Order, Chaos, Irreversibility, Entropy: rethinking an old dualism.*

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Two different conceptions of motion emerge from the attempts to construct a theory of motion in Physics. The very idea of starting on such a task is daunting: we see so many different kinds of motion that it hardly seems possible to regard them as manifestations of some general and simple laws. Nevertheless such a theory was attempted with great success already in antiquity, remaining substantially unchallenged until late in the 19-th century, when it became increasingly clear that the classical analysis could not be sufficient to deal with the variety of phenomena.

1. Regular motion

First I will summarize the classical conceptions of motion, that I will briefly call *regular* with the aim of contrasting them with the modern theories which conceive *also* other kinds of motions that I will call for brevity *chaotic*.

Both are *deterministic*: *i.e.* future motion is uniquely determined by the present or "initial" state. Furthermore both are *reversible*: in the sense that if a *daemon* acted by reversing at a given instant all velocities of the point masses constituting the matter under investigation, leaving the positions unchanged, then a new motion would be obtained retracing *exactly* all previously assumed positions but with opposite velocities.

Therefore neither view solves the contradiction between the microscopic motion, deterministic and reversible, and the macroscopic motions of large assemblies of point particles ("molecules") which present obvious irreversibility phenomena.

The contradiction is overcome by Boltzmann's conception of irreversibility which proposed, essentially at the same time, the first studies of chaotic motions: his solution avoids having recourse to *new laws*, intrinsically irreversible, governing mechanics.

The need for a theory of motion arose in Antiquity from the possibility, and the challenge, of predicting the positions of the Stars: which becomes apparent as soon as the nightly Sky is observed with a minimum of attention. The prediction should find its place in the frame of a general deterministic theory explaining the motions of the Fixed Stars and Sun first, then of the Moon and finally of the five remaining moving Stars (Mercury, Venus, Mars, Jupiter, Saturn).

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The general aim of a theory of motion is, thus, to understand how to predict, or retrodict, the behavior of a given system and such a theory must deal with two kinds of motions: the regular ones of the simplest mechanisms (pendulum, gyroscope, free falling weight, ...) and the irregular ones (like waves on the sea surface under wind).

The first idea, or *model*, of motion was to suppose that all motions could be *decomposed* into circular uniform motions. The idea was proposed and studied in ancient Greece and is fundamental in ARISTOTELES and PLATO. This means that a moving point, like a Star or a Planet, must be imagined, in the simplest case, to run on a circle (called *deferent*) at *constant speed*. When this does not suffice more complicated motions can be obtained by imagining that the point is not on the deferent but rather it moves at constant speed on a circle (called *epicycle*) whose center moves at constant speed on the deferent

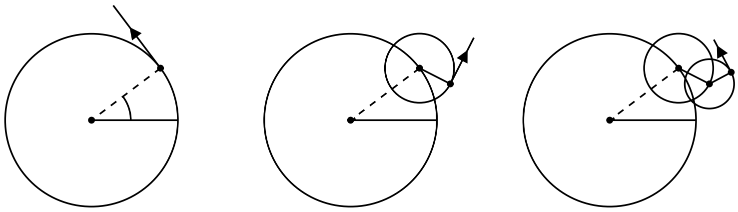

Fig.1 Motion on a deferent, or on an epicycle with center rotating on a deferent, or on an epicycle with center moving on a second epicycle whose center moves on a deferent.

This means that a moving object, like a Star or a planet, should be imagines as uniformly (*i.e.* at constant speed) moving on a circle ("deferent") in the simplest case. In more complicated cases it will move, *still at constant speed* on a circle (*epicycle*) centered on a point moving in turn and at constant speed on the deferent. And if this is not sufficient further epicycles can be added until all "*phenomena are saved*" and Sky ephemerides can be compiled for future observations.

Then a remarkable feature emerges, well known in antiquity: the number of cycles and epicycles and the values of their speeds needed to represent the motions observed in the skies is relatively small (if, as it is natural, rotation speeds that are integer or rational multiples of others are not counted as different).

