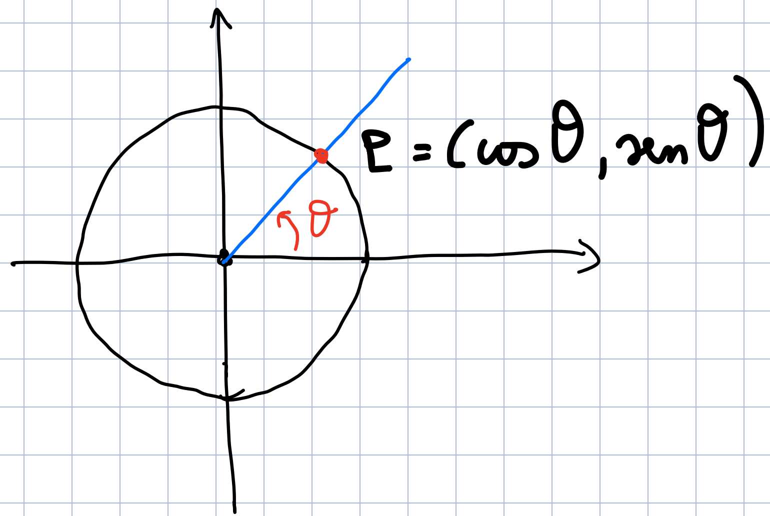
Sulla circonferenza trigonometrica

$x^2+y^2=1$



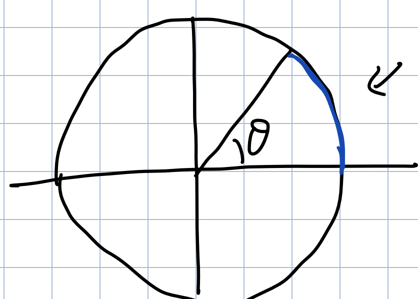
Per definizione $P = (\cos\theta, \text{sen}\,\theta)$

da cui deduco che $\cos^2\theta + \text{sen}^2\theta = 1$

Bisogna sapere.

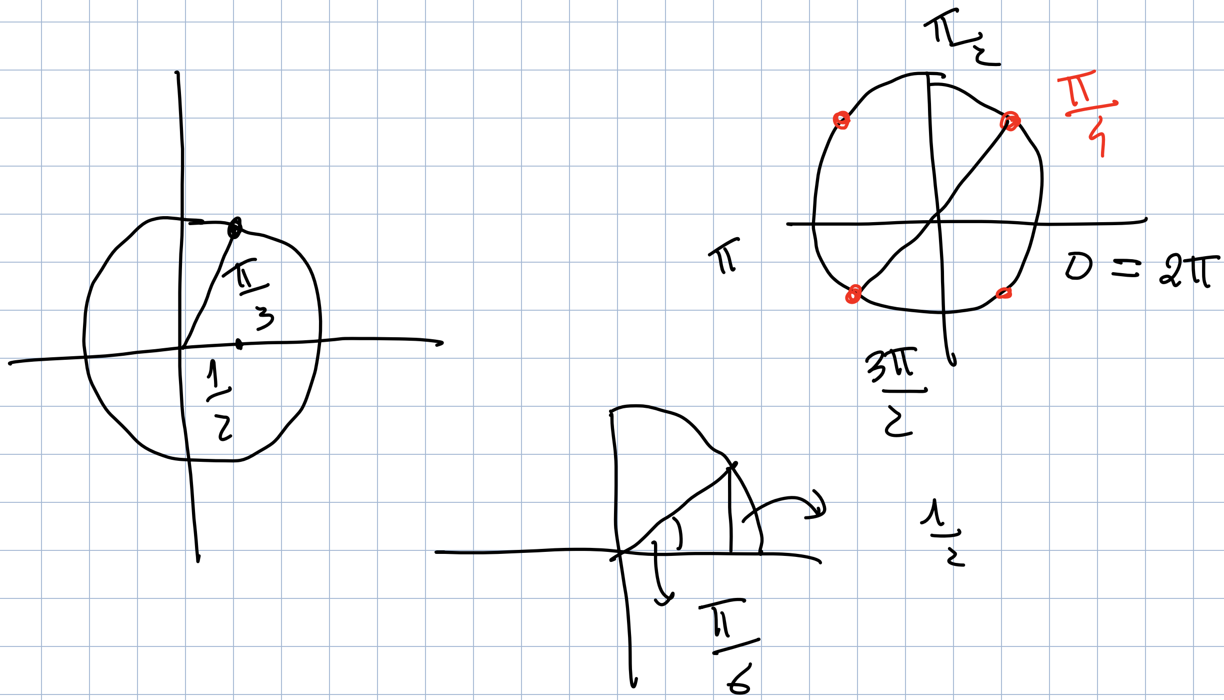
1. Angoli misurati in radianti
2. Sen - Cos notevoli

