Theoretical Mechanics in Italy between 1860 and 1922

Giovanni Gallavotti
Fisica, Roma1; The.N.F.N.

Abstract: *Extended version of the conference on the "Development of Celestial Mechanics in Italy between the end of '800 and the Twenties of the XX century" at the Lincei Academy*

Istoria

Illustrious predecessors were, for instance, GIUSEPPE LODOVICO LAGRANGIA, (1736–1813), FRANCESCO CARLINI, (1783–1862), GIOVANNI PLANA, (1781–1864), GABRIO PIOLA, (1794–1850), GIOVANNI SANTINI, (1787–1877), OTTAVIANO FABRIZIO MOSSOTTI, (1791-1863): astronomers and theoreticians whose work is concluded at the beginning of the period considered here.

A short and preliminary analysis of the development of Mathematical Physics, discipline that I identify with Theoretical Mechanics (point mechanics, n–bodies, continuous, statistical, celestial mechanics...), in the period 1860–1922, from the "Unification of Italy" to the "March towards Rome" or from MOSSOTTI to FERMI, can begin from from

ENRICO BETTI, (1823-1892).

Well known in Algebra and Geometry he was, in fact, mainly a mathematical physicist disciple of MOSSOTTI. He did not refrain to start by translating the *Treatise of elementary Algebra* of BERTRAND that certainly contributed to draw him much closer to the European currents of ideas. He was a source of inspiration and a cultural reference for his contemporaries, among which BELTRAMI. Particularly in the last twenty years of his work he dedicated his research to Mechanics: he contributed to the theory of elasticity by extending the method of images to the biharmonic equation, later followed by CERRUTI, SOMIGLIANA, as well as the methods employed for studying the LAPLACE equation.

His works contain detailed and concrete presentations; mini-treatises, very useful even for today students who happen to have knowledge of their existence, on potential theory, on capillarity, on the heat equation, on elasticity: always attentive to the electrostatic interpretations of the many results.[1]

Among the first in Italy to be aware and to apply the ideas (that he attributes to CLAUSIUS without quoting BOLTZMANN) at the roots of Statistical Mechanics: he deals with the problems of mechanical interpretation of heat in an interesting work on the heat theorem and entropy where he derives a "thermodynamical" interpretation of the states of a system of masses interacting with gravitational potential, [Be888]; or he studies the equilibrium of a mass of rarefied gas isolated in space, [Be880], and more precisely he estimates the quantity of heat that can be contained in a star and how it could increase a because of gravitational contraction reaching undoubtedly modern results of astrophysical interest.[2] He was Senator of the young kingdom of Italy but he did not really get involved into political matters.

FRANCESCO BRIOSCHI, (1824–1897).

---

[1] It is interesting, from a historical viewpoint, to remark that he deals with the problem known today as the Dirichlet problem with the variational method by considering as obvious that the functional has a minimum: a statement which was considered intuitive by RIEMANN and DIRICHLET but which became important to prove after a counterexample by WEIERSTRASS (1880) and HILBERT's solution (1899) and the successive theory of Fredholm: it still continues to constitute a main result in the treatises or lectures on PDE's, [Be863] p. 82 in OP-2.

[2] "*If there has been a time in which in the unit of mass of the Sun was contained in average a just* 1 *calory, the radius of the sphere occupied by it was, at the time, equal a 6825 times the distance of the Sun from Neptune*", (see original quote in the Appendix)

Illustrious mathematical physicist who, however, dedicated himself to Analysis, mainly to the theory of equations, of elliptic functions and of differential equations. But his volcanic production, always dedicated to technically demanding questions, greatly influenced many young students and formed a generation of analysts and mathematical physicists. I fraction of the his works properly in Mathematical Physics is rather small and almost exclusively dedicated to problems in hydraulics, very concrete and of direct interest for engineering, [Br866], (unless we consider the theory of elliptic functions as part of the Mechanics, which would be an easily defensible position). Particularly relevant for Theoretical Mechanics is the paper on the equilibrium configurations of a rotating fluid, [Br861], which is a subject on which all Mathematical Physicists of this period were involved (as well as in earlier times, like for instance PLANA): this paper was continued by PADOVA who has the merit of having extended the analysis to the cases of rotating ellipsoidal configurations with axes periodically oscillating, [Pa871]. He had important academic and political charges (he founded the Milan Polytechnic Al School, was President of Accademia dei Lincei and was Senator). He was teacher and then colleague of

LUIGI CREMONA, (1830-1903),
who influenced Mathematical Physics although he was mostly contributing to Geometry. A trace of his direct involvement in questions of applied Mathematical Physics can be found in his membership in a committee for a referee report (together with BETOCCHI, BLASERNA, BELTRAMI) for the possible publication on the Rendiconti Lincei of a somewhat strange paper of Mr. Colonel PIETRO CONTI, which very likely "had" to be accepted, [Cr875].[3] His successor was VALENTINO CERRUTI, (1850–1909), also a mathematical physicist who gave important contributions to elasticity theory. CREMONA influenced, together with BRIOSCHI,

EUGENIO BELTRAMI, (1835–1900).
Around him and BETTI gravitated the Italian Mathematical Physicists until the dawn of the XX century, when the exceptional flourishing of VOLTERRA and LEVI-CIVITA took place.

His most known contribution has been, perhaps, the realization of a portion of surface with constant negative curvature in a 3 dimensional space: the *pseudosphere*, figure of rotation associated with the *tractrix*, around 1868, [BGT998]. The achievement was obtained through considerations and methods typical of the Mathematical Physics,[4], by studying the concrete question of representing without deformation a surface on another by making use of the instruments provided by GAUSS' and LOBACHEVSKY's theories. With an attitude of an experimental physicist he also built paper models to illustrate properties of non Euclidean geometry and derived from them idea of the possible validity of new properties of such geometry, yet unknown, which he proceeded to prove.

BELTRAMI, a mathematician by formation, also studied quite diverse questions of mechanics, well beyond the properties of the geodesic motions on curved surfaces and their interpretation in terms of the geometric conceptions of GAUSS and LOBACHEVSKY, inno-

[3] "*We cannot refrain from remarking that from the 135 experiments quoted in the Memory one can well extract arguments to casts doubts on the results obtained by* COULOMB *and by* MORIN *but one can doubt whether they suffice to convince on the truth of the laws stated from Sig.* CONTI*, even more so as by his process it is not evident the possibility of reaching some conclusive result."* .... closing by "*... it seems desirable to us that the Memory be published, so that a fruitful discussion on it is generated and more competent judges can pronounce an appropriate sentence*".

[4] His qualification as a Mathematical Physicist is paradigmatically described by a quote from his several works on potential theory "*thus the generic functions that we shall meet in the following must always be assumed to be endowed apart from the explicitly stated properties of all those, like integrability, differentiability or other, that are necessary for the legitimacy of the operations performed on them*, p. 574 of [Be880], words that should be kept in mind by contemporaries who associate Mathematical Physics with "weak" or not constructive solutions of important equations.

vative at the time: the more than actual importance of the LAPLACE-BELTRAMI operator and of the differential parameters of a surface is a witness. He was an attentive to what was being developed in Europe in the domains of Analysis, of Geometry as well as in Physics. He did not hesitate to even enter into considerations on the phenomenology of electrodynamics and on FARADAY's ideas (in a letter to CESÁRO, [Be889]). He dedicated much time to electrodynamics and to electrostatics: solving several problems of potential theory and studying fluidodynamics in detail trying to clarify the connection with MAXWELL's equations and reaching negative conclusions about their possible elastic nature (a work revisited by SOMIGLIANA, [So907]). I four monographs on fluidodynamics, [Be874], can be considered a treatise of theory of EULER's fluids, with interesting new exact solutions of fluid motions, both planar and not planar (like the Hamiltonian motions of vortices or the helicoidal motions of incompressible fluids) and with several other natural problems posed in a form that still today we would call modern. Elasticity also attracted his curiosity and he contributed to the diffusion in Italy of ST. VENANT's principle, for instance, by criticizing some of its aspects (about the quantity that it would be necessary to estimate in order to guarantee the cohesion of an elastic material, where his answer differs from ST. VENANT's (the work was revisited by LEVI-CIVITA, [LC901]): and elasticity, as well as electromagnetism, attracted his attention also because of their relation with the problems about action at distance (by studying for instance the representation of the Newtonian or electromagnetic forces by means of elastic forces).

Quite remarkable is his work on entropy, [Be882], that shows a deep knowledge of the ideas of CLAUSIUS and that probably stimulated the interest that also BETTI, [Be888], showed to the matter. He develops on the basis of the ideas of CLAUSIUS and HELMHOLTZ a "theorem of the heat" verified by a system of n charged conductors when one changes the "state", *i.e.* the form, the charge and the spatial position: by associating with a each state suitable electrostatic quantities to be called (we use here, for brevity, the notation of BOLTZMANN instead of his) U, energy, T temperature, p pressure, V volume he shows that such quantities have the property that the variations of U and of V, corresponding to an arbitrary variation of the state, are such that

$$\frac{dU + pdV}{T} = \text{exact differential}$$

i.e. the quantity in the the l.h.s. is the variation of a suitable function S of the state. This is a"*mechanical analogy of heat*" and introduces a quantity analogous to the notion of *entropy*: it followed an idea that in those years BOLTZMANN, with a work culminating in 1884 but already begun much earlier in his youth, [B0866], [Bo884], along lines followed also by CLAUSIUS and then HELMOLTZ which led him to the ergodic hypothesis and to the theory of statistical ensembles. In Italy, as elsewhere, the theory of BOLTZMANN has been largely misunderstood or ignored in spite of the developments that led GIBBS to its transformation into the modern statistical mechanics. Hence it interesting to remark that BELTRAMI (and then BETTI and to a less extent also VOLTERRA) have been, until present times, among the few to take into a serious consideration the heat theorem which fascinated him because of the mechanical meaning that it could attach to entropy. It is somewhat surprising, however, that in the paper no reference to CLAUSIUS, HELMOLTZ or BOLTZMANN appears, although he quotes them in other papers: and it seems certain that he knew their work on the subject.

In Mathematical Analysis proper BELTRAMI gave many contributions, quite modern in spirit, among which the development of a function in a trigonometrical series with harmonics of frequencies equal to the roots of the first BESSEL function or of its derivative, following a work of ABEL. He did not get directly involved in Celestial Mechanics in spite of a brief period at the Brera observatory (1863-64) and his acquaintance with SCHIAPARELLI with whom he wrote trigonometric tables for geodesic usage.

The personality of BELTRAMI has been, therefore, very rich and polyhedric although his influence did not materialize in a large school: possibly because of his repeated moves from university to university. Nevertheless he had students in various universities (for instance BURGATTI, cf. below). In his youth (1853) he was expelled from the Collegio Ghislieri under the accusation of having incited unrest against the Rector (at the time Lombardy was under Austrian rule and many found that not acceptable) so that he could not even finish his studies. Nevertheless he could continue to work on research and get, after Unification, a position in the Royal University thanks to the appreciation and interest of BRIOSCHI and CREMONA eventually even becoming a Senator (as several other illustrious mathematical physicists, like BRIOSCHI, BETTI, SIACCI, VOLTERRA) and was given important government tasks.

His work has been important for the influence on Italian research: perhaps mainly because of the dissemination of knowledge of the great themes of Mechanics (from fluids to electrodynamics to elasticity to theoretical Physics) that followed a large number of papers in which he derived, providing remarkable simplifications through new and original methods, important results of European scientists. Furthermore we owe to him the discussion and valorization of SACCHERI's work, [Be889], on non Euclidean Geometry. He was influenced by

DOMENICO CHELINI, (1802–1878).

Older than BELTRAMI, he suffered politically (being an ecclesiastic) and was fired by the University of Bologna "not having" attended a religious ceremony celebrating the "Statuto" (1860); readmitted (1863) and again fired (1864, having refused an oath of allegiance to the Kingdom because of his ecclesiastic status,, although in fact he was personally in favor of Unification of the country). CHELINI has been an important figure who contributed to the development of Mechanics in various ways and *in primis* with his detailed and explicit constructive treatment of POINSOT's motions, which was studied in the rest of Europe, [Ch859]. He therefore continued and contributed to keep alive the interest on Analytical Mechanics on the footsteps of LAGRANGE (and, in Italy, of PLANA, PIOLA, SANTINI and others that followed LAGRANGE by applying and developing his methods in the first part of the XIX century). However in Italy the developments that might have been expected on the theory of rigid motions following CHELINI's contributions, although their study remained at the center of numerous successive researches and of the interests of all mathematical physicists.