HYPPARCUS arranged the observation data known at his time in the above scheme: at his time it was likely to be already clear that this was equivalent to assuming existence of a small number of *motors* which by rotating at constant speed would put in motion an array of levers forming clockwork that would carry the planet into a motion that could be, therefore, thought as a "composition" of circular motions: the following figure is an illustation of a motion controlled by an *equant*

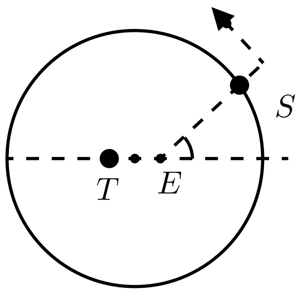

Fig.2: *T=Earth, S=Sun, E="equant point": a lever ES rotates at uniform angular velocity pushing S.*

PTOLEMY made in fact wide use of the equivalence leading to believe that he had abandoned the Aristotelean principle that *any* motion can be regarded as a composition of circular uniform motions.

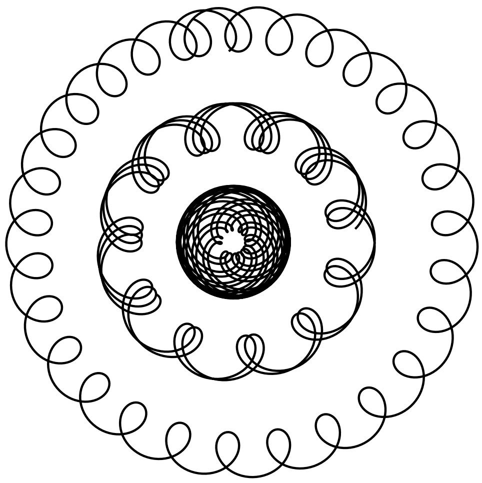

Fig.3: *Geocentric motion of the external planets (Mars, Jupiter and Saturn) as seen from Earth with respect to the fixed Stars: Ptolemaic system.*

It is not really clear why, with the decadence of the Hellenistic culture, the equivalence between arrays of rotating levers and arrays of epicycles was "forgotten" and, worse, the very elaborate theoretical analysis behind Ptolemaic astronomy and tables left no traces. The grave consequence was that in about a thousand years after Ptolemy (~ 200 AD) no one was able to build *ab initio* a theory of the simplest motions (*i.e.* of the Stars and Planets) not even for the purpose of correcting the errors ("*secular variations*") that Time accumulated on the predictions of the *Mathematical Syntaxis* (*i.e.* the *Almagest*).

So COPERNICUS considered that PTOLEMY had abandoned the classical principle, established by ARISTOTELES and PLATO, that all motions were compositions of circular motions. In his *Commentariolus* he declares:

"Nevertheless, what PTOLEMY *and many others legated us here and there about such questions, although mathematically acceptable, did not seem for this reason not to give*

rise to doubts and difficulties ...". "So that such an explanation seemed to be neither sufficiently complete nor sufficiently conform to a rational criterion...". "Having therefore realized this I often meditated on whether, by chance, a more rational system of circles could be found with which it would be possible to explain every possible apparent inequality; [with respect to composition of circular motions] *I mean circles all moving upon themselves with uniform motion* **as demanded** *by the law of absolute motion.*

Here the young COPERNICUS returns to the simple model of circles with centers rotating upon circles abandoning the system of levers, eccentrics and equants of Ptolemaic astronomy and for the first time since a millennium rebuilds *ab initio* a model for the solar system following a "rational" method. At the end of his life and upon completion of his lifetime work he obtains a theory that did not surpass in precision that of PTOLEMY. Nevertheless he developed a method to build systematically a theory of planetary motions and, possibly, to achieve better precision (at the prize of further work which will be done by its successors).