E abituale misurare gli angoli tramite la lunghezza d'arco corrispondente sulla circonf. trigonometrica



lunghezza di questo arco

Dato che sappiamo che la circonferenza unitaria ha perimetro $= 2\pi$



Deduco anche alcune relazioni

per esempio

$$\cos\left(\frac{\pi}{2} - \vartheta\right) = \sin(\vartheta) \quad ; \quad \sin\left(\frac{\pi}{2} - \vartheta\right) = \cos(\vartheta)$$

$$\sin(\pi - \vartheta) = \sin(\vartheta) \quad ; \quad \cos(\pi - \vartheta) = -\cos(\vartheta)$$

etc...

È usuale assegnare un valore a

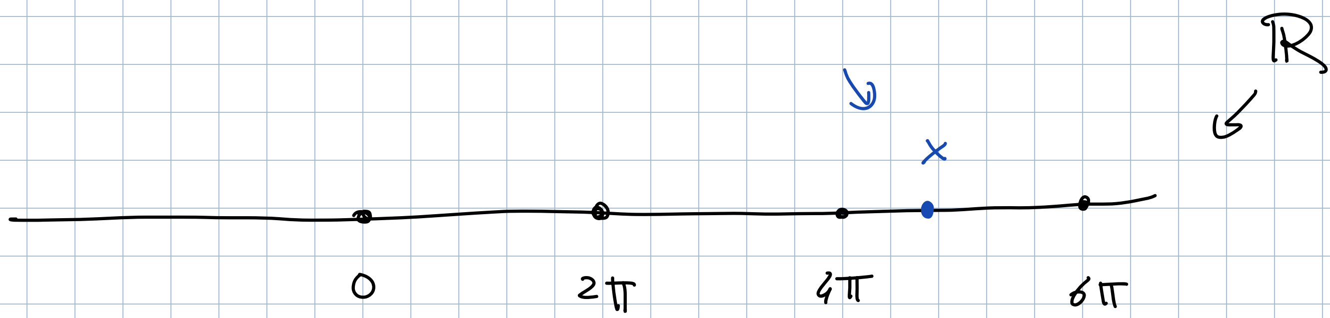
$\sin(x)$, $\cos(x)$ $\quad \forall\, x \in \mathbb{R}$

usando la regola

$$\cos(\theta + 2k\pi) = \cos(\theta) \quad ; \quad \operatorname{sen}(\theta + 2k\pi) = \operatorname{sen}(\theta)$$

