Cohomological decompositions of almost complex manifolds

Let (X, J) be a compact almost complex manifold of dimension $2n$. The almost complex structure J acts in a natural way on the space of 2-forms $A^2(X)$, by setting, for every pair of vector fields u, v on X, $J\alpha(u, v) = \alpha(Ju, Jv)$. In [1] T.-J. Li and W. Zhang introduce the *invariant* and *anti-invariant cohomology groups* on (X, J) defined as respectively

$$H_J^{\pm}(X) := \{[\alpha] \in H^2_{dR}(X; \mathbb{R}) \mid J\alpha = \pm\alpha\}\,.$$

The group $H_J^-(X)$ is related to the tamed symplectic cone on a compact almost complex manifold. In [2] T. Draghici, T.-J. Li and W. Zhang prove that on a compact 4-dimensional almost complex manifold (X, J) the following cohomological decomposition holds:

$$H^2_{dR}(X; \mathbb{R}) = H_J^+(X) \oplus H_J^-(X)\,.$$

In this talk we will focus on the study of $H_J^{\pm}(X)$; in the integrable case, we will consider various kind of cohomological decompositions on compact complex non-Kähler manifolds, involving the de Rham, Bott-Chern and Aeppli cohomologies.

[1] T.-J. Li, W. Zhang, Comparing tamed and compatible symplectic cones and cohomological properties of almost complex manifolds, *Comm. Anal. Geom.* **17** (2009), no. 4, 651–684.

[2] T. Draghici, T.-J. Li, W. Zhang, Symplectic forms and cohomology decomposition of almost complex 4-manifolds, *Int. Math. Res. Not. IMRN* **2010** (2010), n. 1, 1–17.