An important document on CHELINI's stature is his treatise of Rational Mechanics, [Ch860], which seems to me to have remained unequaled until the appearance (1921) of the LEVI-CIVITA and AMALDI treatise (which I would dare say that went beyond only in the theory and applications of canonical maps and variational principles).[5] He applies the theory of rigidi motions computing the terrestrial precession and nutation, that will be also in LEVI-CIVITA and AMALDI but that is not usually discussed in the Italian treatises of the period intermediate between the 1860 and the 1921. It is a syndetic exposition, and yet it is very clear, even if judged according to present day criteria. He discusses the theory of frictionless constraints via the principles of virtual works (called virtual velocities) and of D'ALEMBERT essentially presenting them as a definition of constraint without friction (and setting friction, when present, among the active forces) without trying the a proof "on physical basis" which (quite strangely) was adopted by the Italian schools of Mechanics (cf. the lectures of BURGATTI or of LEVI-CIVITA and AMALDI): the point of view of CHELINI instead has been that of the main treatises

[5] The lectures of GIOVANNI BATTAGLINI, (1826–1894), [Ba873], were also equally interesting. Although being explicitly based on the treatises of TODHUNTER and of others they constitute a compilation of highest level: they are relevant for Celestial Mechanics as the elementary motions (precessions and nutations) of Earth and Moon are elegantly discussed. A important earlier treatise was due to SANTINI, [Sa830]: also quite remarkable for the attention devoted to such questions. Several other treatises appeared between that of CHELINI and that of LEVI–CIVITA AND AMALDI: for instance those of ALESSANDRO DORNA (1825–1866), [Do873], FILIBERTO CASTELLANO, [Ca811].

of the time, see for instance the point of view of APPELL and of VOIGT.[6] CHELINI's interest for matters of principle and for the foundations is always present in his book and culminates in the interesting Appendix "*On the fundamental principles of Mathematics*" where his attention is attracted by the axiomatization needs that in those years pervaded mathematical studies in Europe.

He was actively interested in Celestial Mechanics: see for instance his calculation of the ephemerides of Hind's comet, [Ch847].

Therefore CHELINI appears as a key figure, as also proved by BELTRAMI's appreciation who, at times, called him "dear and illustrious friend"; in the exceptional commemoration that he wanted to read about him he calls CHELINI with the affectionate expression "il buon CHELINI".[7]

FRANCESCO SIACCI, (1839–1907).

Contemporary of BELTRAMI emigrated from Roma to Torino to participate to the Italian unification process becoming a soldier. He become professor of Rational Mechanics (at the Torino university) and of Ballistic (at the School of Applications, Torino). He is an interesting personality who succeeded in performing also research in spite of being urged to work on concrete artillery problems. He revisited the results of CHELINI on POINSOT motions, [Si877]. In his lectures on Rational Mechanics the principle virtual work is presented, essentially, as a postulate: he comes back to the matter in one among his last papers where he tries a simple proof of the principle opening with a methodological statement that show that he was very attentive to fundamental questions and appreciative of their relevance.[8]

The ideal successors of BELTRAMI and BETTI.

were VOLTERRA, a physicist laureate at the Scuola Normale of Pisa, student of BETTI and then successor to BELTRAMI's chair, and TULLIO LEVI-CIVITA, (1873-1941), mathematician, influenced by ERNESTO PADOVA, (1845-1896), and GREGORIO RICCI-CURBASTRO, (1853–1925). Two rather different personalities, both with varied and often common interests: with their work Italian research enters in the flow of European science and competes with it.

VITO VOLTERRA, (1860-1940).

His work is initially dedicated to Mathematical Analysis under the influence of ULISSE DINI, (1845–1918), and undertakes with clarity and without refraining from even very substantial computations, that reflect his background as a physicist, problems of electrodynamics, [Vo891], with attention to the representation of wave solutions via KIRCHOFF's principle. The papers of the first twenty years, until the beginning of the '900, is vast, varied and of of exceptional interest.

Well known are his foundational contributions to Functional Analysis and to the theory of integrals equations and of the "hereditary phenomena" that appear already in his first

[6] Although CHELINI discussed in a in an earlier paper, [Ch847], a "proof" of the principle of the " virtual velocities" going back to AMPÈRE ("... it obviously suffices that the sum of the applied forces is normal normal to M ..."). In this respect it is convenient to recall that the classical treatise of VOIGT was been translated, 1894, into Italian with a forward by BELTRAMI who stresses that the attention to the analytical mechanics of LAGRANGE would have perhaps deserved to be larger.

[7] *i.e.* "the good CHELINI".

[8] "*On the other hand at school, long and complicated proofs should be avoided: the ones which are not rigorous are unacceptable, and should not be given. Not to give them at all ... does not seem to me suitable to a good mathematician.* However this is followed by a proof, [Si905], that he himself immediately after, in a successive paper, remarks to be non rigorous: "*Where rigor fails it is no longer worth caring and whoever might be curious will be easily able to find it on his own.*". The non rigorous proof in reality, it seems to me, contains an error in item 2°, p. 605: an error that I do not see in the proof, quite analogous, in the treatise of CHELINI.

works, [Vo883], [Vo887], see also [Vo912], [Vo914].[9] The theory has a rapid development which makes necessary several review works or papers comparing results with those of other authors (ABEL, LIOUVILLE, BELTRAMI, SCHLÖMILCH, DINI, SONINE, LEVI-CIVITA, PINCHERLE), [Vo897].

Theoretical Mechanics is the subject of several memoirs among which I quote the remarkable extension of HAMILTON-JACOBI's theory to cases in which the time variable is two–dimensional:[10] and he finds in this way properties of families of minimal surfaces already obtained by PADOVA. A modern revisitation of the problems that this memory suggests is certainly desirable.

Celestial Mechanics has been object of intense study, although concentrated within a few years, dedicated, unlike LEVI-CIVITA, to questions of applied Astronomy. In particular he has been attracted, possibly under the influence of PADOVA, [Pa885], by the theory of polar motion: following CHANDLER's precession discovery (1891). In a series of papers he discusses, [Vo895], the possibility that the precession is due to motions internal to Earth, marine currents for instance, that produce a couple $\underline{M}$ constant, or with a slowly varying direction in a earthly frame (with the Earth otherwise regarded as a rigid body). The purpose is finding an explanation of the observed inequality between the CHANDLER's precession and Eulerian precession.[11] He also discusses alternative hypotheses like the plasticity hypothesis proposed by SCHIAPARELLI (*i.e.* variability of the inertia tensor which, instead, remains constant in the internal motions theory, favored by VOLTERRA). His background as a physicist is manifest in the latter papers which show hi ability to attack a difficult and not *a priori* well defined phenomenological problem by drawing inspiration and methods from the rigorous methods of theoretical mechanics, for both the definition and the treatment of the mathematical model. In this respect one can get an idea of how much CHANDLER's discovery had excited the interest of physicists and mathematicians by reading the strong polemics, quite likely not exempt from a priority dispute, in VOLTERRA's letter, [Vo896], to the president of the Accademia dei Lincei BRIOSCHI in answer to comments by GIUSEPPE PEANO, [Pe895], on his theory.

Rigid motions theory, as everyone knows, is a quite contagious entity: hence also VOLTERRA will devote to it a large number of memories after his initial interest for the physical aspects of the polar precession. Hence he studies various mechanical problems, usually related to CHANDLER's precession, among which the theory of polycyclic systems (rigid bodies hosts of systems performing periodic motions of different periods) going back (and developing) HELMOLTZ' theory of monocyclic systems which had had a great importance in the foundations of Statistical Mechanics through the BOLTZMANN's work, [Bo884]. It is interesting that such papers of the founders of Statistical Mechanics,

---

9 An interesting trace of his study of potential theory is also to be found, aside from his early papers, in a series of "exercises" that it would be very useful to =suggest to students, [Vo894].

10 The generalization is roughly described as follows. The coordinates $x_1, \ldots, x_n$ "motion" (with a two–dimensional time) depend on two parameters u, v that vary in a domain Δ; the role of the time derivatives $\dot{x}_1, \ldots, \dot{x}_n$ is played by the determinants $\xi_{i,j} \stackrel{def}{=} \frac{\partial(x_i,x_j)}{\partial(u,v)}$, $i < j$ and the "Lagrangian" is now a function $\mathcal{L}(\xi, x)$. The problem is to minimize $\int_\Delta \mathcal{L}(\xi, x) du dv$ under the condition that x takes given values on the boundary of Δ. The equations "of motion" become $\sum_k \frac{\partial(\xi_{i,k},x_k)}{\partial(u,v)} - \frac{\partial \mathcal{L}}{\partial x_i} = 0$ and having set $p_{i,k} \stackrel{def}{=} \frac{\partial \mathcal{L}}{\partial \xi_{i,k}}$ and $H = p \cdot \xi - \mathcal{L}(\xi, x)$ one finds the "Hamiltonian form": *i.e.* $\frac{\partial(p_{i,k},x_k)}{\partial(u,v)} = -\frac{\partial H}{\partial x_i}$, $\frac{\partial(x_i,x_j)}{(u,v)} = \frac{\partial H}{\partial \xi_{i,k}}$.

11 The model is $I\,\underline{\dot{\omega}} = (I\,\underline{\omega} + \underline{M}) \wedge \underline{\omega} + \underline{\dot{M}}$, with $\underline{M}$ constant (stationary internal motions) or variable (slowly varying internal motions) and with I the inertia matrix, which is integrable, by quadratures, if $\underline{\dot{M}} = \underline{0}$ or, if $\underline{\dot{M}}$ is small, approximately; the model allows us to derive the value of $\underline{M}$ as a function of the observed motion of the poles. He finds the remarkable result that the Eulerian period would change into the observed CHANDLER period if the angular momentum along the Earth axis of the internal motions was $\sim 10^{-3}$ times the total.

known and applied by BELTRAMI and BETTI, as cited above, were known and followed in Italy at the time, unfortunately to be forgotten later. In 1898 he completes and reviews his studies in a monography on the prestigious journal Acta Mathematical, [Vo898].

To Celestial Mechanics he comes back with the study of n–bodies fixed on a line and a $(n+1)$-th body gravitationally attracted by them: a problem that will be further investigated by ARMELLINI, see below.

To very concrete questions related to the interpretation of experiments and of natural phenomena VOLTERRA, as LEVI-CIVITA, RICCI-CURBASTRO and others, has always been attentive: the interest for the polar precession is not the only example. Papers on electrolysis and on analogous phenomena, developed as a Student or as an Academician, provide other examples, [Vo897]; in his theory of the "*seiches*" of Geneva lake where, [Vo898], he proposes the study of the analogous phenomena in the italian lakes and in a brilliantly short note in the appendix he shows that the corresponding theoretical problem can be reduced to an eigenvalue problem ("exceptional values problem").

His continuous interest for the theory of holomorphic functions keeps him in close contact with Mathematical Analysis and Geometry of the constant curvature surfaces: an interest always kept up even in the periods when concrete questions, directly related to Physics experiments or Astronomy, seemed should predominate. This is already manifest in the extension of the holomorphy notion of a functions of several variables, [Vo887].[12] A theory that he will develop into the theory of differential forms of dimension arbitrary obtaining remarkable extensions of previous partial results of RIEMANN and BELTRAMI, among others. And later into the theory of linear differential equations in the complex field, [Vo899]. A really elegant note is his addition theorem for double integrals which shows that VOLTERRA was in synchrony with the interests and the methods that were being developed in Europe (by PICARD in this case), [Vo897].

Elasticity is the subject of several important papers extending to waves propagating in elastic isotropic and anisotropic bodies the notion of characteristic lines (which become characteristic surfaces), [Vo894]. Furthermore he analyzes elastic properties of non simply connected bodies and develops a detailed theory showing that, unlike the case of simply connected bodies, a non singular solution of the elasticity equations presenting internal tensions is possible *in absence of external forces*, [Vo907]. The wave equation, the heat equation and the elasticity equations continue to attract his attention: and he discusses an application of the method of images to the wave equation, [Vo904], writes lectures on the equations of Mathematical Physics, [Vo908], an interesting application of the heat equation is devoted to the rather concrete problem of the temperature inside a mountain, [Vo912], he also studies integrodifferential equations, [Vo912], and hereditary phenomena, [Vo913].

During the war period his scientific production is somewhat affected and it is adapted to his new state of voluntary aeronautics soldier, [Vo916]: he nevertheless manages to complete works on the theory of algebras of integral operators, [Vo916], which continue his 1912 Princeton lectures and which will continue to attract his interest, [Vo920]. These are ideas that have had several kinds of developments, for instance the modern theory of convergence of the virial series and the "*cluster expansion*" algebraic formalism, [Ru969].

The theory of population evolution, important for originality and results, has been mainly developed in the period successive to the one considered here.

He has been very active in shaping scientific policies in Italy and, particularly after 1922, abroad. He achieved important results in the organization of research and soon developed interests for the applications of Mathematics to biological and social sciences.

[12] A pair of complex functions on the regular and closed, or ending on the boundary of a given domain of a 3–dimensional surface, curves defined by line integrals of a pair of differential forms are *Riemann–related* if the curls of the coefficients of the forms are everywhere proportional. Like the case of a bidimensional surface when two complex functions of the points of the surface are *Riemann–related* if the infinitesimal variations with respect to the value taken in a arbitrary point and any of its infinitesimally close neighbors have a ratio which is independent from the variation of the points, [Vo889].

