## INdAM Italian-Korean Meeting on Algebraic Geometry 2015


#### Abstract

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Francesco Bastianelli: On large theta-characteristics with prescribed vanishing Let $\mathcal{M}_{g}^{r}$ be the locus in $\mathcal{M}_{g}$ described by smooth curves $C$ admitting a thetacharacteristic $L$ such that $h^{0}(C, L) \geq r+1$ and $h^{0}(C, L) \equiv r+1(\bmod 2)$. These loci have been introduced by Harris, who proved a sharp bound on their codimension. Besides, Kontsevich and Zorich investigated theta-characteristics depending on the vanishing of global sections. I shall present a joint work with E. Ballico and L. Benzo, where we consider both the viewpoints above at once. Namely, given a sequence of positive integers $\underline{k}=\left(k_{1}, \ldots, k_{n}\right)$ with $\sum_{i} k_{i}=g-1$, we study the loci $\mathcal{G}_{g}^{r}(\underline{k})$ in $\mathcal{M}_{g, n}$ parameterizing $n$-pointed curves $\left(C, p_{1}, \ldots, p_{n}\right)$ such that $L:=\mathcal{O}_{C}\left(\sum_{i} k_{i} p_{i}\right)$ is a theta-characteristic as above. We achieve a general upper bound governing the codimension of $\mathcal{G}_{g}^{r}(\underline{k})$ in $\mathcal{M}_{g, n}$. Moreover, we prove that when the genus $g$ is large enough, any $\mathcal{G}_{g}^{r}(\underline{k})$ has an irreducible component of maximal codimension, so that our bound turns out to be sharp.


Cristiano Bocci: Recent results for the containment problem and Waldschmidt constants

The interest for the containment problem of symbolic powers of ideals in ordinary ones has grown in the last years, producing many interesting papers and arising new related problems. I will mainly focus on some result with B. Harbourne and S. Cooper, about a series of conjectures with different "containment behaviour". In general, in these cases the studied ideals are ideals of zero-dimensional schemes. Another class of interesting ideals is given by square-free monomial ideals, were the conjecture were proved by (Cooper, Embree, H‘a, Hoefel). Here, the authors proposed a different approach to compute the Waldschmidt constant of the ideal based on the symbolic polyedron. I wil show that for square-free monomial ideals associated to a graph we can give a description of the Waldschmidt constant in term of the combinatorial data of the graph.

Cinzia Casagrande: On the birational geometry of Fano 4-folds with large second Betti number

We will review some results and open problems in the study of the geometry of (smooth, complex) Fano 4-folds, in particular in the case of large second Betti number.

## Paolo Cascini: Invariance of plurigenera for foliations

I will survey some recent results on the Minimal Model Program for foliations on algebraic surfaces and I will apply these results to prove a version of invariance of plurigenera for foliations on surfaces. Joint work with E. Floris.

Insong Choe: Understanding Segre invariants via higher secant varieties

The moduli space of $G$-bundles over an algebraic curve is stratified by the generalized Segre invariants, which measures the degree of maximal subbundles. For the cases $G=G L(n), \operatorname{Sp}(2 n)$, and $O(n)$, we observe that the Segre stratification matches with that coming from the higher secant varieties of certain subvarieties in the extension spaces. From this observation, together with the non-defectivity of the secant varieties, we prove the sharp upper bound on the generalized Segre invariants. This talk is based on a series of joint works with George H. Hitching.

## Youngook Choi: Hilbert schemes of smooth curves

Let $\mathcal{M}_{g, d}^{r}$ be the sublocus of $\mathcal{M}_{g}$ whose points correspond to smooth curves possessing $g_{d}^{r}$ and let $\mathcal{I}_{d, g, r}^{\prime}$ be the union of the components of the Hilbert scheme whose general points represent smooth irreducible complex curves of degree $d$ and genus $g$ in $\mathbb{P}^{r}$.

The aim of this talk is twofolds. First, to state the geometry of $\mathcal{I}_{d, g, r}^{\prime}$ when the Brill-Noether number $\rho(d, g, r):=g-(r+1)(g-d+r)=-1$. Second, to show the existence of an additional component of $\mathcal{I}_{d, g, r}^{\prime}$ whose general elements are double covers of curves of positive genus for some $d, g, r$ with $\rho(d, g, r) \geq 0$.

