

Luca Battaglia

List of scientific publications

[29]

Authors: L.B., Yixing Pu, Giusi Vaira.

Title: *Infinitely many solutions for a boundary Yamabe problem.*

Status: submitted (<http://www.arxiv.org/abs/2503.06192>).

Short description: We study the problem of prescribing scalar curvature on the unit ball and mean curvature on its boundary, in the case of negative scalar and positive mean radial non-constant curvature. Using a Ljapunov-Schmidt-type construction, we build infinitely many non-radial solutions.

[28]

Authors: L.B., Francisco Javier Reyes Sánchez.

Title: *Prescribing curvatures on surfaces with conical singularities and corners.*

Status: submitted (<http://www.arxiv.org/abs/2502.15976>).

Short description: We study the problem of prescribing Gaussian and geodesic curvature on compact surfaces with boundary with singularities. Using a new mean field formulation and an asymptotic analysis based on the study of the Morse index of solutions, we get existence of energy-minimizing or min-max solutions.

[27]

Authors: L.B., Isabella Ianni, Angela Pistoia.

Title: *New solutions for the Lane-Emden problem in planar domains.*

Status: submitted (<http://www.arxiv.org/abs/2407.15742>).

Short description: We construct solutions to the Lane-Emden problem on planar domains which have, as the exponent approaches infinity, phenomena of multiple concentration. In particular, both the positive and the negative part concentrate, with different asymptotic profiles.

[26]

Authors: L.B., Angela Pistoia, Luigi Provenzano.

Title: *On the critical points of Steklov eigenfunctions.*

Status: accepted, *Ann. Mat. Pura Appl.* (<http://www.arxiv.org/abs/2402.01190>).

Short description: We study critical points of Steklov eigenfunction on Riemannian manifolds and we prove a formula concerning the sum of the indexes of critical points, which in the case of surfaces gives a precise count of their number. The formula holds true when eigenfunctions are Morse functions, which we also show to be true for a generic choice of a metric.

[25]

Authors: L.B., Rafael López Soriano.

Title: *A mean field problem approach for the double curvature prescription problem.*

Status: accepted, *Commun. Contemp. Math.* (<http://www.arxiv.org/abs/2309.07735>).

Short description: We give a new mean field-type formulation to study the problem of prescribing Gaussian and geodesic curvature on compact surfaces, which is equivalent to a Liouville-type PDE with nonlinear Neumann conditions. We provide three different existence results depending on the sign of the Euler characteristic of the surface.

[24]

Authors: L.B., Sergio Cruz Blázquez, Angela Pistoia.

Title: *Prescribing nearly constant curvatures on balls.*

Status: accepted, Proc. Roy. Soc. Edinburgh Sect. A (<http://www.arxiv.org/abs/2305.09622>).

Short description: We study the problem of prescribing both curvatures in the Euclidean ball, in the case of negative interior curvature and positive boundary curvature. Using a Ljapunov-Schmidt type reduction we get existence of solutions in the case when the prescribed curvatures are perturbation of constants.

[23]

Authors: L.B., Fabio De Regibus, Massimo Grossi.

Title: *On the shape of solutions to elliptic equations in possibly non convex domains.*

Status: published, Discrete Contin. Dyn. Syst. Ser. S 17 (2024), no. 4, 1588-1598 (<http://www.arxiv.org/abs/2301.08098>).

Short description: We study uniqueness of the critical point of a solution to an elliptic equation. In particular, we give sufficient conditions for the Poisson problem on a simply connected open domain and we consider the non-linear problem on a perturbation of a convex domain.

[22]

Authors: L.B., Matteo Cozzi, Antonio J. Fernández, Angela Pistoia.

Title: *Non uniqueness for the nonlocal Liouville equation in \mathbf{R} and applications.*

Status: published, SIAM J. Math. Anal. 55 (2023), no. 5, 4816-4842 (<http://www.arxiv.org/abs/2211.12106>).

Short description: We construct solutions for a nonlocal Liouville-type equation with a non constant potential which bifurcate from the well-known solutions with constant potential. Existence and multiplicity of solutions depend on the nature of critical points of the potential and the half-laplacian of the potential plays a key role. The equation has applications in geometry to the problem of prescribed geodesic curvature and in mathematical physics to the Calogero-Moser equation.

[21]

Authors: L.B., Aleks Jevnikar, Zhi-An Wang, Wen Yang.

Title: *Prescribing Gaussian curvature on surfaces with conical singularity and geodesic boundary.*

Status: published, Ann. Mat. Pura Appl. (4) 202 (2023), no. 3, 1173-1185 (<http://www.arxiv.org/abs/2011.01505>).

Short description: We study a singular Liouville-type PDE on a surface with boundary, corresponding to the problem of prescribing Gaussian curvature with geodesic boundary. Using min-max arguments, we find solutions in the case of disconnected boundary and Euler characteristic not belonging to a discrete set of critical values.

[20]

Authors: L.B., María Medina, Angela Pistoia.