Idea

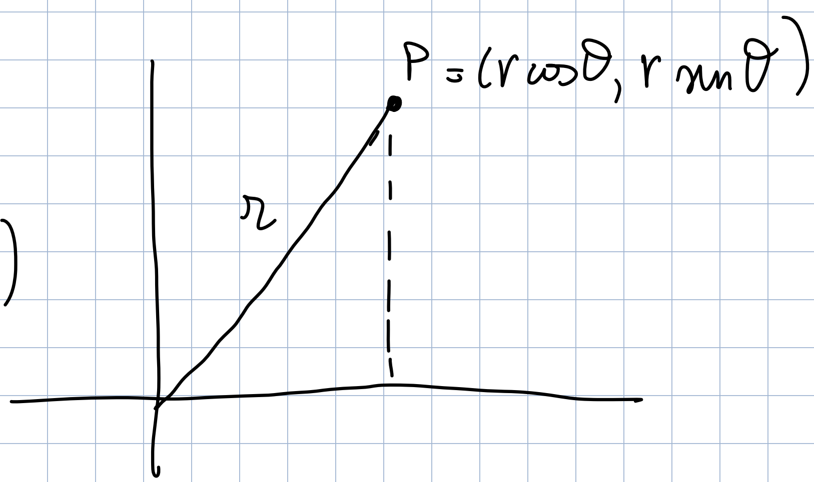
$$0 < x - 4\pi < 2\pi$$



$$\operatorname{sen}(x) = \operatorname{sen}(x - 4\pi)$$

$(x, y) \longrightarrow (r, \theta)$

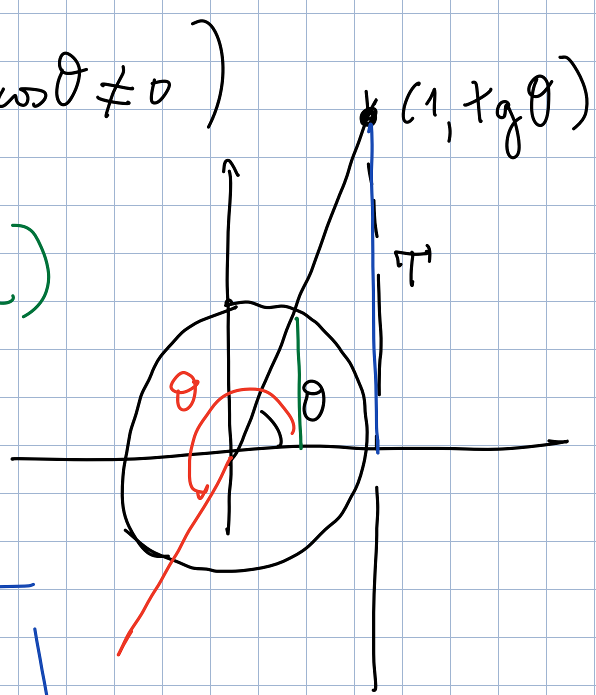
(coordinate polari)



Tangente

$1 : \cos\theta = T : \sin\theta$

$$tg(\theta) = \frac{\sin\theta}{\cos\theta} \qquad (\cos\theta \neq 0)$$



(Teorema di Talete)

NB.

$$tg(\theta+\pi) = tg(\theta)$$

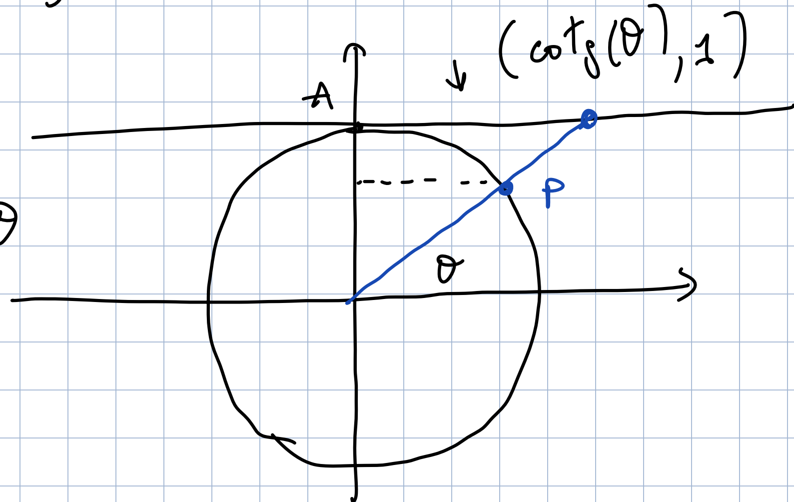
$$\boxed{\begin{array}{l} tg\left(\frac{\pi}{2}\right) = \infty \quad (?) \\ tg\left(\frac{3}{2}\pi\right) = -\infty \quad (?) \end{array}}$$

