Equazioni differenziali e soluzioni- Presi da Marcellini -Sbordone Esercizi

Equazioni omogenee

$x' = 3x$	$x = ce^{3t}$
$x' = 2tx$	$x = ce^{t^2}$
$x' = \dfrac{(t-1)}{t}x$	$x = ce^t/t$
$x' = (\cos t)\, x$	$x = ce^{\sin t}$
$x' = -e^t x$	$x = ce^{-e^t}$
$x' = 2te^{t^2} x$	$x = ce^{e^{t^2}}$
$x' = (\tan t)\, x$	$x = \dfrac{c}{\cos t}$
$x' = -\dfrac{x}{2t}$	$x = \dfrac{c}{\sqrt{t}}$
$x' = \dfrac{2x}{t}$	$x = ct^2$
$x' = -(\cot t)\, x$	$x = \dfrac{c}{\sin t}$
$x' = (\sqrt{t})\, x$	$x = ce^{\frac{2}{3}t^{3/2}}$
$x' = \dfrac{x}{\sqrt{t+5}}$	$x = ce^{2\sqrt{t+5}}$
$x' = \dfrac{(\log t)}{t}x$	$x = ce^{(\log t)^2/2}$
$x' = \dfrac{t}{t^2-1}x$	$x = c\lvert t^2-1\rvert^{1/2}$
$x' = (1+\log t)x$	$x = ct^t$
$x' = \dfrac{x}{t\log t}$	$x = c\log t$
$x' = \dfrac{x}{\sin(t+1)}$	$x = c\tan\left(\dfrac{t+1}{2}\right)$
$x' = tx\sin t$	$x = ce^{\sin t - t\cos t}$
$x' = -(\sin 2t)x$	$x = ce^{\cos^2 t}$
$x' = (\arctan t)x$	$x = ce^{t\arctan t}/\sqrt{1+t^2}$
$x' = (\arcsin t)x$	$x = ce^{t\arcsin t + \sqrt{1-t^2}}$
$x' = g'(t)x$	$x = ce^{g(t)}$

Equazioni non-omogenee

$x' = 3x + 1$	$x = ce^{3t} - \frac{1}{3}$
$x' = ax + b \quad (a \neq 0)$	$x = ce^{at} - \frac{b}{a}$
$x' = x + t$	$x = ce^{t} - t - 1$
$x' = -x + e^{-t}$	$x = e^{-t}(t + c)$
$x' + x = \frac{1}{t}$	$x = \frac{1}{t} + c$
$x' = x + e^{t}$	$x = te^{t} + c$
$x' = 4x - e^{2t}$	$x = ce^{4t} + \frac{1}{2}e^{2t}$
$x' = ax + e^{bt} \quad (a \neq b)$	$x = ce^{at} + \frac{e^{bt}}{b-a}$
$x' = -2tx + te^{-t^2}$	$x = e^{-t^2}\left(c + \frac{t^2}{2}\right)$
$x' = (2x)t + \frac{t+1}{t}$	$x = ct^2 - t - \frac{1}{2}$
$x' = x + t^2 - 1$	$x = e^{t}(t^2 - 2t + 1 + c)$
$x' = x - (x)t$	$x = ce^{-t}(\sin t + c)$
$x' = 2tx + e^{-t^2}\cos t$	$x = e^{t^2}(\sin t + c)$
$x' = -x + e^{-t}t^2$	$x = e^{-t}(t^2 + c)$
$x' + x = 3t^2e^{-t}$	$x = e^{-t}(t^3 + c)$
$x' = e^{-t}\sqrt{t}$	$x = e^{-t}(\sqrt{t} + c)$
$x' = (\cos t)x + \sin t \log t$	$x = e^{\sin t}(t \log t - t + c)$
$x' = (\tan t)x + t^3e^{2t}$	$x = t^3(e^{2t} + c)$
$x' = (1 + t)x + \sqrt{t}$	$x = e^{t + \frac{t^2}{2}}(\sqrt{t} + c)$

Equazioni a variabilis spearabili (generate da chatGPT)

Risolvi le seguenti equazioni differenziali utilizzando il metodo della *separazione delle variabili*. In tutte le equazioni la funzione incognita è $x(t)$.

Esercizio 1

1. $x'(t) = x$
2. $x'(t) = x^2$
3. $x'(t) = \sqrt{x}$
4. $x'(t) = \dfrac{1}{x}$
5. $x'(t) = e^{x}$

Esercizio 2

6. $x'(t) = t\,x$
7. $x'(t) = t^2 x^3$
8. $x'(t) = e^t\,x$
9. $x'(t) = (1+t^2)\sqrt{x}$
10. $x'(t) = \dfrac{t}{x}$

Esercizio 3

11. $x'(t) = \dfrac{x}{t}$
12. $x'(t) = \dfrac{x^2}{1+t^2}$
13. $x'(t) = \dfrac{1+t}{1+x}$
14. $x'(t) = \dfrac{t^2}{x^3}$
15. $x'(t) = \dfrac{e^t}{1+x^2}$

Esercizio 4

16. $x'(t) = \sin t\,\cos x$
17. $x'(t) = \cos t\,\sin x$
18. $x'(t) = \dfrac{\sin t}{\sin x}$
19. $x'(t) = \tan t\,\tan x$
20. $x'(t) = \cot x$

Esercizio 5

21. $x'(t) = e^{t+x}$
22. $x'(t) = e^{t-x}$
23. $x'(t) = (\log t)\,x$
24. $x'(t) = \dfrac{e^t}{x}$
25. $x'(t) = x \log x$

Esercizio 6

26. $x'(t) = a\,x$
27. $x'(t) = a\,x^n$
28. $x'(t) = k(t)\,x$
29. $x'(t) = \lambda(1 + x^2)$
30. $x'(t) = kx(1 - x)$

Esercizio 7

31. $(1 + x^2)x'(t) = t$
32. $(\sin x)x'(t) = e^t$
33. $x\,x'(t) = t^2$
34. $\dfrac{x'(t)}{1 + x^2} = \sin t$
35. $\sqrt{x}\,x'(t) = t$