If the Brill-Noether number $\rho(g, r, d)=-1$, it is known that $\mathcal{M}_{g, d}^{r}$ is irreducible. We prove that if $g$ is odd, and $r, s, d, e(r \neq s)$ are positive integers satisfying $\rho(g, r, d)=\rho(g, s, e)=-1$ and $e \neq 2 g-2-d$, then the supports of $\mathcal{M}_{g, d}^{r}$ and $\mathcal{M}_{g, e}^{s}$ are distinct. As an application, we show that in case $d>g$ there is a unique irreducible component $\mathcal{I}_{d, g, r}^{\prime}$ dominating $\mathcal{M}_{g, d}^{r}$ and that a general member $\mathcal{I}_{d, g, r}^{\prime}$ has no $(d-e)$-secant $(r-s-1)$-plane for $\rho(g, s, e)=-1, e \neq 2 g-2-d$.

On the other hand, Severi claimed that $\mathcal{I}_{d, g, r}^{\prime}$ is irreducible if $d \geq g+r$. His conjecture turned out to be correct for $r=3$ and 4 , while for $r \geq 6$ there have been found counter examples using families of $m$-sheeted covers of rational curves with $m \geq 3$. In this talk, we show the existence of an additional component of $\mathcal{I}_{d, g, r}^{\prime}$ whose general elements are double covers of curves of positive genus.

## Alessio Corti: Mirror symmetry and Fano orbifolds

I give an account of mirror symmetry for Fano orbifolds with special emphasis on the classification problem for Fano orbifolds. I discuss at some length the case of surfaces and give indications in higher dimensions.

## Gabriele Di Cerbo: Birational geometry of complex hyperbolic manifolds

In 1984 Hirzebruch constructed the first examples of smooth toroidal compactifications of ball quotients with non-nef canonical divisor. In this talk, I will show that if the dimension is greater or equal than three then such examples cannot exist. We will use this result to reprove and improve classical theorems, such as boundedness of hyperbolic manifolds, Baily-Borel embeddings and cusps count.

June Huh: A tropical approach to a Hodge conjecture for positive currents
Demailly showed that the Hodge conjecture is equivalent to the statement that any closed current with rational cohomology class can be approximated by linear combinations of integration currents associated to subvarieties, and asked whether
any strongly positive closed current with rational cohomology class can be approximated by positive linear combinations of integration currents associated to subvarieties. Using tropical geometry, we construct a strongly positive closed current on a smooth projective variety that does not satisfy the latter statement. Joint work with Farhad Babaee.

David Hyeon: Regularity and Bridgeland stability of zero dimensional subschemes of the projective plane
In this talk, I will describe a close and precise connection between the CastelnuovoMumford regularity and the Bridgeland stability of zero dimensional schemes on the projective plane. This is a joint work with Jungyoung Park (POSTECH) and Izzet Coskun (University of Illinois at Chicago).

Young-Hoon Kiem: On the Fano visitor problem
One basic problem in algebraic geometry is to study how a variety can be embedded in other varieties. In 2011, Bondal categorified the embedding problem and raised the following question.

Question. (Fano visitor problem) Let Y be a smooth projective variety. Is there a Fano variety X equipped with a fully faithful embedding of the derived category of Y into that of X?

If there is such an X , then Y is called a Fano visitor and X a Fano host of Y . In this talk, I will talk about a joint work with In-Kyun Kim, Hwayoung Lee and Kyoung-Seog Lee in which we proved that every complete intersection is a Fano visitor. I may also talk about a Hodge-theoretic criterion for the existence of a Fano host and discuss related questions.

## Bumsig Kim: Elliptic Quasimap Invariants

We propose and prove a mirror theorem for the elliptic quasimap invariants for (the products of) smooth CY complete intersections in projective spaces and local CY over projective spaces. This is a joint work with Hyenho Lho.

Seonja Kim: Brill-Noether loci with small codimension
We will investigate the existence of a smoothable limit linear series of dimension $r$ and degree $d$ on a nodal curve of compact type such that two curves with general moduli are connected by a chain of elliptic curves. The existence of such a linear series means that the nodal curve belongs to the closure of the Brill-Noether locus $\mathrm{M}(\mathrm{r}, \mathrm{d})$ in $\mathcal{M}_{g}$ whose point corresponds to a smooth curve possessing a linear series of dimension r and degree d . Using this we see that the Brill-Noether loci with the same Brill-Noether number $\rho$ have mutually distinct supports in case $\rho=-1$ or -2 .

## Young-Rock Kim: Geometry and topology of phase tropical hypersurfaces

We will start by giving the definition of phase tropical hypersurface in terms of some degeneration data of smooth hypersurface in $\left(\mathbb{C}^{*}\right)^{n}$. First, we prove that pair of pants are homeomorphic to their degeneration called phase tropical pair of pants. As we know, by Mikhalkin's theorem, smooth hypersurfaces have a decomposition
into by pair of pants, we show the more general situation by gluing these pair of pants. Moreover, we show that there is a symplectic structure on phase tropical hypersurfaces. This is joint work with Mounir Nisse.