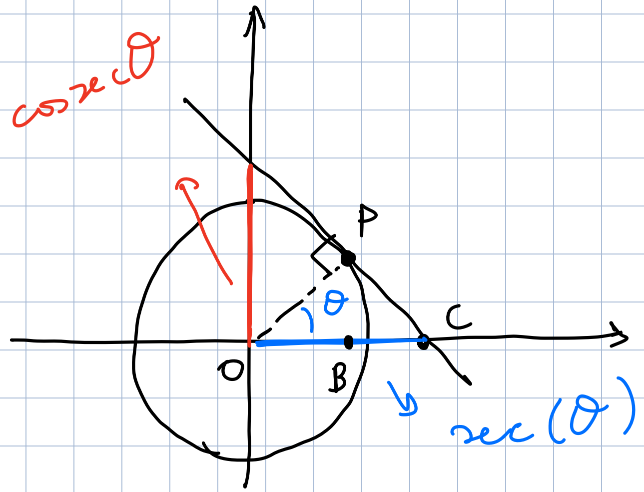
Cotangente

$$cotg(\theta) = \frac{\cos\theta}{\sin\theta} \qquad (\sin\theta \neq 0)$$

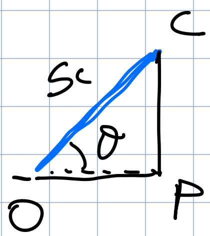
$$cotg(\theta) = \frac{1}{tg(\theta)} = tg\left(\frac{\pi}{2}-\theta\right)$$

$$1 : \sin\theta = AB : \cos\theta$$

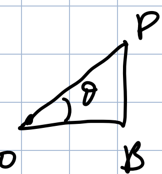




$\sec(\theta) = \frac{1}{\cos\theta}$



è simile a

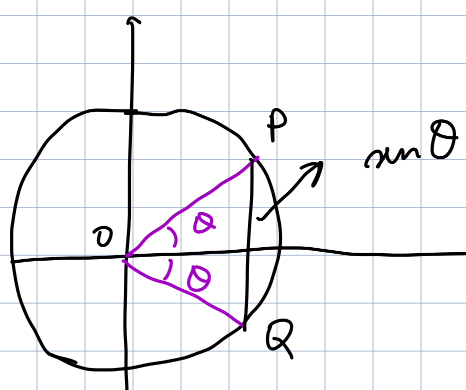


$$Sc : 1 = 1 : \cos\theta$$

Nota bene $\quad 1 = \sec(\theta) \cdot \cos\theta$

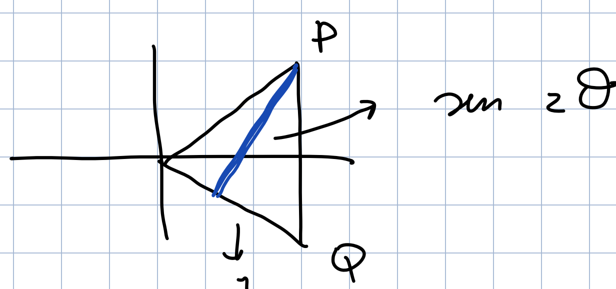
Le formule di addizione

Facciamo la duplicazione



area di

$\widehat{OPQ} = \cos\theta \sin\theta$



area $\widehat{OPQ} = \frac{1}{2}(\sin 2\theta)$

quindi $\quad \sin(2\theta) = 2\sin\theta\cos\theta$

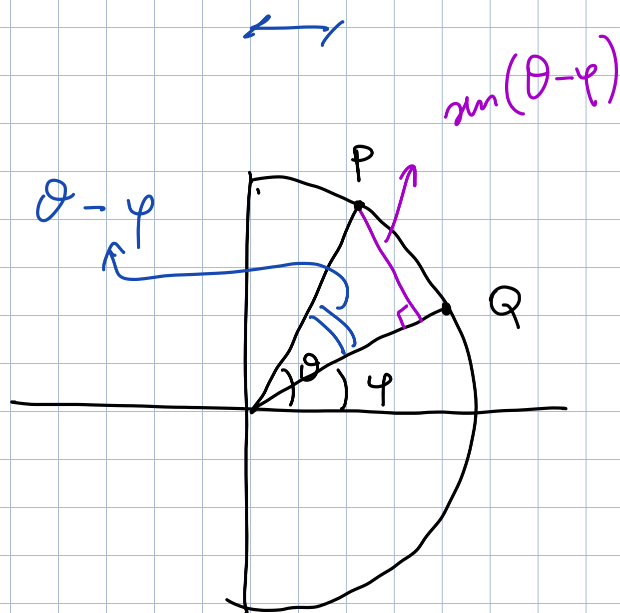
Per calcolare il coseno.

