Youngook Choi

(Yeungnam University)

Hilbert schemes of smooth curves

Let $\mathcal{M}^r_{g,d}$ be the sublocus of $\mathcal{M}_g$ whose points correspond to smooth curves possessing g^r_d and let $\mathcal{I}'_{d,g,r}$ be the union of the components of the Hilbert scheme whose general points represent smooth irreducible complex curves of degree d and genus g in $\mathbb{P}^r$.

The aim of this talk is twofolds. First, to state the geometry of $\mathcal{I}'_{d,g,r}$ when the *Brill-Noether number* $\rho(d,g,r) := g-(r+1)(g-d+r) = -1$. Second, to show the existence of an additional component of $\mathcal{I}'_{d,g,r}$ whose general elements are double covers of curves of positive genus for some d, g, r with $\rho(d,g,r) \geq 0$.

If the Brill-Noether number $\rho(g,r,d) = -1$, it is known that $\mathcal{M}^r_{g,d}$ is irreducible. We prove that if g is odd, and r, s, d, e $(r \neq s)$ are positive integers satisfying $\rho(g,r,d) = \rho(g,s,e) = -1$ and $e \neq 2g-2-d$, then the supports of $\mathcal{M}^r_{g,d}$ and $\mathcal{M}^s_{g,e}$ are distinct. As an application, we show that in case $d > g$ there is a unique irreducible component $\mathcal{I}'_{d,g,r}$ dominating $\mathcal{M}^r_{g,d}$ and that a general member $\mathcal{I}'_{d,g,r}$ has no $(d-e)$-secant $(r-s-1)$-plane for $\rho(g,s,e) = -1, e \neq 2g-2-d$.

On the other hand, Severi claimed that $\mathcal{I}'_{d,g,r}$ is irreducible if $d \geq g+r$. His conjecture turned out to be correct for $r = 3$ and 4, while for $r \geq 6$ there have been found counter examples using families of m-sheeted covers of rational curves with $m \geq 3$. In this talk, we show the existence of an additional component of $\mathcal{I}'_{d,g,r}$ whose general elements are double covers of curves of positive genus.