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Volume forms

Let \mathcal{L} and \mathcal{F} be a rank 1 and respectively a rank *n* locally free sheaf over an *m*dimensional smooth algebraic variety *X*. Consider an exact sequence $0 \to \mathcal{L} \to \mathcal{E} \to \mathcal{F} \to 0$ associated to an element $\xi \in \text{Ext}^1(\mathcal{F}, \mathcal{L})$ and the induced exact sequence:

$$0 \to \bigwedge^{n-1} \mathcal{F} \otimes \mathcal{L} \to \bigwedge^n \mathcal{E} \to \det \mathcal{F} \to 0.$$

If W is an n+1 dimensional subspace of $\operatorname{Ker}(\partial_{\xi} \colon \operatorname{H}^{0}(\mathbf{X}, \mathcal{F}) \to \operatorname{H}^{1}(\mathbf{X}, \mathcal{L}))$ then we can associate to W and ξ a top form $\Omega \in \operatorname{H}^{0}(X, \det \mathcal{E})$ up to liftings of W to $\operatorname{H}^{0}(X, \mathcal{E})$. We study the geometry of these top forms. This theory is a strong generalisation of the analogue one shown in the paper Variations of the Albanese morphisms, J. A. G. 12 (2003), no. 3, 535–572 written with G. P. Pirola. In particular we prove a version of the Adjoint theorem which is valid in any dimension m and for any rank n l.f. sheaf \mathcal{F} ; its original version in the case m = n = 1 is in The Griffiths infinitesimal invariant for a curve in its Jacobian Duke Math. J. 78 (1995), no. 1, 59–88 written by A. Collino and G.P. Pirola.

As a byproduct, we recover the Griffith's infinitesimal Torelli theorem for projective hypersurfaces; that is: Theorem 9.8 (b) of On the Periods of Certain Rational Integrals: I, Ann. of Math. (2) 90 (1969), 460–495, and the Green's infinitesimal Torelli theorem for a generic hypersurface X of a sufficiently ample linear system on a smooth variety Y; see: The period map for hypersurface sections of high degree of an arbitrary variety, Compositio Math. 55 (1985), 135–156.