INTEGRAZIONE PER PARTI

$$\int \underline{f'(x)}\, \underline{g(x)}\, dx = f(x)g(x) - \int f(x) g'(x) dx$$

$$\underbrace{\int \sin^2(x) dx}_{F(x)} = \int \sin(x) \cdot \sin(x) dx = -\sin(x)\cos(x) + \int \cos^2(x)\, dx = -\sin(x)\cos(x)$$

$f'(x) = \sin(x)$, $\quad g(x) = \sin(x)$

$f(x) = -\cos(x)$, $\quad g'(x) = \cos(x)$

$\cos^2 x + \sin^2 x = 1$

$$+ \int (1 - \sin^2 x)\, dx = -\sin(x)\cos(x) + x \underbrace{- \int \sin^2 x\, dx}_{-F(x)}$$

$$F(x) = x - \sin(x)\cos(x) - F(x)$$

$$\underbrace{\cancel{2}F(x)}_{\int \sin^2(x) dx} = \boxed{\frac{x - \sin(x)\cos(x)}{2}} + c \Rightarrow \int \cos^2 x\, dx = \int (1 - \sin^2 x) dx = \underbrace{x}_{\int 1 dx} - \int \sin^2 x\, dx$$

$$= \frac{x + \sin(x)\cos(x)}{2} + c$$

$$\int 1 \cdot \ln x\, dx = x \ln x - \int \underbrace{x \cdot \frac{1}{x}}_{1} dx = x \ln x - x + c$$

$1 = f'(x)$, $\ln x = g(x)$

$f(x) = x$, $\quad g'(x) = \frac{1}{x}$

$(a \neq -1)$

$$\int x^a \cdot \ln x = \frac{x^{a+1}}{a+1} \ln x - \int \frac{x^{a+1}}{a+1} \cdot \frac{1}{x} dx = \frac{x^{a+1}}{a+1} \ln x - \frac{1}{a+1} \int x^a dx = \boxed{\frac{x^{a+1}}{a+1} \ln x - \frac{1}{(a+1)^2} x^{a+1} + c}$$

$x^a = f'(x)$, $\ln x = g(x)$

$f(x) = \frac{x^{a+1}}{a+1}$, $\quad g'(x) = \frac{1}{x}$

$$\int 1 \cdot \operatorname{arctg} x = x \operatorname{arctg} x - \frac{1}{2} \int \frac{\overset{(x^2+1)'}{2x}}{x^2+1} dx = x \operatorname{arctg} x - \frac{1}{2} \ln(x^2+1) + c$$

$f'(x) = 1$, $\quad g(x) = \operatorname{arctg} x$

$f(x) = x$, $\quad g'(x) = \frac{1}{x^2+1}$

$$\frac{x}{x^2+1} = \frac{1}{2} \frac{2x}{x^2+1} \qquad \frac{f'(x)}{f(x)}$$

INTEGRAZIONE PER SOSTITUZIONE

DERIVAZIONE DELLA FUNZIONE COMPOSTA: $\left(F(g(x))\right)' = F'(g(x))\, g'(x)$

PRIMITIVE:

SE $F' = f$

$$\boxed{\int f(y) dy_{\,y = g(x)} = \int f(g(x))\, g'(x)\, dx} \qquad \int f(y) dy_{\,y=g(x)}$$

ESEMPIO

$$\int x \sin(x^2) dx = \frac{1}{2} \int 2x \sin(x^2)\, dx = \frac{1}{2} \int \sin(y)\, dy = -\frac{1}{2} \cos(y)$$

$\boxed{\text{ESEMPIO}}$ $\int x \sin(x^2)dx = \frac{1}{2}\int \underbrace{2x}_{g'(x)} \underbrace{\sin(x^2)}_{f(g(x))} dx = \frac{1}{2}\int \sin(y)\,dy = -\frac{1}{2}\cos(y)$

$f(y) = \sin(y)$

$g(x) = x^2$

$f(g(x)) = \sin(x^2)$

$g'(x) = 2x$

$y = g(x) = x^2 \quad = -\frac{1}{2}\cos(x^2)$

$\boxed{\text{ESEMPIO}}$ $\int \frac{\ln x}{x}dx = \int \underbrace{\frac{1}{x}}_{g'(x)} \cdot \underbrace{\ln x}_{f(g(x))} dx = \int y\,dy = \frac{y^2}{2} + c = \frac{\ln^2 x}{2} + c$

$g(x) = \ln x \quad f(y) = y \quad y = g(x) = \ln x$

$\int \frac{\ln^2 x}{x}dx = \int y^2 dy = \frac{y^3}{3} + c = \frac{\ln^3 x}{3} + c$

