ON THE WAHL MAP OF PLANE NODAL CURVES

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1. INTRODUCTION

Let C be a smooth irreducible curve of genus g and let $\Phi_{\omega_C} : \bigwedge^2 H^0(\omega_C) \to H^0(\omega_C^3)$ be the Wahl map of C. Since Wahl's introduction in 1987, the corank of Φ_{ω_C} has been related to many geometrical properties of the curve, such as the possibility of embedding C on a K3 surface [W1] and on Fano varieties [CLM1], [CLM2], the Clifford index and the existence of linear series on C [BEL], [P], the fact that C has general moduli [CHM], [V], just to mention a few. A recent addition to the above list was made by Wahl in 1995 [W2], where he remarked the importance of the cohomology of the square of the ideal sheaf of a canonical curve and its connection with Green's conjecture. Wahl proved that if $C \subset \mathbb{P}^{g-1}$ is a canonical curve satisfying $(*)$ $H^1(\mathcal{I}_C^2(t)) = 0$ for every $t \geq 3$, then C is extendable if and only if Φ_{ω_C} is not surjective. In the same article he conjectured that $(*)$ holds for large g and for any curve with Clifford index at least 3 and remarked that if Green's conjecture holds for $p = 3$ (that is if $Cliff C > p$ then $C \subset \mathbb{P}^{g-1}$ is projectively normal, its ideal is generated by quadrics and the syzygies are generated by linear polynomials up to the p-th module) then $H^1(\mathcal{I}_C^2(4)) = 0$ for every curve C with $Cliff C > 3$. Inspired by this, in a recent article the first two authors [CL] considered the problem of finding families of curves in $\mathcal{M}_g$ of large dimension, for example $\frac{3}{2}g + 15$, with nonsurjective Wahl map, as

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their general element would be a counterexample to Wahl's conjecture. As [CL] shows this task appears to be one of not so easy solution, and in the present article we confirm this intuition by studying the behaviour of the Wahl map in a very natural family of curves, i.e. plane curves with nodes.

Let $d \geq 1, 0 \leq \delta \leq \binom{d-1}{2}$ be given integers and let $D \subset \mathbb{P}^2$ be an irreducible curve of degree d with δ nodes $P_1, \ldots, P_\delta \in \mathbb{P}^2$ and no other singularity. Denote by $V_{d,\delta}$ the Severi variety parametrizing such curves. For a real number x let $[x]$ be its integer part. Our result is

Theorem (1.1). *Let $D \subset \mathbb{P}^2$ be an irreducible curve of degree $d \geq 15$ with δ nodes and no other singularity, C its normalization. Then the Wahl map of C is surjective if one of the following holds:*

(1.2) D is a general member of the Severi variety $V_{d,\delta}$ and $10 \leq \delta \leq \frac{1}{2}\left[\frac{d}{3}\right](\left[\frac{d}{3}\right]+3)-5$

or

(1.3) D is any member of the Severi variety $V_{d,\delta}$ such that the nodes $P_1, \ldots, P_\delta$ do not lie on a cubic and $10 \leq \delta \leq \left[\frac{d}{3}\right]+5$.

The surjectivity of the Wahl map of C is obtained in the following way. Let $\epsilon : X \to \mathbb{P}^2$ be the blow-up of $\mathbb{P}^2$ at the nodes $P_1, \ldots, P_\delta$ with exceptional divisors $E_1, \ldots, E_\delta$. Let l be the divisor of a line in $\mathbb{P}^2$ and $H = \epsilon^* l$. By the standard diagram

$$
\begin{array}{llcl}
 & \bigwedge^2 H^0(X, \mathcal{O}_X(K_X+C)) & \stackrel{\Phi_{K_X+C}}{\longrightarrow} & H^0(X, \Omega^1_X(2K_X+2C)) \\
 & & & \downarrow \phi \\
(1.4) & \downarrow p & & H^0(C, \Omega^1_X(2K_X+2C)_{|C}) \,. \\
 & & & \downarrow \psi \\
 & \bigwedge^2 H^0(C, \omega_C) & \stackrel{\Phi_{\omega_C}}{\longrightarrow} & H^0(C, \omega_C^3)
\end{array}
$$

it follows that Φ_{ω_C} is surjective as soon as the same holds for $\psi \circ \phi$ and Φ_{K_X+C}. In section two we show that the surjectivity of the first map is related to the cohomology of a certain sheaf of differentials on X with logarithmic poles along C and we prove that it does hold in most cases. The Gaussian map Φ_{K_X+C} is studied in section three, employing a technique introduced in [CLM3], mainly an application of the Kawamata-Viehweg vanishing theorem. We show that this method is successful if the line bundle $[\frac{d}{3}]H - \sum_{j=1}^{\delta} E_j$ is very ample on X, hence giving the bound on δ in Theorem (1.1).