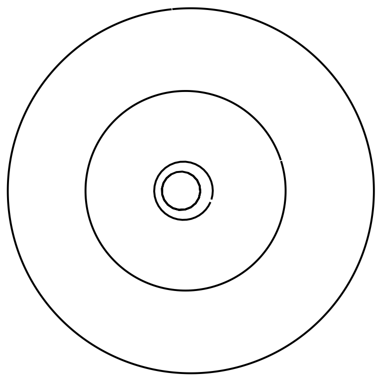

Fig.4. *External planets in Heliocentric theory. In spite of the evident greater simplicity* Copernicus*' theory was not more precise than* Ptolemy*'s. Here the central orbit is Earth's orbit, (in scale).*

We do not know the theoretical methods followed by PTOLEMY to conceive and construct the levers and eccentric circles systems which, rotating uniformly, generated the relatively complex motions which, as soon as one refines the observation precision, are observed in the Skies: therefore this was a major contribution of COPERNICUS, opening the path followed by TYCHO BRAHE, KEPLER.

It is useful to stress, in parenthesis, that although in the *Mathematical Syntaxis* (*i.e. Almagest*) there is no trace of the methods used to achieve the Ptolemaic constructions (like the vituperated equant) such methods had to exists because it is difficult to think that PTOLEMY had proposed solutions so apparently complex and nevertheless so precise (a precision that COPERNICUS did not surpass) while proceeding by trial and error, purely empirically. In a sense PTOLEMY's book is an "ephemerides catalog" (like the modern "*American Astronomical Almanac*") in which the underlying theory is not explained other than through (very detailed) "prescriptions" whose origin is not discussed nor justified; but following them one "saves the phenomena".

The marvelous precision of modern ephemerides, as seen in voluminous books or CDs and DVDs, would not easily allow a reconstruction of the Newtonian theory on which the ephemerides are built. It seems possible, therefore, to think that the scientific decadence following the departure from Alexandria of the Hellenistic scientists due to the tragic political events of the V century (Jews' expulsion, assassination of HYPATHIA, religion driven riots, decline of the Roman rule ...) had the consequence that only what was considered "useful", according to criteria that we hear again as a sinister knell at the onset of the XXI century, was preserved: a prominent example being PTOLEMY's *Mathematical Syntaxis*, while all that was necessary to build it up rationally, together with the preceding works which it "made obsolete", has been lost.

2. Chaotic motions

Returning to motion as decomposable into deferents and epicycles it is important to stress that the fundamental discoveries of KEPLER and the successive theories of NEWTON, based on the Copernican methodology and to GALILEO's work, did not diminish its value.

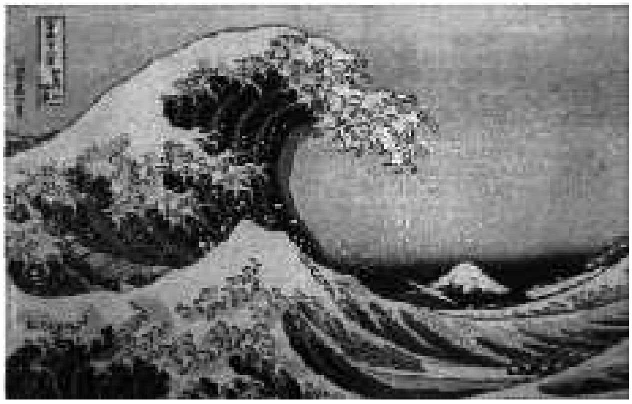

Fig.5: *Even though it might seem difficult to maintain until recent times it has been proposed that this type of motion could be seen as composed by uniform circular motions.*

On the contrary, as shown by LAPLACE, in his "*Mécanique Celèste*" the entire astronomical theory, in spite of being governed by *a priori* laws, was still described or at least equivalently describable in terms of cycles and epicycles with the essential feature that their properties were no longer computed on the basis of experimental data but were (in principle and to a great extent in practice) consequences of universal laws.

It should be remarked that in antiquity it was believed that *any motion* could be represented in this way: the reason why waves on a windy sea seemed far more complex compared to the motion of Stars (already very complex) was simply to be attributed to a far larger number of cyclic motions involved in composing the actually observed one, compared to the number of "motors" responsible for the strange Stars motions.