As a Senator he exercised strong influence on Italian science, at least until the advent of fascism, which he opposed with firm coherence until his ousting from the university and the academies which took place well before the racial laws.[13] For an understanding of his care for high school education and research it is useful to read his relation on the teaching of dynamics in "scuole industriali", [Vo921], or the slightly later relation on the teaching of Mathematical Physics in the university, [Vo920].

TULLIO LEVI-CIVITA, (1873-1941).
With LEVI-CIVITA Analytical Mechanics receives a boost with the proof of new and original propositions. Among his first papers one finds a classification of the geodesic motions on surfaces and the problem of isomorphism of continuous systems dynamical (as we call it today) is set with clarity and studied in some detail, [LC886]. Soon his interest for Analytic Mechanics and his knowledge of Celestial Mechanics problems lead him to the "LEVI-CIVITA*'s regularization*" of the restricted three body problem in a well known series of papers, which begin in [LC903] with the discovery of the canonical change of coordinates acting the position coordinates (x, y) of the restricted problem as $(x, y) \to (\xi, \eta)$ with $x + iy = (\xi + i\eta)^2$, and which is accompanied and followed by several problems related to Celestial Mechanics which puts LEVI-CIVITA among the few authors of the period considered here who, in Italy, were involved into theoretical questions relative to the tabulation of the perturbation theory functions. For instance the study of inversion of KEPLER's equation where he shows that the inversion of $u - e \sin u = \zeta$ by series, for arbitrary real ζ, can be achieved for all eccentricities $0 < e < 1$ via a power series in the parameter $\eta = \frac{e \exp \sqrt{1-e^2}}{1+\sqrt{1-e^2}}$ with radius of convergence 1, [LC904]. The study of KEPLER's equation has been a recurring theme in Italy since LAGRANGE, at least, as in the papers of CARLINI or of CHIÓ, [Ch846], and LEVI-CIVITA's is an original and innovative continuation of the earlier studies.. Also his work on implicit functions inversion, which does not seem well known, is likely to have been generated by the problems that he was attacking on the representation of the perturbing function in Celestial Mechanics: he proposes using a summation method of the series expressing an implicitly defined function; in modern language, this is a "resummation" of a (convergent) power series, [LC907]. A method that has been recently adapted and applied to small divisors and divergent series problems (resummations of mass diagrams in quantum field theory and in KAM theory).
An interesting theorem on "stationary motions",[14] allows us to discuss in a unified way all motions integrable by quadratures known at the time in several systems, relevant for Mechanics and mainly for Celestial Mechanics, which were not completely reducible to quadratures, like the heavy rigid body with a fixed point, the three bodies problem and more.
"Mathematician" by formation, as witnessed by his first papers and by his recurring interest for subtle questions like the prime numbers theory, the infinitesimals, the transfinite numbers or for the absolute differential calculus, learned from RICCI-CURBASTRO

[13] In this respect one often hears that his refusal of the oath of allegiance to the fascist regime took place when he was about to retire, 1932: and one omits to add that VOLTERRA, since the beginning, 1922, had to suffer the consequences of his opposition to the regime and one forgets also that exclusion from the society of peers at an old age is harder to overcome than by people who, like VOLTERRA fifteen years earlier and at the height of his forces and fame, can choose to emigrate.

[14] consider a n degrees of freedom system with Hamiltonian $H(\pi, \kappa, p, q)$ where π, κ are k canonically conjugate coordinates and p, q are the remaining $n - k$. If $I_r(\pi, \kappa, p, q) \stackrel{def}{=} \pi_r - f_r(\kappa, p, q)$ and $I_r = 0$, $r = 1, \ldots, k$ are "invariant relations", *i.e.* if they remain zero at all time if they are zero at the initial time and if furthermore hey are in involution, then setting $H'(\kappa, p, q) = H(f(\kappa, p, q), \kappa, p, q)$ it follows that also the relations $\partial_p H'(\kappa, p, q) = 0$ and $\partial_q H'(\kappa, p, q) = 0$ are invariant relations; expressing through them $p = P(\kappa), q = Q(\kappa)$ in terms of κ and solving the "only k–dimensional" equations $\dot{q} = \partial_\pi(f(\kappa, P(\kappa), Q(\kappa)), \kappa, P(\kappa), Q(\kappa))$ one obtains ∞^k motions. The result was reconsidered and extended by BURGATTI, [Bu912], and then by LEVI-CIVITA himself.

and then developed in collaboration with him, [RL900], LEVI-CIVITA rapidly became a Theoretical Physicist through his interests for Mechanics and the Physics of continua starting with elasticity and the theory of fluids and, through Geometry and together with RICCI-CURBASTRO, has been among the founders of the mathematics of General Relativity. He remained always close and grateful to RICCI as testified by his commemoration at the Accademia dei Lincei, [LC925], where he attributes almost completely the creation of absolute differential calculus to RICCI, recalling also how little RICCI had been appreciated during most of his scientific life .[15]

As a theoretical physicist he has been, beyond doubt, the most prominent in Italy in the considered period: his multiple interests lead him to study with great attention the most varied questions: in a letter to SIACCI (who was already a Senator, but not yet artillery General) he explains his attempt at explaining resistance met by a body in motion in an ideal fluid that he develops into an ambitious theory of wakes and fluid friction.[16] He is strongly interested to concrete electromagnetism problems, on request of experimental physicists like RIGHI, for interpretation of laboratory experiments. His rigorous treatment of the (non linear) waves at the free surface of a bidimensional fluid (begun in [LC907] and continued on several later papers) is still today rightly well known.

In other papers he studies problems suggested by industrial enterprises on coaxial cables construction,[17] or from ballistics.[18]

He was always particularly interested by electromagnetism: both for for special theoretical questions and for matters of principle. His interest culminates, perhaps, precisely in the period in which "Mr. EINSTEIN" was developing the relativistic revolution: this is witnessed by the analysis, admirable by its lucidity and methodological rigor, of the theory of electron and of its electromagnetic mass presented Italian Physical Society (S.I.F.) meeting in Pavia in 1901 and rielaborated and published in 1907 stimulated by his papers on the existence and properties of the motions of a density of charge without any inertial mass, [LC907].[19] Electrodynamics and the questions related to the electrons

---

[15] The difficulties met in the academic carrier, remarkably lagging behind that of other less important colleagues, were certain due to lack of appreciation of his work: as it appears from BELTRAMI's statements on the occasion of the decision, 1886, of not assigning to him the "Royal prize for Mathematics" and by the statement, analogous and in analogous occasion, 1901, of BIANCHI, [Bi902]: ..."*Furthermore we feel that the algorithmic aspects receive too much stress often shadowing the essential geometric contents and sometimes leads to state as special results of the calculus geometrically evident properties. ... Rather than in the particularity of the algorithms employed, in the field of ordinary infinitesimal geometry, we recognize the true source of discoveries in the force of geometric intuition, helped by that potent analytic instrument that each mathematician will be able to forge one way or another depending on his individual preferences or habits... It ought to be recognized that the advantages of the absolute calculus procedures are much larger in the field of differential geometry in more dimensions... The results achieved by the Author do not present any substantial novelty.* However it seems that the Committee also had to find a valid excuse for not assigning the prize that would have had to be given to others who, for some accident could not be considered (nothing new!).

[16] He says that the "paradox", [LC901], according to which in an ideal fluid a body in uniform motion does not meet resistance can be overcome by admitting that the velocity field is discontinuous on a (natural) discontinuity surface: and he derives NEWTON's friction (friction proportional to velocity square). In modern language the solution of EULER's equation that he proposes *would not be a weak a solution* (in fact the requirement that a solution should be a weak solution does not have a really clear physical meaning in the case of a perfect fluid); he argues that when the velocity is not small this form of friction dominates over a possible friction due to the viscosity. It is a proposal that still deserves to be investigated, [LC901], [LC907].

[17] *I.e.* "from Pirelli's engineer E. JONA": which leads to a brilliant solution of DIRICHLET's problem, in a planar region with cusps, to evaluate the largest voltage to which the insulating material is subject in a coaxial cable as a function of the number of constituent cables, [LC904]: he expresses the largest voltage as the value of a suitable hypergeometric function.

[18] *I.e.* from Tenente Colonnello CALVI who asks a theoretical explanation of why a projectile shot against target penetrates a distance which does not increase with the closeness to the target, but there is instead an optimal positive distance for the largest penetration, [LC906].

[19] He discusses various theories and supposes that the electron (*i.e.* the cathodic rays particle)is rigid and consists of atoms, showing the inconsistence of such hypothesis. He the analyzes the ABRAHAM's theory showing its consistency with a zero mass assumption for for an electron whose inertia is purely

electromagnetic mass will continue to attract his attention also in the following years: as a side result he finds the Hamiltonian formulation for the motion of a charged particle in a electromagnetic field (*i.e.* the form of "minimal pairing "), [LC910].??

Celestial Mechanics leads him to a general study on the differential equations periodically dependent on time, motivated by the need to provide an existence proof of the average motion of the lunar node assuming that the motion of the heavenly body is described by certain simple models. In modern language establishes the existence of the rotation number for the Hamiltonian periodically forced motions that appear in Moon theory. This follows a theorem by POINCARÉ, [Po885], on the existence of the rotation number for maps of the circle which he improves by eliminating an assumption implicit in the works of LINDSTEDT, ADAMS, HILL and POINCARÉ himself.[20] Furthermore LEVI-CIVITA studies the problem of the form of Saturn rings in the frame of a series of papers on the gravitational properties of thin mass tubes, [LC912]. And he comes back on the problem of a representation of the perturbing function by introducing a new system of canonical coordinates for the n–bodies problem, [LC913], and then (after the general SUNDMAN's solution) again on the restricted three bodies problem simplifying, [LC916], and completing the theory obtaining new results on the regularization of the binary collisions in the non planar problem, [LC918].

But the Einsteinian theory is emerging and, naturally, the active interest of LEVI-CIVITA for the Geometry comes back: a never really neglected interest which reappeard with the introduction of the notion of parallel transport, [LC917], and continues with his original contributions to the new theory, [LC917], that will be summarized in the monograph on the absolute differential calculus published after a long care in 1925, [LC925]. His interest for Relativity goes well beyond the 1922, but it will by no means exhaust his continuous revisitations of problems studied in earlier times, on Celestial Mechanics, on the fluids on elasticity and it will be accompanied and enriched by a strong interest towards the new Mechanics (quantum) to which he will also give contributions showing culturally vast interests that were not to be really imitated by his heirs.[21]

It is interesting to recall here the conclusive words of his *Extension and evolution of Mathematical Physics (in the last fifty years, with special attention to the Italian contributions)*, [LC911]. Here he comments on the success of the atomic hypothesis and the conflicts that one meets in the theory of electromagnetism: "*We can fear that, as knowledge of the laws of nature advances, it will no longer suffice, as it was so far the case, to ask ... but it will become absolutely necessary to develop more detailed analysis, paying attention to the molecular behavior. If so ... the unknown equations will have*

electromagnetic and he evaluates as $r \sim 10^{-12}\, cm$ the electron dimension (*i.e.* a value of the "classical electron radius" which he deduces as $r = \frac{4}{5}\frac{e^2}{mc^2}$ (in modern symbols)). In the concluding remarks he says *the experimental apparata are already now so developed to exceed in exactitude the domain covered by the various kinematical hypotheses"*: and proposes the necessity of attacking the problem by introducing a "cut-off" (as we would call it today) and then removing it at the end of the computations. A task that he recognizes to be outside the present range of knowledge and to which further parallel difficulties are added, as the lack of additivity of the electromagnetic mass; and concludes *To deduce by such principles an electromagnetic explanation of matter does not look encouraging to me; I would rather say that we are close to circular arguments. Will it be possible to avoid them by some change in the way of posing the problem?.* A question that, since, has been first deferred to quantum electrodynamics, then to the standard model, then to quantum gravity and to strings without, however, reaching a satisfactory solution yet.

[20] C.f. footnote 13 in [LC911]. To say the truth it seems to me that the result is already contained in POINCARÉ's paper where it is obtained with the method that will appear also in [Le911]: why neither LEVI-CIVITA nor LEVI go back to the latter paper can possibly be understood if one remarks that only someone familiar with the theorem of POINCARÉ through some expert's lecture or who had studied in detail the large number of pages of the original paper would have really known it. This does not seem to have been the case of LEVI-CIVITA nor of LEVI nor, later, of BURGATTI.

[21] His scientific attitude and curiosity is witnessed by an amusing, as much as learned, paper (that he called an "exercise") on the stability of blackboards, [LC924], that opens a window on the academic life of the time in which the kind of problems that worried a jury for the exams of Rational Mechanics formed together with ARMELLINI and BISCONCINI.

to be looked for ... as statistical relations, generated by myriads of different events, and made eventually manageable thanks to the the law of large numbers." These words appear to indicate that LEVI-CIVITA was conscious of the labor of a Physics that was heading towards a deep revolution.