It should be remarked that this bound on the number of nodes forces the image in $\mathcal{M}_g$ of the Severi variety to have in fact not so large dimension (of the order of g). When the number of nodes is higher our technique does not apply and, as far as we know, the problem of computing the corank of the Wahl map for such curves is still open.

2. THE ROLE OF DIFFERENTIALS WITH LOGARITHMIC POLES

The aim of this section is to study the composition map $\psi \circ \phi$ in diagram (1.4). We start by recording a general result.

Lemma (2.1). *Let S be a smooth irreducible surface, $C \subset S$ a smooth irreducible curve and L a line bundle on S. Consider the maps $\phi : H^0(S, \Omega^1_S \otimes L^2) \to H^0(C, \Omega^1_S \otimes L^2_{|C})$ and $\psi : H^0(C, \Omega^1_S \otimes L^2_{|C}) \to H^0(C, \omega_C \otimes L^2_{|C})$.*
Then $\quad$ *cork* $\psi \circ \phi \le$ *cork* $\psi + h^1(S, \Omega^1_S(logC) \otimes L^2(-C))$.

Proof: The diagram

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & \Omega^1_S \otimes L^2(-C) & \to & \Omega^1_S(logC) \otimes L^2(-C) & \to & \mathcal{O}_C \otimes L^2(-C) & \to & 0 \\
 & & \downarrow \mathrm{id} & & \downarrow & & \downarrow & & \\
0 & \to & \Omega^1_S \otimes L^2(-C) & \to & \Omega^1_S \otimes L^2 & \to & \Omega^1_S \otimes L^2_{|C} & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & \to & \omega_C \otimes L^2_{|C} & \xrightarrow{\mathrm{id}} & \omega_C \otimes L^2_{|C} & \to & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}$$

induces in cohomology

$$\begin{array}{ccccccc}
 & & 0 & & & & \\
 & & \downarrow & & & & \\
 & Ker\psi = & H^0(\mathcal{O}_C \otimes L^2(-C)) & \xrightarrow{\beta} & H^1(\Omega^1_S \otimes L^2(-C)) & \to & H^1(\Omega^1_S(logC) \otimes L^2(-C)) \\
 & & \downarrow & \searrow \alpha & & \searrow \mathrm{id} & \\
H^0(\Omega^1_S \otimes L^2) & \xrightarrow{\phi} & H^0(\Omega^1_S \otimes L^2_{|C}) & \to & Coker\phi & \subseteq & H^1(\Omega^1_S \otimes L^2(-C)) \\
 & & \downarrow \psi & & & & \\
 & & H^0(\omega_C \otimes L^2_{|C}) & & & &
\end{array}$$

Therefore cork $\psi \circ \phi -$ cork $\psi =$ cork $\alpha \le$ cork $\beta \le h^1(S, \Omega^1_S(logC) \otimes L^2(-C))$. ∎

We apply the above lemma to our case $L = K_X + C$ where X is the blow-up of $I\!P^2$ at the nodes of D and $C \in |\mathcal{O}_X(dH - 2\sum_{j=1}^{\delta} E_j)|$ is its normalization.

Proposition (2.2). *Suppose $d \geq 7, \delta \geq 10$ and that the nodes $P_1, \dots, P_\delta$ do not lie on a plane cubic. Then $h^1(X, \Omega^1_X(logC) \otimes \omega^2_X(C)) = 0$ and the map $\psi \circ \phi$ in diagram (1.4) is surjective.*

Proof: Let x, y be local coordinates on $\mathbb{P}^2$ so that the local equation of D is $xy = 0$. On X we have local coordinates ξ, η with $x = \xi, y = \eta\xi$ where $x \neq 0$ and ξ', η' with $y = \xi', x = \eta'\xi'$ where $y \neq 0$. The sheaf $\Omega^1_X(logC)$ is then locally generated by $d\xi, \frac{d\eta}{\eta}$ and $d\xi', \frac{d\eta'}{\eta'}$ on the two open subsets. Let $\mathcal{F}$ be the subsheaf of $\Omega^1_X(logC)$ which coincides with $\Omega^1_X(logC)$ away from the exceptional divisors and in a neighborhood of a point of an exceptional divisor is generated by $d\xi$ and $\xi\frac{d\eta}{\eta}$ (and $d\xi', \xi'\frac{d\eta'}{\eta'}$). By restricting to a local equation of C we get an exact sequence

$$0 \to \epsilon^*\Omega^1_{\mathbb{P}^2} \to \mathcal{F} \to \mathcal{O}_C \to 0 \tag{2.3}$$

