Esercizi su integrali e serie

Calcolare i seguenti integrali:

$$\int_1^{\sqrt{3}} \frac{\arctan x}{x^2}\,\mathrm{d}x;$$

$$\int_0^{\pi} e^{3x}\cos(2x)\,\mathrm{d}x;$$

$$\int_0^1 e^{\sqrt[3]{x}}\,\mathrm{d}x;$$

$$\int_1^{e^{\frac{\pi}{6}}} \frac{1}{x(1-\sin(\ln x))}\,\mathrm{d}x;$$

$$\int_{-1}^1 \frac{x^3+x^2}{1+|x|}\,\mathrm{d}x.$$

Discutere la convergenza delle seguenti serie:

$$\sum_{k=1}^{\infty}\left(e^{\frac{1}{k}}-1\right), \qquad \sum_{k=1}^{\infty}(-1)^k\left(e^{\frac{1}{k}}-1\right);$$

$$\sum_{k=1}^{\infty}\frac{\sin\frac{2}{k^2}}{2+\sin(k^2)}, \qquad \sum_{k=1}^{\infty}(-1)^k\frac{\sin\frac{2}{k^2}}{2+\sin(k^2)};$$

$$\sum_{k=1}^{\infty}\left(1-k\sin\frac{1}{k}\right), \qquad \sum_{k=1}^{\infty}(-1)^k\left(1-k\sin\frac{1}{k}\right);$$

$$\sum_{k=1}^{\infty}\frac{3^k}{2^{k^2}}, \qquad \sum_{k=1}^{\infty}(-1)^k\frac{3^k}{2^{k^2}};$$

$$\sum_{k=1}^{\infty}\frac{(2k)!}{k^k}, \qquad \sum_{k=1}^{\infty}(-1)^k\frac{(2k)!}{k^k}.$$