His treatise of Theoretical Mechanics, in collaboration with UGO AMALDI, published in 1921 has been an important innovation in Italy with a detailed discussion of the implications and applications of Analytical Mechanics and a set up of several problems of perturbation theory in the perspective of its Astronomy applications.

LEVI-CIVITA was hit by the racial laws and therefore he was ousted from university and Accademia dei Lincei.

Contemporary of LEVI-CIVITA and a BELTRAMI's student has been

PIETRO BURGATTI, (1868-1938).

His lectures on Rational Mechanics are set up along a very classical scheme and do not show a trace of the labor generated by the audacious and novel theories of POINCARÉ, although the introduction to the lectures begins with the promising remark (still more than valid today) "*It is quite true - unfortunately! - that at present people want to leave that scientific base that a long experience has show very solid, to look for a lower base more comfortable and more accessible to mediocrity. But there is only fog, I dare say, and the wish for Sun will come back*". Unlike the the texts on Rational Mechanics by CHELINI and SIACCI, where an attempt to a proof of D'ALEMBERT's principle is presented which goes back essentially to —AMPÈRE, here the principle is justified by its consequences and it is presented as parallel to NEWTON's laws starting a tradition that shall have a long life in Italy because it was adopted also by LEVI-CIVITA: one attempts a justification of the principle that is independent of NEWTON's laws presenting in this way to the student two theories for the same phenomenon, each with the proper experimental justification. A tradition that deviates from point of view followed in the main Mechanics treatises of the time, like that of APPELL or of VOIGT (although the latter had been translated into Italian and supported by a foreword by BELTRAMI himself) that give up a discussion of the principle and examine only its consequences.

BURGATTI in the interesting conclusive essay of the Rational Mechanics book, the "*Discourse on the historical development of Mechanics*" he presents a history of Mechanics in which ARISTOTELES is presented as thinking that "to be found among the properties of the circle all that is shown and admirable in the mechanical phenomena" and he gives no analysis not even superficial of Greek Astronomy and of ARISTARCUS, HIPPARCUS or PTOLEMY (the latter is not even quoted) essentially ignoring the *Papers on of ancient Astronomy* of SCHIAPARELLI who had exhaustively clarified, [Sc936], the meaning of cycles and epicycles in terms of the modern FOURIER transform.

He is among the last "Meccanici Razionali" that I consider in this brief review: he had several interests ranging from the mechanics of rigid systems, to potential theory where takes up works of BELTRAMI and VOLTERRA, to elasticity to phenomenology of the Celestial Mechanics. He gets remarkable results in the theory of gyroscopes finding some special motions of the HESS' gyroscopes in which the configurations are described by a differential equation of first order for two angles one of which isochronous: a mathematical problem already studied by LEVI-CIVITA and LEVI in which the existence of the average motion of the not isochronous coordinate is established, [Bu912], [LC911], [Le911].[22] In elasticity he studies the structure of the boundary potentials necessary to reduce the problem of the deformations of an elastic solid with given boundary stresses to an integral equation for functions defined on the surface: a problem studied in wide generality and for equations with variable coefficients in memorable papers by LEVI, [Le907]. In Astronomy he studies phenomenological questions: for example he goes back, [Bu915], to a critique

[22] The average motion, as already remarked, has been in reality studied by POINCARÉ, [Po885], who not is quoted here.

by SCHIAPARELLI to LAPLACE's theory of comets.[23] The interest of the paper lies rather in the (qualitative) discussion of the possibility that a comet influenced from field of the Sun and of another star penetrates the solar system following afterwards an elliptic orbit; a very difficult problem, even today, if one wishes to treat it in a mathematically rigorous way.

Important contributors to continuum Physics, mainly on elasticity theory, often inspired by the works of BETTI, [Be872], were CARLO ALBERTO CASTIGLIANO, (1847–1884), who also wrote an interesting monograph, [Ca879], mainly devoted to applicative aspects but with a effort of syntesis and of reduction to general principles (1870–1922), GIAN ANTONIO MAGGI, (1856–1937), ORAZIO TEDONE, (1870–1922), CARLO SOMIGLIANA, (1860-1955), CARLO SOMIGLIANA, (1860-1955), and ROBERTO MARCOLONGO, (1862–1943)), who contributed with their students and often inspired by VOLTERRA, to the wide development that later Mechanics of continua had in Italy.

Interest for electromagnetism and, perhaps lagging somewhat behind the times or perhaps in contrast with them, the theory of ether has been a (not always hidden) motivation of some papers, [So907], in which SOMIGLIANA, tries to overcome of exceed the results negative of BELTRAMI; and in general si dedicates himself to the study of the mechanics of the Continua ([So906]: going back to BETTI he obtains the representation, a by means of boundary potentials, of the elastic displacements, a"parametric" method that LEVI, [Le907], will apply to very general equations with variable coefficients) and produces continuous and varied studies also on waves theory. Still today "Hoepli manuals" of MARCOLONGO are useful: among which I quote the one on elasticity, [Ma904], and the one on the three bodies problem, [Ma919].

MAGGI several times (if only shadowy) criticizes VOLTERRA providing interesting interpretations of his results on the flexible and inextensible surfaces, [Ma884], which he reduces to "classical" results; or reducing to certain remarks of himself the analysis of VOLTERRA on anholonomous systems, [Ma901]. Later, while expressing admiration for VOLTERRA's famous result on the elastic stresses in not simply connected bodies, he reduces them to little, [Ma905].[24]

TEDONE studies the motion of a fluid an ellipsoidal form, thus following a tradition inspired in last analysis by Celestial Mechanics, [Te893], (see [Ch860], [Br861], [Pa771], [Be872], [Be880], [Vo881]) and presenting a new version of the motions in which the angular velocity is a linear function of the coordinates, alternative t that of VOIGT; very important is his analysis in elasticity theory leading to the extension to elastic vibrations KIRCHOFF's representation of the wave equation solutions, [Te897], [Te902].

Mathematical Analysis, a field that was starting to differentiate from Mathematical Physics also in the applications to the theory ordinary and partial differential equations, continued to generate remarkable contributions to problems of Physico-mathematical nature: for example

ERNESTO CESÁRO, (1859-1906).

---

[23] In fact it seems to me that the critique and, hence, the defense of LAPLACE's theory were not are really needed. Nevertheless, as also ARMELLINI will point out, he arrives at an interesting application of the problem of the two centers of gravitational attraction, [Ar920], one among the few applications in Astronomy of this remarkable system integrable by quadratures, if not the only one. The work has been praised by ARMELLINI who will use it in his confutation of a theory that attributed the distribution of the aphelia of the cometary orbits to a viscous interaction with ether, [Ar916].

[24] "*It seems to me indeed ... that one could keep unchanged the classical theorem that an elastic body occupying a finite region and not being subject to volume and pressures applied on the boundary will be found in the natural state.*" *I.e.* it would have been sufficient to think to the matter! However the interesting feature of VOLTERRA's result has been perhaps that a ring of , for instance, cut and straightened into a cylinder and then wringer on itself and finally glued again at its extremes *can* keep the ring form (among possible others, but not easy to do in practice). In any event VOLTERRA quotes him in his review in [Vo907].

His interest for Analysis (for instance for questions on divergent series), for number theory (his well known "first memory of arithmetic" marks is his strong entrance in European Mathematics, [Ce883]) and for analytic Geometry leaves a not secondary role, and one of increasing relevance, to questions of Mathematical Physics. A theorem on the zeroes of polynomial equation, for example, is formulated and interpreted in mechanical terms , [Ce886]. But it is on the elasticity theory and on the heat equation that he conjugates his expertise in Analysis with analytic Geometry: in a series of papers he studies various questions meant to simplify known results, [Ce901] (BELTRAMI), [Ce902] (POINCARÉ), [Ce906] (VOLTERRA), or a remarkable extensions [Ce889]. His papers on singular curves, *e.g.* with no tangent, are examples treated in a very modern fashion as constructions of self similar fractals, [Ce905]. His memoir on non Euclidean Geometry, which is unfortunately almost his scientific will, shows his increasing interest for intrinsic Geometry, [Ce905].

A further very interesting example is provided by

EUGENIO ELIA LEVI, (1883-1917),

Analyst to list also among the mathematical physicists for his interest for the existence of the average lunar motion and for his contributions to the theory of the elliptic and parabolic equations. Like many others he was fascinated by non Euclidean Geometry and from POINCARÉ's theory of Fuchsian groups. He gives, in Mathematical Physics, important contributions to elliptic equations theory. For instance the proof that the functions that enjoy the property of the average without being *a priori* differentiable are, instead, such necessarily and therefore hence are harmonic inaugurates the tradition particularly developed in Italy of the study of the regularity properties the solutions of elliptic equations. He revisits the lunar node motion problem extending LEVI-CIVITA's result, [LC911], on the existence of the average motion and providing a very elegant proof under much weaker Lipschitzian regularity assumption that prelude to the later works of BIRKHOFF, [Le911]. Particularly relevant for Mathematical Physics are also the papers on the boundary values problems for the elliptic equations of order $2n$ which, for instance, are met in elasticity theory: he reduces the problem to the theory of constant coefficients equations combined with an integral equation of FREDHOLM type, extending FREDHOLM's method for the POISSON's equation.

From Physics and from Astronomy.

Important contributions to Rational Mechanics came from quite different schools, like Physics and Astronomy. I mention here

GIOVANNI CANTONI, (1818–1897).

He is among the major Italian physicists of '800s: he contributed, possibly the first, with experimental skillfulness and theoretical intuition to the correct identification of the Brownian motion nature, [Ca867]: *"In fact, I think that the dancing motion of solid extremely minute particles inside a liquid, could be attributed to the different velocities that must be at a given temperature, both in such solid particles, and in the molecules of the liquid that collide with each other from every side. I do not know if others have already attempted such an explanation of the Brownian motions..."* In his 1867 paper he presents a series of experiments, that he performed, in which he shows evidence for the energy equipartition between suspended particles and solvent molecules and concludes that *"And in this way Brownian motion, so manifested, gives us one of the most beautiful and direct experimental proofs of the fundamental principles of the Mechanical theory of heat, manifesting the assiduous vibrational state that must be both and in liquids and in solids even when their temperature does not change"*.

In Italy atomism, perhaps thanks to AVOGADRO, and later of PIOLA, MOSSOTTI, and of others like SECCHI who wrote an ambitious synthesis of the atomic conceptions and of the theory of Ether, [Se874], has been at the time perhaps more accepted than the rest of Europe (we find traces also in works of several other authors). This work of CANTONI, suitably discussed and brought to light in [Pa982], is particularly interesting if we take into account that it was contemporary of the first works of BOLTZMANN on the heat theorem and on equipartition, [Bo866], and this has been perhaps the highest achievement of the Italian Theoretical Physics after MOSSOTTI and before FERMI.

ENRICO FERMI, (1901-1954).
Just at the end of the period considered the work of FERMI begins with his first works to electrodynamics and general relativity (where he rederives unpublished results of RICCI), [F923]. He investigates the validity of the ergodic hypothesis with an important work that shall have wide influence (on himself in particular) and on which he will come back in his last years with a great contribution to Mathematical Physics, [FPU55]. Since the beginning he imports in Italy echoes of the new Mechanics, [Fe923]. His discussion of the adiabatic invariants, [Fe923], is developed to analyze the BOHR-SOMMERFELD quantization rules, and precedes the interest for such invariants by LEVI-CIVITA who will apply them mainly to Celestial Mechanics problems providing remarkable complements to ARMELLINI theory on the problem of two bodies with variable masses. Certainly it is only in the years following the early twenties that FERMI will emerge as a great theoretical and, at the same time, experimental Physicist with contributions that will go well beyond Mathematical Physics in a strict sense, [BB001].

In Astronomy the year 1860 marks almost exactly the moment when Italian Astronomy becomes mainly positional or descriptive Astronomy and, by privileging the observational aspects with respect to the theoretical Celestial Mechanics. Therefore it does not follow the steps of the astronomers of the early 1800's[25] and employing Mechanics in a form that, with respect to the European studies, appears not too technical and as auxiliary apparatus for the quantitative analysis that follow, without innovation, well established methods as, for example, those of

OTTAVIANO FABRIZIO MOSSOTTI, (1791-1863).
His astronomical work, as his contribution to Physics, has been instead very innovative and provided important im[provements to the ephemerides computation methods, not refraining from entering into elaborate theoretical computations, and stimulated Mathematical Physicists who worked after him (for instance he was teacher to BETTI, as already mentioned). His posthumous paper on the determination of orbital elements from three observations, [Mo866], is an interesting memoir which completes a series of researches, begun in his youth, [Mo817], and which has to be compared with the similar problem treated by GAUSS in the case of asteroids: the work of MOSSOTTI has been highly appreciated in Europe and by GAUSS in particular. Unfortunately MOSSOTTI has been, for a a long period, 1823-1840, an exile (England first and then Argentina and Corfú) because of political reasons. This certainly deprived, at least in the observatory Brera where he was working when he had to leave for England, a generation of a teacher and of an example of a prolific attitude towards research: the consequences will be felt in the following decennies.