Fig.6. *An important example of turbulent motion is given by the red spot on Jupiter.*

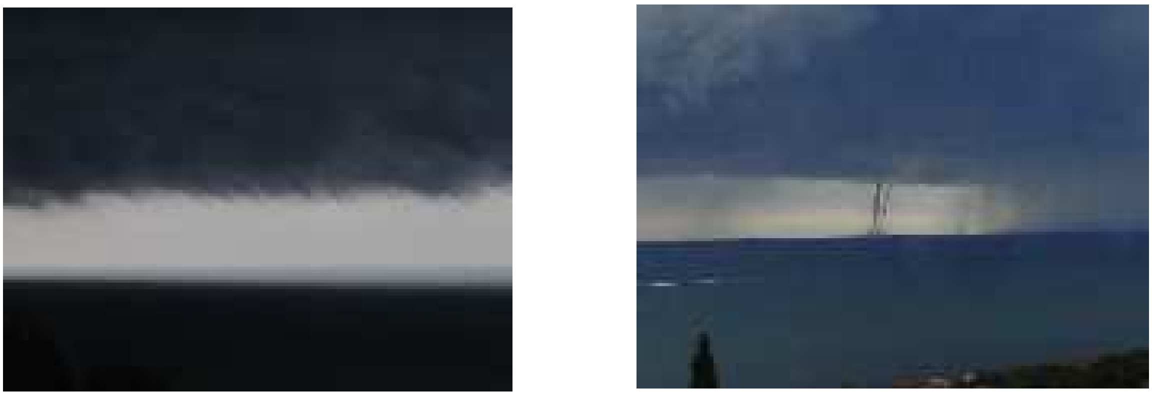

Fig.7. *A tornado on the sea (Tuscany) seems to provide an example of a motion composed by circular motions. In reality it is very difficult to sustain this view in a quantitative form.*

This might seem extreme; however it is, upon reconsideration, not only not impossible (were it only because of the representability of any motion in Fourier series or integral), but it has been considered often and even very recently: as witnessed by a book that is, possibly, the most consulted manual by Physics students, the Treatise of Theoretical Physics by Landau and Lifshitz printed in the 1950's. Turbulence theory is there presented as due to the increasing number of "motors" which, as the force that solicits the motion increases (for instance the wind speed on the sea surface) Start rotating generating a motion which is *apparently* disordered (a theory that has been "modified" in the editions successive to 1970 by the editors of the new editions of the book).

It was just at the moment of the triumph of the quasi-periodic conception of motion, after LAPLACE and the progress in Celestial Mechanics of the XIX Century, that the work of BOLTZMANN and POINCARÉ, on the eve of the XX Century Started bringing evidence that not all motions could be represented as generated by a system of wheels moved at constant rotational speeds by motors and connected by a suitable system of wheels and bars to the material points constituting the systems under study.

One of the characteristic properties of motions described by cycles and epicycles, technically called *quasi periodic*, is their *predictability*. This has the precise meaning that close initial data evolve deterministically remaining close for a long time.

Following two motions of a system starting from neglegibly close initial conditions suppose that we observe a difference of prefixed appreciable size D after a suitable waiting time. Assume as time unit T the further time interval that we have to wait to see the difference increase by a further amount D.

Starting at this moment to count time, the development of a difference, $2D$, requires in a typical regular motion a double time $2T$; to observe a difference of $4D$ a quadruple time is needed; for a difference $8D$ a time $8T$ is needed and so on. The waiting time increases *arithmetically* as visualized in the first two columns of the following table.

quasi periodic motion			chaotic motion	
Difference	time/T		Difference	time/T
$2D$	2		$2D$	1
$4D$	4		$4D$	2
$8D$	8		$8D$	3
$16D$	16		$16D$	4
...	...		...	...
$1024D$	1024		$1024D$	10
...	...		...	...
$\sim 1.000.000D$	$\sim 1.000.000$		$\sim 1.000.000D$	~ 20

With POINCARÉ it became clear (to the few who appreciated this aspect of his work) that *instead* there are motions for which, choosing as unit of time the same time interval T as above, in order to see differences $2D, 4D, 8D, \ldots$ it is necessary to wait *only* times $2T, 3T, 4T, \ldots$ in a *logarithmic* progression as visualized in the table columns 3 and 4.

It is evident that motions of this second kind, called *chaotic*, are unpredictable on the same time scale on which they can be observed because even a small variation in the initial data is geometrically amplified growing rapidly to become too large for a forecast.

