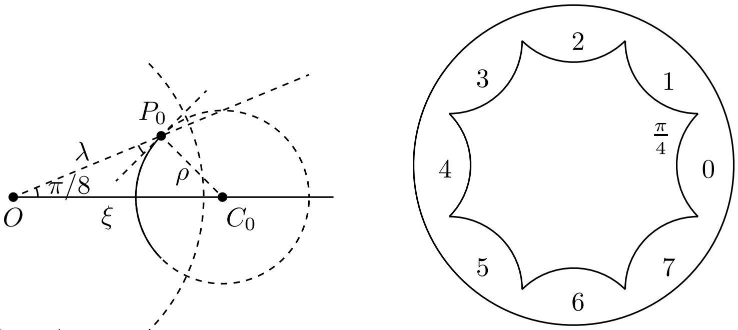

Fig. (5.1.1) *Illustration of the drawings proposed in* [5.1.34] and [5.1.35].