Among the Celestial Mechanics studies of the period around 1860 we must also count ANNIBALE DE GASPARIS, (1819–1892), who has been among the last to study, in the

[25] Like, to example, PLANA, who had attacked in a monumental work, [Pl832], questions like the Moon motion with the methods, then very modern, of Analytical Mechanics, or MOSSOTTI who had developed improvements to GAUSS's method for the computation orbital elements, [Mo866], or of SANTINI and others who engaged themselves *both* in observations and *also* in theoretical questions. See, among SANTINI's works, also the classical and still useful treatise of elementary Astronomy, [Sa830], in which he performed in detail the computation of the periods of the fundamental motions of Earth, Moon, planets, asteroids and comets and often applied Mechanics to the computation of perturbed orbits.

XIX century, problems of Celestial Mechanics understood in the modern sense of theory of orbits and computation of their perturbations. At least until ARMELLINI in Astronomy prevailed attention to observations, accurate but separate from the theory: a tendency principally represented by SECCHI and SCHIAPARELLI.

ANGELO SECCHI, (1818–1878).
A Jesuit father he was still in full vigor after 1860 although, after 1970, his work was strongly affected by the government the Italian Kingdom. An illustrious Physicist and Astrophysicist he left, beyond to the technical and specific works, interesting books on general subjects which became well know in Europe, like his monograph on the Sun, [Se870], accompanied by two other monographs on the Moon and on the Stars, or like the book on the unity of Physics from Thermodynamics and Electricity, dedicated to proving the unit of the physical forces, [Se874]: a very high level text remarkable for its precision in spite of a presentation in which mathematical developments are carefully avoided. Here he presents a completely atomistic conception of matter together with a lucid and deeply believed theory of actions at distance based on "ether", yet open to the possibility of alternative theories.[26] In Astronomy he devoted himself mainly to observations (he was the Director of the Collegio Romano observatory). He coordinated a mission for the observation of the 1870 solar eclipse formally directed by SANTINI which is interesting beyond the collected data also because final relation remains and gives an idea of the attitudes of the astronomers of the timed: some were technical and meticulous observers and others more inclined to contemplation, [Sa872]. Certainly his relatively premature death marked, perhaps, a halt in the development Astrophysics in the sense, much in debt to his pioneering work, continued to be in Europe and in the United States. Italian Astronomy when he was still alive became mainly observational and detached from Celestial Mechanics. The main successor of astronomers like PLANA, SANTINI, MOSSOTTI and SECCHI has been indeed

GIOVANNI SCHIAPARELLI, (1835–1910).
It might seem odd that the work of SCHIAPARELLI did not have a larger impact on Mechanics of the time. I think that it has been so because also SCHIAPARELLI has been more of an experimental astronomer than a theoretician of Mechanics.[27] The work of SCHIAPARELLI is however vast: and since the beginning is devoted to Astronomy of planetary observation. First Mars is object of careful and celebrated studies which take the first two volumes of his *Complete papers*. The two following volumes are dedicated to comets, meteorites and falling stars. In the fifth volume are collected imposing Celestial Mechanics computations, and he must have performed quite a few similar ones in several earlier works (which remained behind the scenes and we only see the results): here I mean the determination of the orbital parameters of the planet Esperia that he discovered in 1861. But the computations were quite standard, after GAUSS, PLANA and MOSSOTTI, and the true discovery consisted in seeing and recognizing to have seen a new errant

[26] "*The purpose of this work is to show how the common phenomena of Physics can be reduced to the general laws of matter motion, believing however that beyond the matter commonly called ponderable we must admit another matter invisible and not subject to gravity, which is called Ether or imponderable, whose motions accompany in various forms those of ponderable matter.*

[27] A surprising information is legated by PORRO in his commemoration of G. DARWIN, astronomer and son of CHARLES DARWIN, in which he also commemorates SCHIAPARELLI and POINCARÉ writing about SCHIAPARELLI: "*.. he did not want to devote himself to read Les Méthodes Nouvelles of* POINCARÉ". I do not know how much credit could be given to such a news, [Po913] (p.54), also because at p. 184 of [Bu913] PORRO claims to know *Les Méthodes Nouvelles* saying in this respect "*... I have followed during one year the course that in Torino gave a great Italian mathematician, Vito Volterra, and I have therefore seen how, for the special purpose of Astronomy, the really remarkable results of Poincaré had a purely negative importance*": a comment that contrasts with the substance of POINCARÉ's commemoration made by VOLTERRA in particular when he mentions the influence POINCARÉ's results on DARWIN's contribution to the theory of satellites formation "a the planet expenses", p.596 of the commemoration .

heavenly body. SCHIAPARELLI discovered the important phenomenon of the slowness of the rotation period of Venus and the large "libration" of Mercury (that today we know be a true rotation but that equally has been the discovery of an interesting anomaly that was hiding an even more surprising one, *i.e.* the resonance $3:2$). His writings on ancient Astronomy are still relevant: there he acutely discusses Greek Astronomy reexplaining and reviving the vituperated epycicles theory and making clear its interpretation in terms of the modern FOURIER series, [Sc936].

Also LORENZO RESPIGHI (1824-1888), has been, mainly, an astronomer working on observations (in spite of his important contributions Mathematical Analysis and had concrete interests for Mechanics (pendulum with friction), in the years preceding 1860, [Ar948]). The same can be said of FRANCESCO PORRO, (1861–1937), whose modern treatise on Astronomy is also remarkable for the quantitative and succinct discussion of the main astronomical periods, [Po920].[28]

Hence SCHIAPARELLI and the other astronomers of the time did not play the role that TISSERAND, THE VERRIER, D'OPPOLZER, GYLDYN, M. LEVY and then POINCARÉ had in Europe; and the absence of a figure that plaid a similar role generated a serious slowing in the development of Mechanics (and not only of Celestial Mechanics) which was not remedied by the, otherwise fundamental, works on Celestial Mechanics by LEVI-CIVITA: who, one would say on the same path as POINCARÉ, studies and solves brilliantly various questions *preparatory* to a systematic analysis of perturbation theory *without* however, quite surprisingly, really getting involved with it.
The gap behind European Celestial Mechanics was not filled either by

GIUSEPPE ARMELLINI, (1887–1958).
His work only starts towards the end of the period considered here, however precisely in this period he gives particularly relevant contributions to Celestial Mechanics. He develops an acute analysis of the problem of two bodies in which the center of attraction has increasing mass $M(t)$ and establishing the result that a *necessary and sufficient* condition in order that the distance between the two bodies could become smaller than an arbitrary prefixed value is that $\liminf_{t\to\infty}\frac{M(t)}{t}>0$, [Ar11]: the theme of the variable masses will be central for his interests and he will come back on it quite often extending results of several authors, as in the review [AR15] and the ambitious application to the eccentricity of Mars.[29]
His studies aim at concrete astronomical applications: not disdaining major perturbative computations and in particular computing the fattening of Jupiter from the very rapid precession of the apses caused by the planet on the V satellite, obtaining a very remarkable result and precision, [Ar912]. The derivation is elaborated as a consequence of a general clear perturbation theory, both for mathematical and for astronomical-phenomenological contents, designed for ephemerides computation, [Ar913]. With the

[28] He must also be remembered for having written in the commemoration of DARWIN, who is there compared with SCHIAPARELLI and POINCARÉ, [Po913] (p.51), "*....I only remark that no progress of the physical and astronomical has been based in an essential way on a work of Poincaré*". The statements on POINCARÉ imply also a comparison between the Italian "race" (*sic*, p. 53) and the French and were followed by a polemics on the same journal between PORRO and BURGATTI (and, on another journal, MASCART). From the polemical writings, [Po913], also emerge remarkable considerations on the interest of POINCARÉ's work which lead PORRO to conclude "*The fact that a French astronomer does not understand me,is natural and legitimate: I would like to be understood by the Italian astronomers, who do not miss models to imitate which are more consonant models to genius of our race*"; at the same time it appears clear that BURGATTI instead fully appreciated the work and the figure of POINCARÉ, [Bu913], while PORRO leaving aside not having grasped the interest POINCARÉ's work shows to possess a seriously narrow conception of the scopes of Celestial Mechanics. Finally PORRO's considerations on race may look strange: but they are perhaps a preview of his evolution into a "fascist since 1919" and in a participant to the March on Roma (and it would be interesting to find news on his political positions after 1926 which, if known, are not mentioned in his obituary). See also the preceding footnote 27.

[29] Which however is not consistent with what is believed today, cfr [La988].

same method he analyzes the perturbations affecting the first Neptune satellite concluding that they cannot be due to a second satellite but they still must be due to a polar flattening of Neptune, [Ar918], although is was not directly observable.

His interest for theoretical questions leads him to the n bodies problem: first with $n-1$ fixed attraction centers[30] and then with the problem of 3 or more mobile bodies mobile where he gets some extension of SUNDMAN's result, [Ar914]. A remarkable contribution, [Ar915], is the observation that in the three bodies problem the regularized solutions of LEVI-CIVITA and SUNDMAN can be obtained by imagining that the three bodies are homogeneous elastic hard spheres of radius r in the limit in which $r \to 0$ for all the finite time prefixed intervals thus obtaining a very concrete interpretation of the regularization: the derivation is a quite simple corollary of SUNDMAN's results but, nevertheless, the work is conceptually interesting (as LEVI-CIVITA remarked) and modern.

The study on Jupiter and Neptune satellites his results on the n bodies problem are perhaps his greater contributions for originality and interest.

About the comets theory he continues the mentioned analysis of BURGATTI and SCHIAPARELLI on why hyperbolic orbit comets are not observed, and argument treated by several authors since LAPLACE.

Conclusions.

From our point of view it must be recognized that Italian Mathematical Physics (hence Mechanics and Celestial Mechanics in particular), in the period 1860-1922 failed in general to become really an element in the development of ideas and of results that prevailed in Europe: personalities like LEVI-CIVITA and VOLTERRA appear as exceptions. Looking at the works, although monumental, of that period we can spot the germs of a provinciality that later led Italian Mathematical Physics to a serious crisis. There are just weak traces of the interest for the great debates that were attracting attention of MAXWELL, THOMPSON, BOLTZMANN, POINCARÉ, ERHENFEST, EINSTEIN, PLANCK, BOHR and were marking the birth of new conceptions and new problems involving the greatest mathematicians and physicists (PAINLEVÉ, GIBBS, HADAMARD, HILBERT, PICARD, ZERMELO, ...) and in Astronomy TISSERAND, LE VERRIER, GYLDYN, POINCARÉ, PAINLEVÉ, LEVY, HILL, NEWCOMB, LINDSTEDT, ADAMS, to quote only a few. Also the attention towards electromagnetism stays on a purely plane technical, although rich of results in potential and waves theory; an exception are the analysis of BELTRAMI, [Be886] (negative), and SOMIGLIANA, [So907] (possiblist), on the interpretation of the equations MAXWELL's equations in terms of elasticity theory (about the ether) and LEVI-CIVITA's the discussions on the electromagnetic mass, [LC907].

With the exception of a some papers of BETTI and BELTRAMI there traces of interest for Statistical Mechanics ware not really relevant: which is also true for the physicists of the time although BOLTZMANN was Accademico Linceo, with the exception of CANTONI.

Astronomy drifted towards observations.[31] In Celestial Mechanics the important contributions by LEVI-CIVITA were true exceptions to the rule: but he too did not enter the

---

[30] Extending remarkably VOLTERRA's result, [Vo899], on absence of collisions when the fixed bodies are aligned while the mobile point has non zero angular momentum with respect to the axis and he finds a way, [Ar913], of expressing the solution via a development in a *convergent series for all times.* He also studies the case in which the fixed points *are not* aligned and collisions are possible and are regularized, [AR915].

[31] An extreme example of observational Astronomy can be found in the relation of the group directed by SANTINI and SECCHI about the Sun eclipse of 1870 in which together with dry and technical relations of some of the members there are others like the one by ALESSANDRO SERPIERI, (1823–1855), where we read "*allow me to say that I deeply felt the high beauty and poetry of my station, with joyful flying flags ...*" (p. 199) and then "*The sea was painted in greysch green, and had lost its transparence. And rapidly changing took a glassy aspect almost taking the image of a solid matter. Waves that rebounded foaming on the shore gave the shadows a very beautiful azure color. In the total darkness someone compared the sea color to the greenish black of bottles*", [Sa872].

field (in spite of his very remarkable and well recognized results on Analytical Mechanics, on the three bodies problem, and on quasi periodic motions) other than marginally trying the questions posed by POINCARÉ's papers or by the new quantum mechanics, although he was open minded and interested in it unlike many of his contemporaries and successors, and although he gave essential contributions to the development of the Einsteinian theory and to differential Geometry. VOLTERRA gave original and revolutionary contributions to several problems closely linked to applications.