$$\cos^2(2\theta) = 1 - \sin^2(2\theta) = 1 - 4\sin^2\theta\cos^2\theta$$

$$= (\cos^2\theta + \sin^2\theta)^2 - 4\sin^2\theta\cos^2\theta$$

$$= \cos^4\theta + \sin^4\theta - 2\sin^2\theta\cos^2\theta = (\cos^2\theta - \sin^2\theta)^2$$

quindi $\quad \cos(2\theta) = \left[\cos^2(\theta) - \sin^2(\theta)\right]$ $\quad$ (perché non ci può essere il $-$?)



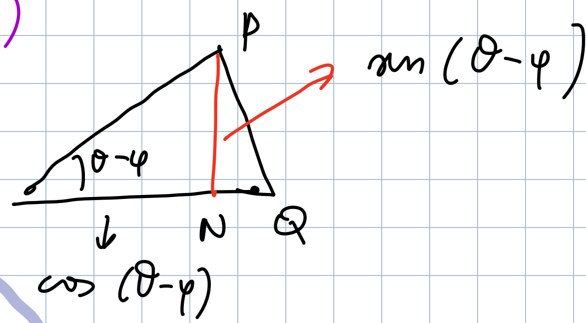
la lunghezza del segmento PQ è (x def)

$$l = \sqrt{(\cos\theta - \cos\varphi)^2 + (\sin\theta - \sin\varphi)^2}$$

$$= \sqrt{\underset{1}{\cos^2\theta} + \overset{2}{\cos^2\varphi} - 2\cos\theta\cos\varphi + \underset{1}{\sin^2\theta} + \overset{2}{\sin^2\varphi} - 2\sin\theta\sin\varphi}$$

(guardiamo L^2 così non scrivo la radice)

$$= 2 - 2(\cos\theta\cos\varphi + \sin\theta\sin\varphi)$$



Daltro conto

$$NQ = 1 - \cos(\theta-\varphi)$$

$$\Rightarrow L^2 = \sin^2(\theta-\varphi) + (1-\cos(\theta-\varphi))^2 =$$

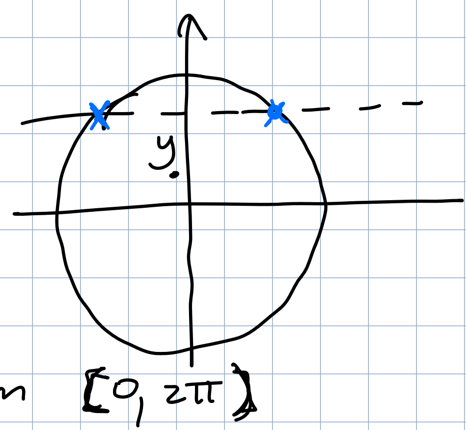
$$\sin^2(\theta-\varphi) + 1 + \cos^2(\theta-\varphi) - 2\cos(\theta-\varphi)$$

uguagliando $\cos(\theta-\varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$

$$\cos(\theta+\varphi) = \cos(\theta)\cos(-\varphi) + \sin\theta\sin(-\varphi)$$

$$= \cos(\theta)\cos(\varphi) - \sin\theta\sin\varphi$$

Funzioni trigonometriche inverse:



$\sin x = y_0$

per $-1 < y_0 < 1$

ha 2 soluzioni in $[0, 2\pi]$

chiedo che

se prendo $-\frac{\pi}{2} < x < \frac{\pi}{2}$ 1 sola soluzione

questa la chiamo $x_0 = \arcsin y_0$

N.B. $\cos x_0 = \sqrt{1 - y_0^2}$ $(\sqrt{\ } \geq 0 !)$

$\arcsin y : [-1, 1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$ in e su.