WORKING GROUP
"The log minimal model program for the moduli space of curves"

1 The log minimal model program for $\overline{M}_g$

1.1 An introduction to the log minimal model program

References: [HH09, Appendix].

1.2 The moduli space $\overline{M}_g^{\mathrm{ps}}$ of pseudo-stable curves (via GIT of 3 and 4-canonical curves)

References: [Sch91], [HM10].

1.3 $\overline{M}_g^{\mathrm{ps}}$ as the first divisorial contraction

References: [HH09].

1.4 The moduli space $\overline{M}_g^{\mathrm{hs}}$ (resp. $\overline{M}_g^{\mathrm{cs}}$) of h-stable (resp. c-stable) curves (via GIT of 2-canonical curves)

References: [HH13] (see also [AH12]).

1.5 $\overline{M}_g^{\mathrm{hs}}$ as the first flip

References: [HH13].

1.6 Towards the second flip
References: [ASvdW], [AS].

1.7 Heuristics for the successive steps

References: [FS12, Sec. 4.4], [AFS].

1.8 GIT of canonical curves and finite Hilbert stability

References: [AFS13], [FJ13] (see also [MS11], [HHL10], [AH12]).

1.9 Aside 1: weakly proper stacks and their good moduli spaces

References: [ASvdW], [Alp13], [AS] (see also [Alp12], [Alp])

1.10 Aside 2: modular birational models of $\overline{M}_{g,n}$

References: [FS12, Sec. 2], [Smy13].

1.11 Aside 3: the variety of stable limits

References: [Has00].

2 Related topics

2.1 Log Canonical models of $\overline{M}_g$ for small genera $(g = 2, 3, 4, 5)$

References: [Has05], [HL07], [HL09], [HL10a], [HL07], [Fed13], [JCML12], [JCML13], [FS13].

2.2 Log Canonical models of the moduli space $\overline{H}_g$ of stable hyperelliptic curves.

References: [HL10b], [Hye], [Fed].

2.3 Log Canonical models of $\overline{M}_{0,n}$

References: [FS12, Sec. 4.2], [FS11], [AS12], [KM10], [Moo1], [Moo2], [GJM], [GJMS].

2.4 Log Canonical models of $\overline{M}_{1,n}$

References: [FS12, Sec. 4.3], [Smy11a], [Smy11b].

2.5 Moduli spaces of weighted pointed stable curves as log canonical models of $\overline{M}_{g,n}$

References: [FS12, Sec. 2.2], [Has03], [Fed11], [Moo3].

References

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