Yongnam Lee: On subfields of the function field of a general surface in $\mathbb{P}^{3}$
In this talk, we treat dominant rational maps from a very general surface $X$ of degree $\geq 5$ in $\mathbb{P}^{3}$ to smooth projective surfaces $Y$. Based on the classification theory of algebraic surfaces, Hodge theory, and deformation theory, we show that there is no dominant rational map from $X$ to $Y$ unless $Y$ is rational or $Y$ is birational to $X$. I will also discuss on rational maps from the product of two very general curves. It is a joint work with Gian Pietro Pirola.

## Gianluca Occhetta : A characterization of complete flag manifolds

Let $G$ be a semisimple algebraic group and $B$ a Borel subgroup; the complete flag manifold $G / B$ is a Fano manifold whose elementary contractions are smooth $\mathbb{P}^{1}$ fibrations. In particular the number of such fibrations is equal to $\rho(G / B)$, the Picard number of the manifold.

I will show how these manifolds can be characterized by this property, namely that a smooth complex projective manifold $X$ of Picard number $n$ which admits $n$ contractions $\pi_{i}: X \rightarrow X_{i}$ which are smooth $\mathbb{P}^{1}$-fibrations is isomorphic to a complete flag manifold $G / B$, and how this characterization can be used to recognize some homogeneous manifolds from their varieties of lines by a point.

This is a joint work with Roberto Muñoz, Luis E. Solá Conde, Kiwamu Watanabe and Jaroslaw A. Wisniewski.

Jihun Park: Cylinders in del Pezzo surfaces
For an ample divisor H on a del Pezzo surface S , an H-polar cylinder on a del Pezzo surface S is the complement of the support of an effective $\mathbb{Q}$-divisor $D$ such that $D$ is $\mathbb{Q}$-linearly equivalent to H and $S \backslash \operatorname{Supp}(D)$ is isomorphic to $\mathbb{A}^{1} \times Z$ for some affine variety $Z$. In this talk I present various cylinders on del Pezzo surfaces and explain why some ample classes do not carry cylinders. This is a joint work with I. Cheltsov and J. Won.

Francesco Polizzi: Surfaces with $p_{g}=q=2$ and an irrational fibration
We discuss several examples of surfaces with $p_{g}=q=2$ and maximal Albanese dimension that are endowed with an irrational fibration. This is a joint work with M. Penegini.

## Francesco Zucconi: Volume forms

Let $\mathcal{L}$ and $\mathcal{F}$ be a rank 1 and respectively a rank $n$ locally free sheaf over an $m$ dimensional smooth algebraic variety $X$. Consider an exact sequence $0 \rightarrow \mathcal{L} \rightarrow \mathcal{E} \rightarrow$ $\mathcal{F} \rightarrow 0$ associated to an element $\xi \in \operatorname{Ext}^{1}(\mathcal{F}, \mathcal{L})$ and the induced exact sequence:

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0 \rightarrow \bigwedge^{n-1} \mathcal{F} \otimes \mathcal{L} \rightarrow \bigwedge^{n} \mathcal{E} \rightarrow \operatorname{det} \mathcal{F} \rightarrow 0
$$

If $W$ is an $n+1$ dimensional subspace of $\operatorname{Ker}\left(\partial_{\xi}: \mathrm{H}^{0}(\mathrm{X}, \mathcal{F}) \rightarrow \mathrm{H}^{1}(\mathrm{X}, \mathcal{L})\right)$ then we can associate to $W$ and $\xi$ a top form $\Omega \in H^{0}(X, \operatorname{det} \mathcal{E})$ up to liftings of $W$ to $H^{0}(X, \mathcal{E})$.

We study the geometry of these top forms. This theory is a strong generalisation of the analogue one shown in the paper Variations of the Albanese morphisms, J. A. G. 12 (2003), no. 3, 535-572 written with G. P. Pirola. In particular we prove a version of the Adjoint theorem which is valid in any dimension $m$ and for any rank $n$ l.f. sheaf $\mathcal{F}$; its original version in the case $m=n=1$ is in The Griffiths infinitesimal invariant for a curve in its Jacobian Duke Math. J. 78 (1995), no. 1, 59-88 written by A. Collino and G.P. Pirola.

As a byproduct, we recover the Griffith's infinitesimal Torelli theorem for projective hypersurfaces; that is: Theorem 9.8 (b) of On the Periods of Certain Rational Integrals: I, Ann. of Math. (2) 90 (1969), 460-495, and the Green's infinitesimal Torelli theorem for a generic hypersurface $X$ of a sufficiently ample linear system on a smooth variety $Y$; see: The period map for hypersurface sections of high degree of an arbitrary variety, Compositio Math. 55 (1985), 135-156.