Note that $2K_X + C \sim (d-6)H$ on X, hence $H^1(\mathcal{O}_X(-K_X - C)) = H^1(\mathcal{O}_X((d-6)H))^* = 0$. Tensoring (2.3) by $\mathcal{O}_X((d-6)H)$ we deduce $H^1(\mathcal{F}((d-6)H) = 0$ since $H^1(\Omega^1_{\mathbb{P}^2}(d-6)) = 0$ and $H^1(\mathcal{O}_C((d-6)H)) = H^0(\mathcal{O}_C(-K_X))^* = H^0(\mathcal{O}_X(-K_X))^* = 0$ by hypothesis. Now the natural map $\Omega^1_X(logC) \to \bigoplus_{i=1}^{\delta} \Omega^1_X(logC)_{|E_i} \to \bigoplus_{i=1}^{\delta} \Omega^1_{E_i}(2)$ gives

$$0 \to \mathcal{F} \to \Omega^1_X(logC) \to \bigoplus_{i=1}^{\delta} \mathcal{O}_{E_i} \to 0$$

and therefore $H^1(\Omega^1_X(logC)((d-6)H)) = 0$. Finally by the definition of ψ we get $Coker\psi \subseteq H^1(\mathcal{O}_C(2K_X + C)) = H^1(\mathcal{O}_C((d-6)H)) = 0$, hence we conclude applying Lemma (2.1). ∎

Remark (2.4). The hypothesis that the nodes do not lie on a cubic is of course necessary, otherwise the map ψ is not surjective in most cases, hence the same holds for Φ_{ω_C}, by diagram (1.4). This is for example the case of a smooth plane curve [CM] (see also [K] for the case of one or two singular points).

3. AN APPLICATION OF KAWAMATA-VIEHWEG'S VANISHING THEOREM.

Let S be a smooth irreducible surface and L a line bundle on it. We start by recalling the techniques of [CLM3] to ensure the surjectivity of the Gaussian map $\Phi_L : \bigwedge^2 H^0(S, L) \to$

$H^0(S, \Omega^1_S \otimes L^2)$. Let Y be the blow-up of $S \times S$ along its diagonal Δ, E the exceptional divisor and for every sheaf $\mathcal{G}$ on S let us denote by $\mathcal{G}_i, i = 1, 2$, its pull-back via the map $Y \to S \times S \xrightarrow{p_i} S$ where p_i is the i-th projection. As is well-known [W3], a sufficient condition for the surjectivity of Φ_L is the vanishing of $H^1(S \times S, p_1^* L \otimes p_2^* L \otimes \mathcal{I}^2_\Delta) \cong H^1(Y, L_1 + L_2 - 2E)$. As $L_1 + L_2 - 2E \sim K_Y + (L - K_S)_1 + (L - K_S)_2 - 3E$ the idea is to apply Kawamata-Viehweg's vanishing theorem to the line bundle $(L - K_S)_1 + (L - K_S)_2 - 3E$, provided this is big and nef. However in our case on the blow-up X of $\mathbb{P}^2$ we have $L = K_X + C$ and it is easily seen that $C_1 + C_2 - 3E$ has a negative intersection with curves contained on the strict transform of $E_i \times E_i$, so Kawamata-Viehweg's theorem does not apply as it is and we will need some extra work to prove our result on the surjectivity of $\Phi_{K_X + C}$. The proof will be divided in two parts, reducing a cohomological statement to one of a more geometrical nature.

Lemma (3.1). *Let Y be the blow-up of $X \times X$ along its diagonal Δ, E the exceptional divisor and $Y_i \subset Y$ the transform of $E_i \times X$, for $i = 1, \ldots, \delta$. Let $M = dH - 3 \sum_{j=1}^{\delta} E_j$ and suppose that*

(3.2) $M_1 + M_2 - 3E$ is big and nef on Y;

(3.3) $[(C - \sum_{j=1}^{i} E_j)_1 + (C - \sum_{j=1}^{i} E_j)_2 - 3E]_{|Y_i}$ is big and nef on Y_i for $i = 1, \ldots, \delta$.