From POINCARÉ's work and from the work of mathematicians, and then physicists, of the XX century emerges that it is precisely chaotic motion that dominates the phenomena involving mechanical systems, even simple ones. They even appear in the planetary motions which, on astronomical time scales (*i.e.* of millions of years), are subject to inequalities completely unexpected on the basis of the Ptolemaic-Laplacean theories.

3. Reversibility: a paradox?

The quasi periodic conception of motion was well adapted with the determinism postulated by NEWTON's laws for all systems, be they constituted of few particle or of many. But Newtonian physics, which predicts both chaotic and regular motions on a case by case basis, implies that also chaotic motions are deterministic, although it is difficult to predict the future from the present. Furthermore both kind of motions share the property that the *reversed* motions, *i.e.* motions in which velocities are systematically opposite to the ones observed in a given motion are also perfectly possible.

Therefore both conceptions are in sharp contrast with certain very familiar aspects of reality. For instance we know well that certain phenomena develop only in one direction: the well known example of the coffee cup falling from a table is just a typical example. Although according to Newtonian physics it is (in principle) possible to organize the motion of the pieces into which it broke so that it comes up from the floor and reassembles itself, nevertheless no one expects to see such a marvelous event.

If a daemon changed the velocity of the eight Skies (carrying the seven Planets and the fixed Stars) by inverting it, it is quite certain that no one would be surprised seeing the motion of the skies proceeding backwards: no problem for our understanding of reality would arise. However seeing half of a bucket of uniform water become warmer and cooler in the other half would seem strange, to say the least: *but, strictly speaking, it would not contradict the laws of Newtonian mechanics.*

In conclusion *the ancient and the modern conceptions of motion* appear to be incompatible with certain easy empirical observations, in spite of their strong predictive power and of being sources of important applications, entirely based on the deterministic Newtonian physics (one can think to the complex machines that can be found in airplanes, (space-)ships, trains, ...).

The contradiction became of central interest for BOLTZMANN to the point that he dedicated his entire life to the problem. Even though he moved from a conception in which all motions were periodic even avoiding the necessity of quasi periodic motions, he wanted to derive the observed macroscopic irreversibility (everyday observed in phenomena) from microscopic motions which are, instead, reversible (in the ancient as well as in the modern theories of motion).

And BOLTZMANN succeeded in the seemingly impossible achievement of deriving laws governing irreversible phenomena and leading to the establishment of thermal equilibrium based on microscopic reversible and, at least at the beginning of his studies, even periodic motions.

Few really understood the truly revolutionary ideas of BOLTZMANN (and of his contemporaries MAXWELL and THOMSON): even POINCARÉ has to be counted among his critics. It is possible to see how to avoid a paradox by following ideas that cost hard criticism to BOLTZMANN, Starting in particular from the mathematician ZERMELO and surprisingly continuing to be brought up by contemporary physicists and philosophers.

To understand the ideas and the Physics of BOLTZMANN it is necessary to keep in mind that deal with the understanding of the process to reach thermodynamic equilibrium for a system and to derive its Thermodynamical properties. The aim of the theory is to explain why a substance, water for instance, warmer is a corner of its (isolated) container and cooler in another evolves to a state in which temperature is constant and which are its eventual properties. This is a special irreversible phenomenon although by far not a general one.

The key point of BOLTZMANN is that in systems constituted by many particles (molecules obeying NEWTON's laws) it is true that motions can develop in the two time directions: however much more can be stated. Namely a motion with a given initial datum evolves assuming cyclically all possible microscopic states compatible with the energy (which is always conserved, hence constant, in isolated systems. The latter is the *ergodic hypothesis*.But the great majority of microscopic states corresponds to macroscopically indistinguishable states..

It is possible (*in principle*) to adjust the initial data so that evolution produces surprising results, like a spontaneous creation of a temperature difference between two halves of a water container Starting from a state which seems to have a well defined homogeneous temperature.