Others (like PICCIATI, PADOVA, CESÁRO, LEVI) prematurely disappeared leaving an indelible trace but an unfulfilled work. Mechanics splitting from Analysis and Geometry looked, in Italy, to goals that eventually were not really fertile: towards studies about more and more detailed properties, in the end not very interesting, on rigid motions or on structure of continua.

Perturbation theory, that POINCARÉ had so forcefully indicated as bearer of great novelties, was not really studied in Italy and practice of Celestial Mechanicsin in the innovative sense of European (and American) scientists was essentially abandoned. This has not been a short lived episode but, also for "political" reasons, went on and influenced the evolution in the following half century and has been certainly at the origin of the progressive emargination of Mathematical Physics from university curricula that has grown today to undeserved contempt with the consequent cancellation from many majors because "useless". Mathematical Physics has nevertheless received new ideas from Theoretical Physics thanks to FERMI and his school and from Mathematical Analysis thanks to the contributions of several outstanding analysts.

In connection with the above remarks one raises, from time to time, the question of why "*In spite of the concentration in Roma of a large part of the best mathematicians of the time (Roma has been by far the most prestigious university in the first half of the XX century) it was not possible to start the process that led to the creation of mathematical school in Pisa. Naturally there have been good mathematicians who were formed in Roma, thanks to the teaching of maestri that worked there, but their number and their quality cannot stand the comparison with those that were formed with Betti and Dini between 1860 and 1900: the pisan "miracle" did not come up again.*"
(http://www.math.unifi.it/matematicaitaliana/pannelli/XII.html)
But perhaps the question is a rhetorical one, or has an obvious answer and it is related to the tragic political (and consequently academic) events that followed 1922. Or perhaps the problem already preexisted and the political events were irrelevant? I think that they were not at all irrelevant. It is true that the oath of allegiance (1932) and the racial laws (1938) came much later: but it is also true that, much before the laws on the race, in the University and in the research institutions persons like GUGLIELMO MARCONI, as President of an "ad hoc" committee, agreed and decided the ousting of VOLTERRA from the instituzions of which he was a member contributing to the growth of a suspicion and fear climate.

And it is hard to see how such a climate could host a school in which LEVI–CIVITA and many others could reap the fruits of the work developed in the years before 1922 and to influence the younger generations with the freedom that permeates the academic and scientific choices in the developed societies.

It will be said: "but did we not, in any case, speak of a provinciality of Italian Theoretical Mechanics?": in reality Italy was, still in 1922, a young country emerged from about two centuries during which it had been divided and weak. A suitable time lapse is necessary to form solid schools: these can be realized in short times only by random fluctuations (like the "miracle" of Pisa): in Mathematical Physics appeared LEVI–CIVITA and VOLTERRA and they were at the apex of their scientific development, and already FERMI was moving the first steps: but the first two were emarginated (together with many other) and also the third was finally forced to leave Italy. An authoritarian climate grew up not very healthy for research work by whom was sensitive to what was happening and it is not surprising that the germs of provinciality prospered. In the culturally potent Germany much less time was needed to develop the converse process and then Science "ceased to speak German" in the brief interval of a few months, [An003].

Today we can hope, given the ease of motion and of contacts with other schools, in a positive evolution in which research in Mechanics could again to the heights that it reached only rarely in the period considered here, through personalities like BETTI, BELTRAMI, LEVI-CIVITA, VOLTERRA. String theory promises new ideal links with the more abstract Mathematics and it will again come *perhaps* (?) the happy time when Physicists like BETTI and VOLTERRA gave very important contributions to Mathematics or Mathematicians like BELTRAMI and LEVI-CIVITA gave as important contributions to the physical sciences.

One should not however hide the hovering severe difficulties due to a the research work being directed by people of a quite different stature than BRIOSCHI, BELTRAMI or VOLTERRA. Who ignore that the blossoming of research in Physics and Mathematics was based (and this is just an example by no means isolated) on people like RICCI–CURBASTRO who for his habilitation as a teacher in the high school presented in 1875 a dissertation about a generalization of a problem on hypergeometric functions studied by RIEMANN , or in 1877 summarized in a paper on Nuovo Cimento the forces of electromagnetism discussing and relating works of NEUMANN, RIEMANN, CLAUSIUS, MAXWELL, BETTI and with this work he obtained a scholarship to study abroad and, upon his return, finally obtaining just a non tenured assistantship to the chair of Calculus: a carrier and a series of choices that today would be considered of little interest.

I thank professor G. Foderá for the invitation to present a relation to the meeting "Hundred years of Astronomy in Italy, 1860–1960", at the Accademia dei Lincei, Roma, 26–28 march 2003 which gave me the occasion to begin studying (in a necessarily preliminary fashion) this interesting period for Italian science. And also for having informed me about the polemics on Poincaré that are quoted above, [Po913], [Bu913].

Bibliografia: Opere scelte

Armellini, G.: *Selecta. Memorie e note*, Accademia Nazionale dei Lincei, Roma, 1974.

Beltrami, E.: *Opere Matematiche di E. Beltrami*, Hoepli, Milano, 1902, Volumi 1-4.

Betti, E.: *Opere*, Hoepli, Milano, 1913. Vol. 1-2.

Cesáro, E.: *Opere scelte*, Cremonese, Roma, 1965. Volumi 1_1, 1_2, 2.

Cremona, L.: *Opere Matematiche*, Hoepli, Milano, 1914. Volumi 1-3.

Somigliana, C.: *Memorie scelte*, Lattes, Torino, 1936.

Brioschi, F.: *Opere matematiche di Francesco Brioschi*, Hoepli, Milano, 1901. Volumi 1-5.

Burgatti, P.: *Memorie scelte*, Zanichelli, Bologna, 1951.

Levi-Civita, T.: *Opere Matematiche. Memorie e note*, Zanichelli, Bologna, 1956. Volumi 1-6.

Levi, E.E.: *Opere*, Cremonese, Roma,1959. Volumi 1-2.

Maggi, G.A.: *Selecta. Raccolta di scritti matematici dal 1880 al 1931*, E.S.T., Milano, 1932.

Siacci, F.: *Scritti scientifici*, Provveditorato dello Stato, Roma, 1928, Volumi 1-2.

Schiaparelli, G.V.: Le opere di Schiaparelli, Hoepli, Milano, 1929. Volumi 1-6.

Mossotti, O.F.: *Scritti*, Domos Galilaeana, Pisa, 1942-1956. Volumi 1,2_1,2_2.

Tedone, O.: *Opere scelte*, Cremonese, 1956.

Somigliana, C: *Opere scelte*, Lattes, Torino, 1936.

Volterra, V.: *Opere Matematiche. Memorie e note*, Accademia Nazionale dei Lincei, Roma, 1954. Volumi 1-5.

Fermi, E.: *Note e Memorie*, Accademia Nazionale dei Lincei, 1962. Vol. 1-2.

Peano, G.: *Opere scelte*, Cremonese, Roma, 1959. Volumi 1-4.

Bibliografia: Articoli e monografie

Con il simbolo OP-n si indica il volume $n \in Z^1_+$ *della raccolta di "Opere" dell'autore citato.*

[Mo817] Mossotti, O.F.: *Nuova analisi del problema di determinare le orbite dei corpi celesti*, note nelle: Appendici alle Effemeridi Astronomiche, Milano, 1817/18/19. OP-1: p. 1–166.

[Pl832] Plana, G.: *Théorie du mouvement de la Lune*, Torino, 1832.

[Ch846] Chió, F.: *Ricerche sopra la serie di Lagrangia*, p. 1–57, Pomba, Torino, 1846.

[Sa830] Santini, G.: *Elementi di Astronomia con le applicazioni alla Geofisica, nautica, gnomonica e cronologia*, Tipografia del Seminario, Padova, 1830.

[Ch847] Chelini, D.: *Principio delle velocità virtuali*, Raccolta Scientifica di Fisica e Matematiche, **III**, 145–152, 1847.

[Ch847] Chelini, D.: *Effemeride della cometa di Hind*, Raccolta scientifica di Fisica e di Matematiche, **III**, 115–117, 1847.

[Mo851] Mossotti, O.F.: *Lezioni di Meccanica Razionale*, s.e., Firenze, 1851.

[Ch859] Chelini, D.: *Determinazione analitica della rotazione dei corpi liberi secondo i concetti di Poinsot*, Memorie dell'Accademia delle Scienze dell'Istituto di Bologna, **X**, 583–620, 1859.

[Ch860] Chelini, D.: *Elementi di Meccanica Razionale con appendice sui principi fondamentali delle matematiche*, Legnani, Bologna, 1860.

[Ch861] Chelini, D.: *Sulla legge onde un ellissoide eterogeneo propaga la sua attrazione*, Memorie dell'Accademia delle Scienze dell'Istituto di Bologna, **I** (serie II), 1861.

[Br861] Brioschi, F.: *Dé veloppements relatifs au §3 des réchèrches de Dirichlet sur un problème d'hydrodynamique*, Journal für reine und angewandte Mathematik, **LIX**, 63–73, 1861. OP-5: 293.

[Be863] Betti, E.: *Teorica delle forze che agiscono secondo la legge di Newton e sue applicazioni alla elettricità statica*, 1863, Op-2: 45.

[Be866] Betti, E.: *Sopra la teoria della capillarità*, Annali delle università toscane, **IX**, 5–24, 1866. OP-2: 161.

[Bo866] Boltzmann, L.: *Über die mechanische Bedeutung des zweiten Haupsatzes der Wärmetheorie*, in "Wissenschaftliche Abhandlungen", ed. F. Hasenöhrl, vol. I, p. 9–33, reprinted by Chelsea, New York.

[Mo866] Mossotti, O.F.: *Sopra la determinazione dele orbite dei corpi celesti per mezzo di tre osservazioni*, OP-2_1: 345.

[Ca867] Cantoni, G.: *Su alcune condizioni fisiche dell' affinità e sul moto browniano*, Il Nuovo Cimento, **27**, 156–167, 1867.

[Be868] Betti, E.: *Sopra la determinazione della temperatura nei corpi solidi omogenei*, Memorie della Società ita;iana delle scienze (detta dei XL),**I**, 165–190, 1868. OP-2: 216.

[Be868] Beltrami, E.: *Saggio di interpretazione della geometria non euclidea*, Giornale di matematiche, **VI**, 284–322, 1868. OP-1: 374.

[Se870] Secchi, A.: *Le Soleil. Exposé des principales découvertes modèrnes sur la structure de cet astre, son influence dans l'Univers et ses rélations avec les autres corps celéstes*, Gauthiers–Villars, Parigi, 1870.

[Pa871] Padova, E.: *Sul moto di un ellissoide fluido e omogeneo*, Annali della Scuola Normale di Pisa, **I**, 1–88, 1871.

[Be872] Betti, E.: *Teoria dell'elasticità*, serie di articoli sul Nuovo Cimento (1972–1973). OP-2: 291.

[Sa872] Santini, G.: *Rapporti sulle osservazioni dell'eclisse totale di Sole*, Lao, Palermo, 1872.

[Do873] Dorna, A.: *Lezioni di Meccanica Razionale*, Arnaldi, Torino, 1873.

[Be874] Beltrami, E.: *Ricerche sulla cinematica dei fluidi*, Memorie dell'Accademia delle Scienze di Bologna, 1871-1874. OP-2: 202–379.

[Se874] Secchi, A.: *L'unità delle forze fisiche. Saggio di filosofia naturale*, Treves, Milano, 1874.

[Cr875] Cremona, L. et al.: *Relazione intorno ad una memoria del Sig. Colonnello Pietro Conti avente titolo "Sulla resistenza d'attrito" della Commissione composta da Betocchi, Baserna, Beltrami e Cremona*, Atti dell'Accademia dei Lincei, Memorie, **II**, 3–15, 1874/75. OP-3: 369–383.

[Si877] Siacci, F.: *Della rotazione dei corpi liberi*, Memorie della Società italiana delle scienze (detta dei XL), **III**, 1877. Op-2: 171.

[Be878] Beltrami, E.: *Sulle funzioni potenziali di sistemi simmetrici intorno ad un asse*, Rendiconti del Regio Istituto Lombardo, **XI**, 13 p., 1878.

[Ca879] Castigliano, A.: *Théorie de l'équilibre des systmes élastique et ses applications*, A.F. Negro, Torino, 1879.

[Be880] Beltrami, E.: *Sulla teoria dell'attrazione degli ellissoidi*, Memorie dell'Accademia delle Scienze dell'Istituto di Bologna, p. 573–616, 1880. OP-3: 269

[Be880] Betti, E.: *Sopra l'equilibrio di una massa di gas rarefatto isolata nello spazio*, Il Nuovo Cimento, **7**, 26–33, 1880.

[Vo881] Volterra, V.: *Sul potenziale di un'ellissoide omogenea sopr se stessa*, Il Nuovo Cimento, **IX**, 221–229, 1881. OP-1: 1.

[Be882] Beltrami, E.: *Sulla teoria dei sistemi di conduttori elettrizzati*, Rendiconti del Regio Istituto Lombardo, **XV**, 400–407, 1882. OP-3: 413.