$\frac{1}{x} = g'(x) \quad y = g(x) = \ln x$

$f(g(x)) = \ln^2 x$

$f(y) = y^2$

$\int f(\sin x)(\sin x)'dx = \int f(y)dy$ $\quad y = \sin x$

$\boxed{\text{ESEMPIO}}$ $\int \cos^3 x\,dx = \int \cos(x)\cdot\cos^2 x\,dx = \int \underbrace{\cos(x)}_{(\sin x)'} \underbrace{(1-\sin^2 x)}_{f(\sin x)} dx = \int (1-y^2)dy$

$f(y) = 1-y^2 \qquad \underline{y = \sin x}$

$= y - \frac{y^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c$

(IDEM POTENZE DISPARI $\int \cos^{2n+1} x\,dx$, $\int \sin^{2n+1} x\,dx$)

$= \int \sin(x)\sin(x)^{2n}dx = \int \underbrace{\sin(x)}_{-(\cos x)'} \underbrace{(1-\cos^2 x)^n}_{f(\cos x)} dx$

$\frac{x^3}{-2x} = \frac{-x^2}{2}$

$\boxed{\text{ESEMPIO}}$ $\int x^3 e^{-x^2}dx = \int \underbrace{-2x}_{g'(x)} \cdot \underbrace{\frac{-x^2}{2} e^{-x^2}}_{f(g(x))} dx = \int \frac{y}{2}e^y dy = \frac{1}{2}\int y e^y dy$

$g(x) = -x^2$

$g'(x) = -2x$

$f(y) = \frac{y}{2}e^y$

$= \frac{1}{2}\left(ye^y - \int e^y dy\right) = \frac{1}{2}(ye^y - e^y) \overset{y=-x^2}{=} \frac{1}{2}\left(-x^2 e^{-x^2} - e^{-x^2}\right) + c$

$\int f(x)dx = \int f(g(t))\,g'(t)\,dt$ | UTILE IN ALCUNI CASI PARTICOLARI.

$$\int f(x)\,dx = \int \underbrace{f(g(t))\,g'(t)}\,dt \qquad t = g^{-1}(x)$$

UTILE IN ALCUNI CASI PARTICOLARI.

FUNZIONI RAZIONALI CON RADICI ($\sqrt[a]{a+bx}$)

$$\int \frac{x}{1+\sqrt{1+x}}\,dx \qquad t=\sqrt{1+x}$$

$$t^2 = 1+x$$

$$x = t^2-1 = g(t)$$

$$g'(t) = 2t$$

$\int f(g(t))\,g'(t)$ NON CONTIENE RADICI (FUNZIONE RAZIONALE) => PIÙ SEMPLICE DI $\int f(x)dx$

||

$$\int \underbrace{\frac{t^2-1}{1+t}}_{f(g(t))} \cdot \underbrace{2t}_{g'(t)}\,dt = \int (t-1)2t\,dt = \int (2t^2-2t)\,dt = \frac{2}{3}t^3 - t^2 + c$$

$$t^2-1 = (t+1)(t-1)$$

$$= \boxed{\frac{2}{3}(1+x)^{\frac{3}{2}} - (1+x) + c}$$

FUNZIONI RAZIONALI DI SIN, COS

FORMULE DI DUPLICAZIONE

$$\sin x = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\sin^2\left(\frac{x}{2}\right)+\cos^2\left(\frac{x}{2}\right)} = \frac{2\,\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}{\frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}+1} = \frac{2\tan\frac{x}{2}}{\tan^2\frac{x}{2}+1}$$

$$\cos x = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = \frac{\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)} = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

POSSO SCRIVERMI SIN X, COS X IN FUNZIONE SOLO DI $t = \tan\frac{x}{2}$

$$\sin x = \frac{2t}{1+t^2} \;(*)$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow \int f(\sin x, \cos x)\,dx = \int f\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)\frac{2}{1+t^2}\,dt$$

VOGLIO $x = g(t)$ $\qquad \tan\frac{x}{2} = t \qquad g'(t) = \frac{2}{1+t^2}$

$$\frac{x}{2} = \arctan t$$

$$x = 2\arctan t = g(t)$$

SE f È FUNZIONE RAZIONALE (FRAZIONE CON SIN, COS) OTTENGO UNA

SE f È FUNZIONE RAZIONALE (FRAZIONE CON SIN, COS) OTTENGO UNA FUNZIONE RAZIONALE IN t => SO FARE LA PRIMITIVA

$$\int \frac{1}{1+\sin x}\,dx = \int \frac{1}{1+\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt = 2\int \frac{1}{1+t^2+2t}\,dt = 2\int \frac{dt}{(1+t)^2}$$

$$= 2\left(-\frac{1}{1+t}\right) = -\frac{2}{1+\tan\frac{x}{2}} + c$$

$$\int \frac{dt}{(1+t)^2} = \int (1+t)^{-2}\,dt$$

$$-\frac{1}{1+t} = \frac{(1+t)^{-1}}{-1}$$