However such "anomalous" (with respect to common sense) states have an extremely short life and very soon are transformed into states with the usual equilibrium properties (constant temperature in the last example). Subsequently the system continues its microscopic evolution (very complex but effectively observable only on time scales beyond our sensory capabilities and, with a few exceptions, even escaping detection through microscopes) without any interesting event. This will last for a duration really unimaginable (measured in a simple example, brought up by Thomson and later by Boltzmann, by a multiple of *the age of the Universe* represented by 1 followed by more than a billion of billions of 0's!) at the end of which it the system will be seen to come back to the anomalous initial state to leave it immediately and for a subsequent equally long period of time (all this *provided* the system was kept in a perfect container, able to screen its contents form external actions, for instance from the ring of a phone. This is of course unthinkable as well, as after such a time not only the container will not exist any longer but the entire Solar System and the entire Milky Way will have left non traces in the Universe, if any).

Therefore irreversibility (of the approach to equilibrium) manifests itself because reversibility is a phenomenon which is possible but which takes place on time scales which are not observable, because of their length: reversible dynamics can generate phenomena that are *apparently* irreversible.

4. Chaos

Is then everything clear? not really, because return to equilibrium of a system initially out of equilibrium is just a special case of irreversible behavior: and it can be studied thanks to very special properties of the microscopic models of isolated matter: namely the Hamiltonian symmetry and the ergodicity of the motions.

These are properties which not only lead to develop a descriptive and qualitative theory of the approach to equilibrium but also generate a detailed, surprisingly accurate, theory allowing us to make quantitative predictions on a variety of equilibrium phenomena, like phase transitions (like water $\longleftrightarrow$ vapor or water $\longleftrightarrow$ ice) or on phenomena that occur while a system approaches equilibrium.

For out of equilibrium phenomena it is desirable to have a theory to describe the motions that a system undergoes when it is in a macroscopic stationary state or close to it: for instance motions on the sea surface under a standing wind blow provide an instance of a *non equilibrium* stationary state (equilibrium would be still sea in absence of wind) and it is *stationary* because its surface, although continuously changing, does so always in the "same way" so that it is impossible to determine how long ago the phenomenon Started or for how long it will continue. The statistical properties remain unchanged while the instantaneous properties do change.

For out of equilibrium phenomena the qualitative explanation of irreversibility by BOLTZMANN remains unchanged, hence we should consider solved by him the contradiction between microscopic reversibility and macroscopic irreversibility; but we are still missing a *quantitative theory* for studying from a fundamental viewpoint the new phenomena that become manifest in out of equilibrium systems. Quantitative equation analogous, for instance, to those that are technically called the "*Boltzmann–Gibbs statistics*" or the "*Boltzmann equation*".

On this problem the attempts at a rational understanding of turbulence phenomena in fluids and at a development of a quantitative theory extending Thermodynamics to gases or general many particle systems have found a common ground.

It goes back to the early 1970's a proposal by D. RUELLE that there are only two paradigms, *i.e.* two simple models, capturing the essential aspects of the possible motions. The first paradigm is the classical one of the uniform rotations, which leads to consider motions which are decomposable into uniform rotations by imagining them as obtained by connecting several levers each of which rotates at constant speed, as in the case of the Ptolemaic (or Copernican or Keplerian ...) motions.

The second is much less familiar, but well understood mathematically, and is given by the so called *hyperbolic motions*: in this case motion can still be imagined as described by composing the motion of several connected levers or gears, each of which goes through a motion which is chaotic in the sense empirically described in the previous table. It is not appropriate to give here a precise description of the hyperbolic motions: it will suffice to say that their key property is that they are *apparently* random. Because they can be simulated by throwing a dice per lever and advancing the levers forward or backward by an amount depending on the outcomes shown by the dies. The resulting motion will then appear as a realization of *one among the possible motions* of the system. It is very easy to simulate such movements by making use of a random number generator (usually present on all computers).

The *chaotic hypothesis* essentially says that although motions distinct from the two "extreme" kinds just mentioned exist, nevertheless such two models of motion are sufficient to describe equilibrium and non equilibrium phenomena of extended mechanical systems.

A consequence is that chaotic motions should be thought as randomly generated: it is, however, only an apparent randomness since they are deterministic exactly as deterministic is the sequence of random numbers generated by a computer (*which is always the same for a given seed* that initializes the sequence). We get quite close to another ancient conception of motion, held by DEMOCRITUS who "the world randomly sets" ("che 'l mondo a caso pone", DANTE).