Title: *A blow-up phenomenon for a non-local Liouville-type equation.*

Status: published, J. Anal. Math. 149 (2023), no. 1, 343-367 (<http://www.arxiv.org/abs/2011.01883>).

Short description: We construct family of solutions to a non-local Liouville-type PDE on the unit circle. Such solutions correspond to flat metrics on the disk with prescribed geodesic curvature. Other than conditions which are known to be necessary, only generic assumptions are made on the prescribed curvature.

[19]

Authors: L.B., María Medina, Angela Pistoia.

Title: *Large conformal metrics with prescribed Gaussian and geodesic curvatures.*

Status: published, Calc. Var. Partial Differential Equations 60 (2021), no. 1, 39
(<http://www.arxiv.org/abs/2006.12900>).

Short description: We construct family of solutions to an elliptic PDE with nonlinear Neumann conditions of exponential type. Such solutions correspond to conformal metrics on the unit disk with prescribed Gaussian and geodesic curvatures. Other than some conditions which are known to be necessary, we only make generic assumptions on the prescribed curvatures.

[18]

Authors: L.B., Francesca Gladiali, Massimo Grossi.

Title: *Asymptotic behavior of minimal solutions to $-\Delta u = \lambda f(u)$ as $\lambda \rightarrow -\infty$.*

Status: published, Discrete Contin. Dyn. Syst. 41 (2021), no.2, 681-700
(<http://www.arxiv.org/abs/1911.03152>).

Short description: We study the problem $-\Delta u = \lambda f(u)$ with Dirichlet conditions on a bounded open domain, with $\lambda < 0$ and f non-negative and non-decreasing. We show existence and uniqueness for fixed λ fissato and asymptotic behavior for $\lambda \rightarrow -\infty$.

[17]

Authors: L.B., Rafael López Soriano.

Title: *A double mean field equation related to a curvature prescription problem.*

Status: published, J. Diff. Equations 269 (2020), no. 4, 2705-2740
(<http://www.arxiv.org/abs/1906.10934>).

Short description: We study a double mean field type equation on a surface with boundary, namely a PDE with a mean field-type nonlinearity and a nonlinear Neumann condition of mean field-type. We prove: blow-up analysis, Moser-Trudinger type inequality, existence of min-max solutions in the non-coercive case.

[16]

Authors: L.B., Massimo Grossi, Angela Pistoia.

Title: *Non-uniqueness of blowing-up solutions to the Gelfand problem.*

Status: published, Calc. Var. Partial Differential Equations 58 (2019), no. 5, Paper No. 163, 28 pp. (<http://www.arxiv.org/abs/1902.03484>).

Short description: We construct families of blowing-up solutions to the Gelfand problem on open planar domains. In particular, if blow up occurs at a degenerate critical points of the reduced functional, we can construct multiple families of blowing-up solutions.

[15]

Authors: L.B.

Title: *Uniform bounds for solutions to elliptic problems on simply connected planar domains.*

Status: published, Proc. Amer. Math. Soc. 147 (2019), no. 10, 4289-4299
(<http://www.arxiv.org/abs/1809.05684>).

Short description: We show an a priori estimate for solutions to Liouville and Hénon-Lane-Emden equations with one singularity: any solution on a simply connected planar domain must have uniformly bounded mass. The same argument applies to some systems and more general nonlinearities.

[14]

Authors: L.B.

Title: *A general existence result for stationary solutions to the Keller-Segel system.*

Status: published, Discrete Contin. Dyn. Syst. 39 (2019), no. 2, 905-926
(<http://www.arxiv.org/abs/1802.02551>).

Short description: We study a Liouville-type equation related to the Keller-Segel model in chemotaxis. We show existence of solutions, via variational and Morse-theoretical methods, for a suitable choice of the parameters.

[13]

Authors: L.B., Angela Pistoia.

Title: *A unified approach of blow-up phenomena for two-dimensional singular Liouville systems.*

Status: published, Rev. Mat. Iberoam. 34 (2018), no. 4, 1867-1910(<http://www.arxiv.org/abs/1607.00427>).

Short description: We construct a family of solutions for a general 2×2 Liouville system on a bounded planar domain. This family blows up at the origin and the local mass is a given quantity depending on the parameters of the system through an explicit formula involving Chebyshev polynomials.

[12]

Authors: L.B., Jean Van Schaftingen.

Title: *Groundstates of the Choquard equations with a sign-changing self-interaction potential.*

Status: published, Z. Angew. Math. Phys. 69 (2018), no. 3, 69:86
(<http://www.arxiv.org/abs/1710.04406>).

Short description: We consider a Choquard equation with a general unbounded potential. We show the existence of groundstate solutions by solving an approximating problem and then proving convergence of such approximating solutions.

[11]

Authors: L.B., Francesca Gladiali, Massimo Grossi.

Title: *Nonradial entire solutions for Liouville systems.*

Status: published, J. Diff. Equations 263 (2017), no. 8, 5151-5174
(<http://www.arxiv.org/abs/1701.02948>).

Short description: Using bifurcation theory we prove the existence of nonradial solutions for some 2×2 Liouville systems on the plane. In particular, we show the existence of branches of solutions bifurcating from the trivial solution where the two components coincide.