Then the Gaussian map $\Phi_{K_X + C}$ is surjective.

Proof: Consider the divisors $Z_i \subset Y$ transform of $X \times E_i, F_i \sim (E_i)_1 + (E_i)_2 \sim Y_i + Z_i$ and $F = \sum_{i=1}^{\delta} F_i$. Set $\mathcal{L} = K_Y + C_1 + C_2 - 3E$, so that $\mathcal{L} - F = K_Y + M_1 + M_2 - 3E$ and $H^1(Y, \mathcal{L} - F) = 0$ by Kawamata-Viehweg's vanishing theorem and (3.2). Set $F_0 = 0$. We claim that

$$H^1(Y, \mathcal{L} - \sum_{j=0}^{i-1} F_j) = 0 \quad \text{for} \quad i = 1, \ldots, \delta + 1. \tag{3.4}$$

Of course (3.4) implies the lemma since the case $i = 1$ gives the required surjectivity of $\Phi_{K_X + C}$ (by [W3]). To see (3.4) we proceed by induction on $\delta + 1 - i \geq 0$, the first case having already being done. Suppose then $\delta - i \geq 0$ and consider the exact sequence

$$0 \to \mathcal{O}_Y(\mathcal{L} - \sum_{j=0}^{i} F_j) \to \mathcal{O}_Y(\mathcal{L} - \sum_{j=0}^{i-1} F_j) \to \mathcal{O}_{F_i}(\mathcal{L} - \sum_{j=0}^{i-1} F_j) \to 0$$

By induction (3.4) follows if we show

$$H^1(F_i, \mathcal{O}_{F_i}(\mathcal{L} - \sum_{j=0}^{i-1} F_j)) = 0 \quad \text{for} \quad i = 1, \ldots, \delta. \tag{3.5}$$

Let us denote by U_{ij} the strict transform on Y of $E_i \times E_j$. Notice that $U_{ij} \cong E_i \times E_j$. By the notation introduced above, we have $F_i = Y_i \cup Z_i$ and if we set $W_i = Y_i \cap Z_i \sim U_{ii} + E_{|Y_i}$ we have

$$0 \to \mathcal{O}_{Y_i}(\mathcal{L} - W_i - \sum_{j=0}^{i-1} F_j) \to \mathcal{O}_{F_i}(\mathcal{L} - \sum_{j=0}^{i-1} F_j) \to \mathcal{O}_{Z_i}(\mathcal{L} - \sum_{j=0}^{i-1} F_j) \to 0. \tag{3.6}$$

As $W_i \cap F_j = \emptyset$ for $j \leq i-1$, on Y_i we also have the exact sequence

$$0 \to \mathcal{O}_{Y_i}(\mathcal{L} - W_i - \sum_{j=0}^{i-1} F_j) \to \mathcal{O}_{Y_i}(\mathcal{L} - \sum_{j=0}^{i-1} F_j) \to \mathcal{O}_{W_i}(\mathcal{L}) \to 0 \tag{3.7}$$

therefore by (3.7), (3.6), the definition of Z_i and symmetry, we will be done if we prove

$$H^1(\mathcal{O}_{W_i}(\mathcal{L})) = 0 \quad \text{for} \quad i = 1, \ldots, \delta \tag{3.8}$$

and

$$H^1(\mathcal{O}_{Y_i}(\mathcal{L} - W_i - \sum_{j=0}^{i-1} F_j)) = 0 \quad \text{for} \quad i = 1, \ldots, \delta. \tag{3.9}$$

Set $E_i' = E_{|Y_i}$, so that $E_i' \cong I\!P\mathcal{E}$ with $\mathcal{E} \cong N^*_{\Delta_{E_i}/E_i \times X}$. From the normal bundle sequence

$$0 \to N_{\Delta_{E_i}/E_i \times E_i} \to N_{\Delta_{E_i}/E_i \times X} \to N_{E_i \times E_i/E_i \times X|\Delta_{E_i}} \to 0$$