In spite of the simplification induced by the chaotic hypothesis, which reduces enormously the possibly different kinds of motions, it is not easier to interpret irreversibility. The qualitative explanations is still the one proposed by BOLTZMANN for the equilibrium phenomena: *i.e.* also in stationary systems out of equilibrium the system goes through all possible microscopic states while macroscopically nothing changes or, better, every fluctuation takes place on super-astronomic time scales.

The novelty is that in phenomena out of equilibrium irreversibility also manifests itself because the rare fluctuations of observables, which bring the system apparently out of stationarity, no longer occur symmetrically about their average values even when they would do so in equilibrium, but may have strong biases. One of the successes of the chaotic hypothesis has been to deduce that for certain fluctuations the probability of occurrence follows a universal law, with no free parameters to adjust to interpret the results of the experiments (the *Fluctuation Theorem*).

And the frequency (*i.e.* probability) of occurrence is controlled by a quantity that has been related to the *entropy creation rate*, *i.e.* to sum of the heat amounts ceded per unit time to the thermostats with which the system is necessarily in contact (to prevent indefinit heaing and eventual "melting" of the system) divided by their respective temperatures.

In models the entropy creation rate receives a purely mechanical interpretation which, if confirmed through various experimental checks, conceivable but not yet done, might have an importance comparable to the identification between temperature and molecular kinetic energy. The enthusiasm with which some accept the validity of the chaotic hypothesis is therefore explained, as is the disgust that others exhibit towards it: in the end this will have the merit of having at least brought up a problem that remained dormant for decades: is it possible to define entropy in systems out of equilibrium (but stationary)?

The *ergodic hypothesis* proposed that an isolated system evolves in time visiting all possible microscopic states. Building upon it BOLTZMANN reached the reconciliation of determinism and reversibility of Newtonian mechanics with the macroscopic observations which instead suggest a privileged direction of time, a privileged *arrow of time* and the perpetual flow towards a future different from the past..

It is therefore important that the ergodic hypothesis is not in contrast with, but it rather appears as a consequence of, the chaotic hypothesis: the latter is an extension of the former to a field in which, surprisingly, conceptual developments have been missing in the last half century. It allows to extend "as is", unaltered, the conceptual scheme developed by BOLTZMANN reconciling microscopic determinism and reversibility with apparent macroscopic randomness and irreversibility.

It is not inconceivable to hope for important future developments if the chaotic hypothesis will be recognized as a valid principle (at the moment it still raises doubts and criticism which cannot be commented here). However it will remain of mainly conceptual importance and it will not be comparable to the importance, equally theoretical, of the ergodic hypothesis. The latter has controlled the development of equilibrium statistical mechanics to a point that it can be said it offers a conceptual frame even to the applications whose concrete nature might induce to think that they have nothing in common with questions of substantially philosophical nature, as the ones discussed here.

Bibliographic note

A guide to the vast bibliography can be found in my monographs

[1] *Statistical Mechanics. A short treatise*, p. 1-345, Springer–Verlag, Berlin, 1999
[2] *Fluid Mechanics*, p. 1-529, Springer–Verlag, Berlin, 2001
[3] *Aspects of the ergodic, qualitative and statistical properties of motion*, p. 1-435, Springer–Verlag, Berlin, 2004, in collaboration with F. Bonetto and G. Gentile.

and to my paper

[4] *Quasi periodic motions from Hypparchus to Kolmogorov*, Rendiconti Accademia dei Lincei, Matematica e applicazioni, **12**,125–152, 2001, and chao-dyn/9907004.

where the bibliographic references are motivated together with a more technical exposition of the ideas discussed here. The monographs and the paper (some also in Italian) can be found and freely downloaded on the web site below.

The *Fluctuaton Theorem* was first introduced and derived in

[5] G. Gallavotti and E. G. D. Cohen: *Dynamical ensembles in nonequilibrium statistical mechanics*, Physical Review Letters, **74**, 2694–2697, 1995.

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