[Be882] Beltrami, E.: *Sull'equilibrio delle superfici flessibili e inestendibili*, Memorie dell'Accademia delle Scienze dell'Istituto di Bologna, **III**, 217–265, 1882. OP-4: 420.

[Vo883] Volterra, V.: *Sopra alcuni problemi di teoria del potenziale*, Annali della Scuola Normale di Pisa, **III**, 207–270, 1882. OP-1: 140.

[Ce883] Cesáro, E.: *Sur diverses questions d'arithmetique*, Mémoires de la Societé royale des sciences de Liège, **10**, 1883. Op-1_1: 10.

[Vo884] Volterra, V.: *Sopra un problema di elettrostatica*, Nuovo Cimento, **XVI**, 49–57, 1884. OP-1: 188.

[Bo884] Boltzmann, L.: *Über die Eigenshaften monzyklischer und anderer damit verwandter Systeme*, in "Wissenshafltliche Abhandlungen", ed. F.P. Hasenöhrl, vol. III, p. 122–152, Chelsea, New York, 1968, (reprint).

[Ma884] Maggi, G.A.: *Sopra l'equilibrio delle superfici flessibili e inestendibili*, Rendiconti dell'Istituto Lombardo, **XVII**, 1884. OP: 37.

[Be885] Beltrami, E.: *Sulla rappresentazione delle forze newtoniane per mezzo di forze elastiche*, Rendiconti del Regio Istituto Lombardo, **XVII**, p. 9, 1884.

[Pa885] Padova, E.: *Sul moto di rotazione di un corpo rigido*, Atti dell'Accademia delle Scienze di Torino, 38–47, 1885.

[Po885] Poincaré, H.: *Sur les courbes définies par les équations differentielles*, Journal de Mathématiques pures et appliquées, **I**, 1885. In *Oeuvres Complètes*, **1**, lavoro n. 57: si veda p. 156 del lavoro n. 57, Gauthiers–Villars, Paris 1934–1954.

[Be886] Beltrami, E.: *Sull'interepretazione meccanica delle formole di Maxwell*, Memorie dell'Accademia delle Scienze dell'Istituto di Bologna, **VII**, 1–38, 1886. OP-4: 190.

[LC886] Levi-Civita, T.: *Sulle rasformazioni delle equazioni dinamiche*, Annali di Matematica, **XXIV**, 255–300, 1896. OP-1: 207.

[Vo887] Volterra, V.: *Sopra una estensione della teoria di Riemann sulle funzioni di variabile complessa*, Rendiconti della Accademia dei Lincei, **III**, 281–287, 1887. Op-1: 329.

[Vo887] Volterra, V.: *Sopra le funzioni che dipendono da altre funzioni*, Rendiconti della Regia Accademia dei Lincei, **III**, 97–105, 1887. OP-1: 294.

[Vo887] Volterra, V.: *Sopra le funzioni dipndenti da linee*, Rendiconti della Regia Accademia dei Lincei, **III**, 225–230, 1887. OP-1: 315.

[Ce887] Cesáro, E.: Nouveax Annales de Mathématiques, **6**, 36–43, 1887. OP-2: 400.

[Be888] Betti, E.: *Sopra l'entropia di un sistema newtoniano in moto stabile*, Rendiconti della Regia Accademia dei Lincei, **IV**, 113–115, 1888. OP-2: 488.

[Br866] Brioschi, F.: Serie di lavori sull'idraulica. Pubblicati si "Il Politecnico" fra il 1866 e il 1873. OP-5: 409–510.

[Vo889] Volterra, V.: *Sur une généralization de la théorie des fonctions d'une variable imaginaire*, Acta Mathematica, **12**, 233–286, 1889. OP-1: 363.

[Be889] Beltrami, E.: *Lettera a Cesáro*, Rendiconti del circolo matematico di Palermo, **III**, 67–79, 1889. OP-4: 320.

[Be889] Beltrami, E.: *Un precursore italiano di Legendre e Lobachevsky*, Rendiconti della Regia Accademia dei Lincei, **V**, 411–448, OP-4: 348.

[Ce889] Cesáro, E.: *Sulle variazioni di volume nei corpi elastici*, Rendiconti della Accademia dei Lincei, **V**,259–264, 1889.

[Vo890] Volterra, V.: *Sopra una estensione della teoia di Jacobi-Hamilton del calcolo delle variazioni*, Rendiconti della Accademia dei Lincei, **VI**, 127–138, 1890. Op-1: 464.

[Vo891] Volterra, V.: *Sopra le equazioni fondamentali dell'elettrodinamica*, Nuovo Cimento, **XXIX**, 147–154, 1891. OP-1: 496.

[Vo892] Volterra, V.: *Sur les vibrations lumineuses dans les milieux biréfrangents*, Acta Mathematica, **16**, 153–215, 1892. OP-1: 514.

[Si892] Siacci, F.: *Meccanica Razionale*, Torino, 1892.

[Te893] Tedone, O.: *Sul moto di un fluido contenuto in un involucro ellissoidico*, Rendiconti della Regia Accademia dei Lincei, **II**, 123-130, 1893. OP: 1.

[Vo894] Volterra, V.: *Sur les vibrations des corps elastiques isotropes*, Acta Mathematica, **18**, 161–232, 1894. OP-2: 19.

[Vo894] Volterra, V.: *Esercizi di Fisica Matematica*, Rivista di Matematica, **IV**, 1–14, 1894. OP-2: 74.

[Vo895] Volterra, V.: *Sulla teoria dei movimenti del polo terrestre*, Astronomische Nachrichten, **138**,33–52, 1895. OP-2: 87.

[Vo896] Volterra, V.: *Replica ad una nota del Prof. Peano*, Rendiconti dell'Accademia dei Lincei, **V**, 4–7, 1896. OP-2: 213.

[Pe895] Peano, G.: *Il pricipio delle aree e la storia di un gatto*, Rivista di matematiche, **V**, 31–32, 1895. OP-3: 285.

[Pe895] Peano, G.: *Sopra lo spostamento del polo e della Terra*, Atti dell'Accademia delle Scienze di Torino, **XXX**, 515–523, 1895. OP-3: 288.

[Pe895] Peano, G.: *Sul moto di un sistema nel quale sussitstono moti interni variabili*, Rendiconti dell'Accademia dei Lincei, **IV**, 280–282, 1895. OP-3: 304.

[Pe896] Peano, G.: *Sul moto del polo terrestre*, Rendiconti dell'Accademia dei Lincei, **V**, 163–168, 1896. OP-3: 309.

[Te897] Tedone, O.: *Sulle vibrazioni dei corpi solidi omogenei e isotropi*, Memorie della Reale Academia delle Scienze di Torino, **XLVII**, 181–258, 1897. OP: 16.

[Vo897] Volterra, V.: *Sopra alcune questioni di inversione di integrali definiti*, Annali di Matematica pura e applicata, **XXV**, 139–178, 1897. OP-2: 279.

[Vo897] Volterra, V.: *Sulla scarica elettrica nei gas e sopra alcuni fenomeni di elettrolisi*, Rendiconti dell'Accademia dei Lincei, **VI**, 389–401, 1897. OP-2: 317.

[Vo897] Volterra, V.: *Un teorema sugli itegrali multipli*, Atti dell'Accademia delle Scienze di Torino, **XXXII**, 859–868, 1897. OP-2: 329.

[Vo898] Volterra, V.: *Sul fenomeno delle "seiches"*, Conferenza al congresso SIF di Torino del 1898, OP-2: 370.

[Vo898] Volterra, V.: *Sur la théorie des variations des latitudes*, Acta Mathematica, **22**, 201-357, 1898. OP-2: 452.

[Vo899] Volterra, V.: *Sui fondamenti della teoria delle equazioni differenziali lineari*, Memorie della Società italiana delle scienze (detta dei XL), 1899, OP-2: 383.

[Vo899] Volterra, V.: *Sopra alcune applicazioni della rappresentazione analitica delle funzioni del Prof. Mittag–Leffler*, Atti dell'Accademia delle Scienze di Torino, **XXXIV**, 492–494, 1898-99. OP-3: 591.

[RL900] Levi-Civita, T., Ricci-Curbastro, G.: *Méthodes de calcul différentiel absolu et leurs applications.* Mathematische Annalen, **IV**, 125-201, 1900. OP-1: 479.

[Ma901] Maggi, G.A.: *Di alcune nuove forme delle equazioni della dinamica applicate ai sistemi anolonomi*, Rendiconti dell'Accademia dei Lincei, **X**, 1901. OP: 64.

[Te902] Tedone, O.: *Saggio di una teoria generale delle equazioni dell'equilibrio elastico per un corpo isotropo*, Memorie della Reale Accademia delle Scienze di Torino, **VIII**, 129–180, 1902. OP: ??.

[Bi902] Bianchi, L., Dini, U., Cerruti, V., D'Ovidio, E., Veronese, G.: *Relazione sul concorso al premio reale, del 1901, per la Matematica*, Rendiconti delle adunanze solenni dell'Accademia dei Lincei, 142–151, 1902.

[LC901] Levi-Civita, T.: *Sul massimo cimento dinamico dei sistemi elastici*, Il Nuovo Cimento, **II**, 188–196, 1901. OP-2: 145.

[LC901] Levi-Civita, T.: *Sopra la determinazione di soluzioni particolari di un sistema canonico quando se ne conosce qualche integrale o relazione invariante*, Rendiconti dell'Accademia dei Lincei, **X**, 1–9, 1901 e **X**, 35–41, 1901_1. OP-2: 87.

[LC901] Levi-Civita, T.: *Sulla resistenza dei mezzi fluidi*, Rendiconti dell'Accademia dei Lincei, **X**, 3–9, 1901_2. OP-2: 129.

[Ce901] Cesáro, E.: *Sopra un'equazione funzionale trattata da Beltrami*, Rendiconti dell'Accademia delle Scvienze di Napoli, **7**, 284–289, 1901. OP-2: 432.

[Ce902] Cesáro, E.: *Intorno ad una limitazione di costanti nella teoria analitica del calore*, Rendiconti dell'Accademia delle Scvienze di Napoli, **8**, 31–38, 1902.

[LC903] Levi-Civita, T.: *Condition de choc dans le problème restreint des trois corps*, Comptes rendu de l'Acadmie des Sciences de Paris, **CXXXV**, 221-223, 1903. OP-2: 275.

[LC904] Levi-Civita, T.: *Sopra l'equazione di Kepler*, Rendiconti dell'Accademia dei Lincei, **XIII**, 260–268, 1904. OP-2: 321.

[LC904] Levi-Civita, T.: *Sopra un problema di elettrostatica che si è presentato nella costruzione dei cavi*, Rendiconti del Circolo Matematico di Palermo, **XIII**, 173–228, 1904. OP-2: 339.

[Ma904] Marcolongo, R.: *Teoria matematica dell'elasticità*, Hoepli, Milano, 1904.

[Vo904] Volterra, V.: *Note on the application of the method of images to problems of vibrations*, Proceedings of the London Mathematical Scociety, **2**, 327–331, 1904. OP-3: 55.

[Si905] Siacci, F.: *Sul principio dei lavori virtuali*, Rendiconti della Regia Accademia delle Scienze Fisiche e Matematiche di Napoli, **11**, 1905. OP-2: 601.

[Ma905] Maggi, G.A.: *Sull'interpretazione del nuovo teorema di Volterra sulla teoria dell'elasticità*, Rendiconti dell'Accademia dei Lincei, **XIV**, 1905. OP: 255.

[Ce905] Cesáro, E.: *Remarques sur la courbe e Von Koch*, Atti della Accademia delle Scienze di Napoli, **12**, 1905. OP-2: 464.

[Ce905] Cesáro, E.: *Fondamento intrinseco della pangeometria*, Memorie dell'Accademia dei Lincei, **5**, 155–183, 1905. OP-2: 341.

[LC906] Levi-Civita, T.: *Sulla penetrazione dei proiettili nei mezzi solidi*, Atti dell'IStituto Veneto di Scienze, Lettere ed Arti, **LXV**, 1149–1158, 1905-06. OP-2: 505.

[Ce906] Cesáro, E.: *Sulle formole del Volterra fondamentali nella teoria delle distorsioni elastiche*, Rendiconti dell'Accademia delle Scienze di Napoli, **12**, 311–321, 1906.

[So906] Somigliana, C.: *Sulla teoria Maxwelliana dell'azione a distanza*, Rendiconti dell'Accademia dei Lincei, **XVI**, 1907. OP: 325.

[LC907] Levi-Civita, T.: *Scie e legge di resistenza*, Rendiconti del Circolo Matematico di Palermo, **XXIII**, 1-37, 1907. OP-2: 519.

[LC907] Levi-Civita, T.: *Sur le mouvement de l'électricité sans liaisons ni forces extérieures*, Comptes Rendus de l'Academie des Sciences, **CXLV**, 417-420, 1907. OP-2: 583.

[LC907] Levi-Civita, T.: *Sulla massa elettromagnetica*, Nuovo Cimento, **XIV**, 387-412, 1907. OP-2: 587.