[10]

Authors: L.B., Jean Van Schaftingen.

Title: *Existence of groundstates for a class of nonlinear Choquard equations in the plane.*

Status: published, Adv. Nonlinear Stud. 17 (2017), no. 3, 581-594
(<http://www.arxiv.org/abs/1604.03294>).

Short description: We show the existence of a groundstate solution of a class of Choquard equations, under general hypotheses on the nonlinearity (nontriviality, growth, subcriticality).

[9]

Authors: L.B.

Title: *B_2 and G_2 Toda systems on compact surfaces: a variational approach.*

Status: published, J. Math. Phys. 58 (2017), no. 1, 011506, 25 pp.
(<http://www.arxiv.org/abs/1512.07566>).

Short description: We study the B_2 and G_2 Toda systems using variational methods. We obtain a general existence and multiplicity result for surfaces of positive genus, like in [5]. Some of the results are extended to generic Liouville systems.

[8]

Authors: L.B.

Title: *Ground states solutions for a nonlinear Choquard equation.*

Status: published, Rend. Sem. Mat. Univ. Politec. Torino Vol. 74, 2 (2016), 53-60 (<http://www.arxiv.org/abs/1701.02376>).

Short description: This is a survey on my talk at Bru-To PDE's conference. The results are from [10] and a paper by Moroz-Van Schaftingen proving the same results in dimension $N \geq 3$.

[7]

Authors: L.B., Andrea Malchiodi.

Title: *Existence and non-existence results for the $SU(3)$ singular Toda system on compact surfaces.*

Status: published, J. Funct. Anal. 270 (2016), no. 10, 3750-3807 (<http://www.arxiv.org/abs/1508.00929>).

Short description: We give both existence and non-existence results for the $SU(3)$ singular Toda system on compact surfaces of arbitrary genus. Existence results are obtained using variational methods, non-existence results are obtained using blow-up analysis and Pohožaev-type identity.

[6]

Authors: L.B.

Title: *Moser-Trudinger inequalities for singular Liouville systems.*

Status: published, Math. Z. 282 (2016), no. 3-4, 1169-1190 (<http://www.arxiv.org/abs/1410.4994>).

Short description: We extend the Moser-Trudinger inequality from the previous work [2] with Malchiodi to general singular Liouville system. We characterize the parameters which yield coercivity and we provide sharp inequalities under some assumptions on the coefficients.

[5]

Authors: L.B., Aleks Jevnikar, Andrea Malchiodi, David Ruiz.

Title: *A general existence result for the Toda system on compact surfaces.*

Status: published, Adv. Math. 285 (2015), 937-979 (<http://www.arxiv.org/abs/1306.5404>).

Short description: We prove a general existence result for the singular Toda system on compact surfaces with positive genus. The existence of solutions is proved through the non-contractibility of lower sublevels of the energy and a standard min-max scheme. This result also extends to the case of Sinh-Gordon equation.

[4]

Authors: L.B., Gabriele Mancini.

Title: *A note on compactness properties of the singular Toda system.*

Status: published, Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl. 26(3):299-307, 2015 (<http://www.arxiv.org/abs/1410.4991>).

Short description: We consider blow-up for solutions of singular Toda system. In particular, we show how quantization results for local blow-up limits imply compactness of solutions for almost any values of the parameters.

[3]

Authors: L.B.

Title: *Existence and multiplicity result for the singular Toda system.*

Status: published, J. Math. Anal. Appl. 424 (2015), no. 1, 49-85
(<http://www.arxiv.org/abs/1404.1970>).

Short description: We extend the existence result obtained with Jevnikar, Malchiodi and Ruiz [5] to the case of vortices having arbitrary sign. We also prove a generic multiplicity result, that is we give an estimate from below to the number of solutions, using a Morse-theoretical argument, for a generic choice of the data.

[2]

Authors: L.B., Andrea Malchiodi.

Title: *A Moser-Trudinger inequality for the singular Toda system.*

Status: published, Bull. Inst. Math. Acad. Sin. (N.S.) 9 (2014), no. 1, 1-23
(<http://www.arxiv.org/abs/1307.3921>).

Short description: We prove a Moser-Trudinger-type inequality for the singular Toda system. This result is a natural extension of Moser-Trudinger inequalities for the Liouville equation and for the regular Toda system. A straightforward consequences is the characterization of the parameters which give boundedness from below and coercivity for the energy functional.

[1]

Authors: L.B., Gabriele Mancini.

Title: *Remarks on the Moser-Trudinger inequality.*

Status: published, Adv. Nonlinear Anal. 2 (2013), no. 4, 389-425
(<http://www.arxiv.org/abs/1307.0746>).

Short description: We extend the Moser-Trudinger inequality to any Euclidean domain which satisfies Poincaré's inequality. We find out, showing counterexamples, that the same equivalence does not hold in general for conformal metrics on the unit ball. We also study the existence of extremals for the Moser-Trudinger inequalities on unbounded domains, proving it for the infinite planar strip.

Rome, March 12th, 2025