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Preface

Preface to the Second English edition (2007).[©]

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In 2007 I recovered the Copyright. This is a new version that follows closely the first edition by Springer-Verlag. I made very few changes. Among them the Gauss' method, already inserted in the second Italian edition, has been included here. Believing that my knowledge of the English language has improved since the late '970's I have changed some words and constructions.

This version has been reproduced electronically (from the first edition) and quite a few errors might have crept in; they are compensated by the corrections that I have been able to introduce. This version will be updated regularly and typos or errors found will be amended: it is therefore wise to wait sometime before printing the file; the versions will be updated and numbered. The ones labeled 2.* or higher will have been entirely proofread at least once.

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On the web site I also put the codes that generate the non trivial figures and which provide rough attempts at reproducing results whose originals are in the quoted literature. Discovering the phenomena was a remarkable achievement: but reproducing them, having learnt what to do from the original works, is not really difficult if a reasonably good computer is available.

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Giovanni Gallavotti

Roma 18, August 2007

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Preface to the first English edition

The word "elements" in the title of this book does not convey the implication that its contents are "elementary" in the sense of "easy": it mainly means that no prerequisites are required, with the exception of some basic background in classical physics and calculus.

It also signifies "devoted to the foundations". In fact, the arguments chosen are all very classical, and the formal or technical developments of this century are absent, as well as a detailed treatment of such problems as the theory of the planetary motions and other very concrete mechanical problems. This second meaning, however, is the result of the necessity of finishing this work in a reasonable amount of time rather than an a priori choice.

Therefore a detailed review of the "few" results of ergodic theory, of the "many" results of statistical mechanics, of the classical theory of fields (elasticity and waves), and of quantum mechanics are also totally absent; they could constitute the subject of two additional volumes on mechanics.

This book grew out of several courses on "Meccanica Razionale", i.e., essentially, Theoretical Mechanics, which I gave at the University of Rome during the years 1975-1978.

The subjects cover a wide range. Chapter 2, for example, could be used in an undergraduate course by students who have had basic training in classical physics; Chapters 3 and 4 could be used in an advanced course; while Chapter 5 might interest students who wish to delve more deeply into the subject, and fit could be used in a graduate course.

My desire to write a self-contained book that gradually proceeds from the very simple problems on the qualitative theory of ordinary differential equations to the more modern theory of stability led me to include arguments of mathematical analysis, in order to avoid having to refer too much to existing textbooks (e.g., see the basic theory of the ordinary differential equations in §2.2-§2.6 or the Fourier analysis in §2.13, etc.).

I have inserted many exercises, problems, and complements which are meant to illustrate and expand the theory proposed in the text, both to avoid excessive size of the book and to help the student to learn how to solve theoretical problems by himself. In Chapters 2-4, I have marked with an asterisk the problems which should be developed with the help of a teacher; the difficulty of the exercises and problems grows steadily throughout the book, together with the conciseness of the discussion.

The exercises include some very concrete ones which sometimes require the help of a programmable computer and the knowledge of some physical data. An algorithm for the solution of differential equations and some data tables are in Appendix O and Appendix P, respectively.

The exercises, problems, and complements must be considered as an important part of the book, necessary to a complete understanding of the theory.

In some sense they are even more important than the propositions selected for the proofs, since they illustrate several aspects and several examples and counterexamples that emerge from the proofs or that are naturally associated with them.

I have separated the proofs from the text: this has been done to facilitate reading comprehension by those who wish to skip all the proofs without losing continuity. This is particularly true for the more mathematically oriented sections. Too often students tend to confuse the understanding of a mathematical proposition with the logical contortions needed to put it into an objective, written form. So, before studying the proof of a statement, the student should meditate on its meaning with the help (if necessary) of the observations that follow it, possibly trying to read also the text of the exercises and problems at the end of each section (particularly in studying Chapters 3-5).

The student should bear in mind that he will have understood a theorem only when it appears to be self-evident and as needing no proof at all (which means that its proof should be present in its entirety in his mind, obvious and natural in all its aspects and, if necessary, describable in all details). This level of understanding can be reached only slowly through an analysis of several exercises, problem, examples, and careful thought.

I have illustrated various problems of classical mechanics, guided by the desire to propose always the analysis of simple rather than general cases. I have carefully avoided formulating "optimal" results and, in particular, have always stressed (by using them almost exclusively) my sympathy for the only "functions" that bear this name with dignity, i.e., the C^∞ -functions and the elementary theory of integration ("Riemann integration").

I have tried to deal only with concrete problems which could be "constructively" solved (i.e., involving estimates of quantities which could actually be computed, at least in principle) and I hope to have avoided indulging in purely speculative or mathematical considerations. I realize that I have not been entirely successful and I apologize to those readers who agree with this point of view without, at the same time, accepting mathematically non rigorous treatments.

Finally, let me comment on the conspicuous absence of the basic elements of the classical theory of fluids. The only excuse that I can offer, other than that of non pertinence (which might seem a pretext to many), is that, perhaps, the contents of this book (and of Chapter 5 in particular) may serve as an introduction to this fascinating topic of mathematical physics.

The final sections, §5.9-§5.12, may be of some interest also to non students since they provide a self-contained exposition of Arnold's version of the Kolmogorov-Arnold-Moser theorem.

This book is an almost faithful translation of the Italian edition, with the addition of many problems and §5.12 and with §5.5, §5.7, and §5.12 rewritten.

I wish to thank my colleagues who helped me in the revision of the manuscript and I am indebted to Professor V. Franceschini for providing (from his files) the very nice graphs of §5.8.

I am grateful to Professor Luigi Radicati for the interest he showed in inviting me to write this book and providing the financial help from the Italian printer P. Boringhieri.

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Giovanni Gallavotti
Roma, 27 December 1981

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