[LC907] Levi-Civita, T.: *Sullo sviluppo delle funzioni implicite*, **XVI**, 2–12, 1907. OP-2: 573.

[LC907] Levi-Civita, T.: *Sulle onde progressive di tipo permanente*, Rendiconti dell'Accademia dei Lincei, **XVI**, 777–790, 1907. OP-2: 615.

[Le907] Levi, E.E.: *Sulle equazioni lineari totalmente ellittiche alle derivate parziali*, Rendiconti del Circolo Matematico di Palermo, **XVI**, 1907. OP-2: 28.

[So907] Somigliana, C.: *Sopra alcune formole fondamentali della dinamica dei mezzi isotropi*, Serie di note. OP: 273,298,307.

[Vo907] Volterra, V.: *Sur l'équilibre des corps élastiques multiplement connexes*, Annales scientifiques de l'École Normale Superieure, **XXIV**, 401–518, 1907. OP-3: 153.

[Vo908] Volterra, V.: *Drei vorlesungen über neuere fortschritte der mathematischen physik*, Archiv der mathematik und Physik, **XXI**, 97–181, 1908. OP-3: 389.

[Pe908] Peano, G.: *Sul livro V di Euclide*, Bollettino di matematiche (Conti), Bologna, **V**, 87–91, 1908. OP-3: 377.

[Ca911] Castellano, F.: *Lezioni di Meccanica Razionale*, G.U. Cassone, Torino, 1911.

[LC910] Levi-Civita, T.: *Sur les équations générales du mouvement d'un corpuscule dans un champ magnétique et un champ électrique superposés*, Archiv for Mathematik og Naturvidesnskab, **XXXI**, 3–7, 1910. OP-3: 199.

[LC911] Levi-Civita, T.: *Sur les équations linéaires à coefficients periodiques et sur le mouvement moyen du noeud lunaire*, Annales de l'École Normale Superieure, **28**, 325–376, 1911. OP-3: 205.

[LC911] Levi-Civita, T.: *Estensione ed evoluzione della Fisica Matematica (nell'ultimo cinquantennio, con speciale riguardo al contributo italiano)*, Atti della Società italiana per il progresso delle scienze, **V**, 237–254, 1911. OP-3: 275.

[Le911] Levi, E.E.: *Sur les équations différentielles périodiques*, Comptes Rendus de l'Academie des Sciences, **153**, 1911. OP-1: 224.

[Ar911] Armellini, G.: *Il problema dei due corpi nell;ipotesi di masse variabili*, Rendiconti dell'Accademia dei Lincei, **XX**, 682–687, 1911. OP-1: 107.

[LC912] Levi-Civita, T.: *Sulla gravitazione di un tubo sottile con applicazione all'anello di Saturno*, Rendiconti del Circolo Matematico di Palermo, **XXXII**, 354–374, 1907. OP-3: 294.

[Vo912] Volterra, V.: *Sulle temperature nell'interno delle montagne*, **IV**, 111–126, 1912. OP-3: 471,

[Vo912] Volterra, V.: *L'applicazione del calcolo ai fenomeni di eredità*, OP-3: 554–568.

[Vo912] Volterra, V.: *Sur les equations intégro-differentielles et leurs applications*, Acta mathematica, **35**, 295–356, 1912. OP-3:487.

[Bu912] Burgatti, P.: *Osservazioni sul moto dei giroscopi*, Accademia delle Scienze del-l'Istituto di Bologna, **IX**, 281–296, 1911-12.

[Ar912] Armellini, G.: *Determinazione matematica dello schiacciamento polare di Giove*, Rendiconti della Accademia dei Lincei, **XXI**, 334–341, 1912. OP-1: 285.

[Bu913] Burgatti, P.: *Giudizi su Poincaré*, Rivista di Astronomia e Scienze affini (Bollettino della Società Astronomica Italiana), **VII**, p. 180–182, 1913. Seguito da replica di Porro, p. 182–184.

[Vo913] Volterra, V.: *Sui fenomeni ereditari*, Rendiconti dell'Accademia dei Lincei, **XXII**, 529–539, 1923.

[Ar913] Armellini, G.: *Teoria delle perturbazioni (applicazioni a satelliti, comete e pianeti)*, Memorie della Società italiana delle Scienze (detta dei XL), **XVIII**, 3–41, 1913. OP-1: 201.

[Ar913] Armellini, G.: *Sul moto di un punto attratto da più centri fissi*, Rendiconti dell'Accademia dei Lincei, **XXII**, 70–76, 1913. OP-1: 71.

[LC913] Levi-Civita, T.: *Nuovo sistema canonico di elementi ellittici*, Annali di Matematica, **XX**, 1–17, 1913. OP-3: 341.

[Po913] Porro, F.: *Giorgio Darwin*, e *Un'ultima parola su Enrico Poincaré e sui suoi apologisti*, Rivista di Astronomia e Scienze affini (Bollettino della Società Astronomica Italiana), **VII**, p. 50–64 e p. 327-334, 1913.

[Ar914] Armellini, G.: *Un théorème géneral sur le problème des n corps*, Comptes rendus de l'Academie des Sciences de Paris, **158**, 680–684, 1914. OP-1: 43.

[Vo913] Volterra, V.: *Sulle equazioni alle derivate funzionali*, Rebdiconti dell'Accademia dei Lincei, **XXIII**, 393–399, 1914. OP-4: 5.

[Ar915] Armellini, G.: *Sul moto di un punto attratto da più punti fissi*, Memorie della Società italiana delle Scienze (detta dei XL), **XIX**, 1–23, 1915. OP-1: 84.

[Ar915] Armellini, G.: *Estensione della soluzione del Sundman dal caso di corpi ideali, al caso di sferette elastiche omogenee*, Rendiconti dell'Accademia dei Lincei, **XXIV**, 184–190, 1915.

[Bu915] Burgatti, P.: *Osservazioni sull'origine delle comete*, Atti dell'Accademia delle Scienze dell'Istituto di Bologna, **2**, 3-10, 1914-15. OP-1: ???.

[Ar915] Armellini, G.: *Il problema dei due corpi di massa variabile*, Memorie della Società italiana delle Scienze (detta dei XL), **XIX**, 3–24, 1915. OP-1: 121.

[Vo916] Volterra, V.: *Metodi di calcolo degli elementi di tiro per artiglieria aeronautica*, Rendiconti Isituto centrale aeronautico, **V**, 1916. OP-4: 200–248.

[Vo916] Volterra, V.: *Teoria delle potenze e dei logaritmi delle funzioni di composizione*, Memorie dell'Accademia dei Lincei, **XI**, 167–250, 1916. OP-4: 118.

[Ar916] Armellini, G.: *Sopra un'ipotesi del Pickering relativa alla frequenza degli afeli delle orbite cometarie nelle vicinanze dell'antiapice*, Rendiconti dell'Accademia dei Lincei, **XXV**, 622–627, 1916. OP-1: 273.

[LC916] Levi-Civita, T.: *Sopra due trasformazioni canoniche desunte dal moto parabolico*, Rendiconti dell'Accademia dei Lincei, **XXV**, 446–458, 1916. OP-3: 573.

[LC917] Levi-Civita, T.: *Nozione di parallelismo in na varietà qualunque e conseguente specificazione geometrica della curvatura riemanniana*, Rendiconti del Circolo Matematico di Palermo, **XLII**, 173–215, 1917. OP-4: 1.

[LC917] Levi-Civita, T.: *Sulla espressione analitica spettante al tensore gravitazionale nella teoria di Einstein*, Rendiconti dell'Accademia dei Lincei, **XXVI**, 381–391, 1917. OP-4: 47.

[Ma919] Marcolongo, R.: *Il problema dei tre corpi da Newton ai nostri giorni*, Hoepli, Milano, 1919.

[LC918] Levi-Civita, T.: *Sur la regularization du problème des trois corps*, Acta Mathematica, **42**, 99–144, 1918. OP-4: 211.

[Ar918] Armellini, G.: *Sur les perturbations du satellite de Neptune*, Mémoires du Bullettin astronomique, **XXXV**, 97–101, 1918. OP-1: 259.

[Vo920] Volterra, V.: *Functions of composition*, The Rice Institute Pamphlet, **VII**, 181–251, 1920. OP-4: 312.

[Vo920] Volterra, V.: *Sur l'enseignement de la Physique Mathématique et de quelques points d'analyse*, Comptes rendus du Congrès International des Mathématiques, p. 81–97, 1920. OP-4: 370.

[Vo921] Volterra, V.: *Relazione sull'insegnamento della dinamica nelle scuole industriali*, Rivista d'ottica e mecccanica di precisione, **1**, 4–31, 1921. OP-4: 478.

[Ar920] Armellini, G.: *Osservazioni sopra le comete secolari*, Rendiconti dell'Accademia dei Lincei, **XXIX**, 183–186, 1920. OP-1: 279.

[Po920] Porro, F.: *Trattato di Astronomia*, Zanichelli, 1920.

[Fe923] Fermi, E.: *Sul peso dei corpi elastici*, Memorie dei Lincei, **14**, 114–124, 1923. OP-1: 61.

[Fe923] Fermi, E.: *Beweis, dass ein mechanisches normalsystem im allgemeinen quasi–ergodish ist*, Physikalische Zeitschrift, **24**, 261–265, 1923.OP-1: 79.

[Fe923] Fermi, E.: *Il principio delle adiabatiche ed i sistemi che non ammettono coordinate angolari*, Nuovo Cimento, **25**, 171–175, 1923. OP-1: 88.

[LC925] Levi-Civita, T.: *Lezioni di Calcolo diffeenziale assoluto*, Stock, Roma, 1925.

[LC925] Levi-Civita, T.: *Sulla stabilità delle lavagne a cavalletto*, Periodico di Matematiche, **IV**, 59–73, 1925. OP-4: 333.

[LC925] Levi-Civita, T.: *Commemorazione del Socio Nazionale Prof. G. Ricci-Curbastro*, Memorie dell'Accademia dei Lincei,**I**, 556–564, 1925. OP-4: 391.

[Bu935] Burgatti, P.: *Sopra un metodo per studiare l'azione prodotta da un mezzo resistene sul moto dei corpi celesti*, Rendiconti delle sessioni della Regia Accademia delle Scienze dell'Istituto di Bologna, 3–12, 1934-35.

[Sc936] Schiaparelli, G.: *Scritti sulla storia dell'astronomia antica*, Zanichelli, Bologna, 1926.

[Ar948] Armellini, G.: *Lorenzo Respighi nel sessantesimo della morte*, Quanderni di ROma, 1948. OP-1: 483.

[FPU55] Fermi, E., Pasta, J., Ulam, S.: *Studies of nonlinear problems*, Los Alamos report LA-1940, 1955, printed in E., Fermi, *Collected papers*, p. 978–988, vol. **2**, 1965.

[Ru969] Ruelle, D.: *Statistical Mechanics*, Benjamin, 1969.

[Pa982] Pais, A.: *Subtle is the Lord*, Oxford, 1982.

[La988] Laskar, J.: *Secular evolution of the solar system over* 1 *million years*, Astronomy and astrophysics, **198**, 341–362, 1988.

[BGT998] Boi,L., Gicardi, L., Tazzioli, R.: *La découverte de la géometrie non euclidienne sur la pseudosphère*, Blanchard, Paris, 1998.

[BB001] Bernardini, C., Bonolis, L.: *Conoscere Fermi, nel centenario della nascita*, Compositori, Bologna, 2001.

[An003] Antoci, S.: *Quando la Fisica parlava tedesco*, INDAM-CNR, Firenze, 2003.

Commemorazioni

Domenico Chelini: (1802–1878, Beltrami), *Della vita e delle opere di Domenico Chelini*, estratto dal volume in commemorazione di Domenico Chelini, Hoepli, Milano, 1881.

Ottaviano Fabrizio Mossotti: (1791–1863, Betti) Giornale di matematiche ad uso degli studenti, **I**, 92, 1863. OP-2: 154.

Enrico Betti: (1823-1892, Volterra), Nuovo Cimento, **32**, 5–7, 1892. OP-1: 600.

Eugenio Beltrami: (1835–1900, Cremona), OP-3: 470.

Giuseppe Picciati: (1868-1908, Levi-Civita) Nuovo Cimento, **XV** 363–368, 1908. OP-3: 15.

Valentino Cerruti: (1850-1909, Levi-Civita) Rendiconti dell'Accademia dei Lincei, **XVIII**, 565–575, 1909. OP-3: 135.

Gregorio Ricci-Curbastro: (1853-1925, Levi-Civita) Memorie dell'Accademia dei Lincei,**I**, 556–564, 1925. OP-4: 391.

Henri Poincaré: (1854–1912, Volterra) OP-3: 578.

Lorenzo Respighi: (1824-1888, Armellini) Quaderni di Roma, 1948. OP-1: 483.

Francesco Porro de Somenzi, (1861-1937, Barbieri), in Annuario dell'Università di Genova, p. 397–399, 1936-1937.