and the isomorphisms $N_{\Delta_{E_i}/E_i \times E_i} \cong \mathcal{O}_{I\!P^1}(2), N_{E_i \times E_i/E_i \times X|\Delta_{E_i}} \cong \mathcal{O}_{I\!P^1}(-1)$ we easily see that $\mathcal{E} \cong \mathcal{O}_{I\!P^1}(1) \oplus \mathcal{O}_{I\!P^1}(-2)$ and $\mathcal{L}_{|E_i'} \cong \mathcal{O}_{I\!P\mathcal{E}}(2C_0 + 2f)$ where C_0 is a divisor in $|\mathcal{O}_{I\!P\mathcal{E}}(1)|$ and f is a fiber of $I\!P\mathcal{E} \to I\!P^1$. Also the intersection B_i between U_{ii} and E_i' is isomorphic to Δ_{E_i}, whence it is a divisor of type $C_0 + bf$ on $I\!P\mathcal{E}$, for some b. Therefore $h^1(\mathcal{O}_{E_i'}(\mathcal{L})) = 1, h^2(\mathcal{O}_{E_i'}(\mathcal{L} - B_i)) = 0$. From the exact sequence

$$0 \to \mathcal{O}_{E_i'}(\mathcal{L} - B_i) \to \mathcal{O}_{W_i}(\mathcal{L}) \to \mathcal{O}_{U_{ii}}(\mathcal{L}) \to 0$$

we deduce $H^2(\mathcal{O}_{W_i}(\mathcal{L})) = 0$ since $\mathcal{O}_{U_{ii}}(\mathcal{L}) \cong \mathcal{O}_{\mathbb{P}^1\times\mathbb{P}^1}(-1,-1)$. Now applying this to

$$0 \to \mathcal{O}_{U_{ii}}(\mathcal{L} - B_i) \to \mathcal{O}_{W_i}(\mathcal{L}) \to \mathcal{O}_{E_i'}(\mathcal{L}) \to 0$$

we deduce (3.8) because $\mathcal{O}_{U_{ii}}(\mathcal{L} - B_i) \cong \mathcal{O}_{\mathbb{P}^1\times\mathbb{P}^1}(-2,-2)$ hence $h^1(\mathcal{O}_{U_{ii}}(\mathcal{L} - B_i)) = 0, h^2(\mathcal{O}_{U_{ii}}(\mathcal{L} - B_i)) = 1$.

To prove (3.9) observe that, as a divisor on Y_i, we have $W_i \sim [(E_i)_2]_{|Y_i}$, hence $\mathcal{O}_{Y_i}(\mathcal{L} - W_i - \sum_{j=0}^{i-1} F_j) \cong K_{Y_i} + [(C - \sum_{j=0}^{i} E_j)_1 + (C - \sum_{j=0}^{i} E_j)_2 - 3E]_{|Y_i}$ and therefore (3.9) and the lemma follow by (3.3) and Kawamata-Viehweg's vanishing theorem. ∎

To take care of the geometrical statements (3.2) and (3.3) we will use an idea of L. Ein, contained in [CLM3]. We denote by X_i the blow-up of $\mathbb{P}^2$ at $P_1, \dots, P_i$, for $i = 1, \dots, \delta, X = X_\delta$ and we use the same notation H for the pull-back of a line and E_j for the exceptional divisors on X_i.

Lemma (3.10). *Suppose that $\delta \geq 1$ and that the line bundle $[\frac{d}{3}]H - \sum_{j=1}^{i} E_j$ is very ample on X_i, for $i = 1, \dots, \delta$. Then (3.2) and (3.3) hold.*

Proof: As $M \sim dH - 3\sum_{j=1}^{\delta} E_j$, by hypothesis there are three very ample line bundles A_1, A_2, A_3 of type $a_k H - \sum_{j=1}^{\delta} E_j, a_k \geq [\frac{d}{3}]$ such that $M \sim A_1 + A_2 + A_3$ and $h^0(A_k) \geq 4$ for $k = 1, 2, 3$. We recall from [CLM3] that if A_k is very ample then the linear system $|A_{k1} + A_{k2} - E|$ on Y has a sublinear system defining the morphism $Y \to \mathbb{G}(1, \mathbb{P}H^0(A_k)^*)$ associating to $(x, y) \in Y$ the linear span of $\phi_{A_k}(x)$ and $\phi_{A_k}(y)$ (note that this still makes sense if $(x, y) \in E$ since we can think of (x, y) as a pair with $x \in X$, $y \in \mathbb{P}T_{X_{|x}}$). Therefore $A_{k1} + A_{k2} - E$ is nef and also big since the image of X in $\mathbb{P}H^0(A_k)^*$ is non degenerate. Hence we get (3.2) as $M_1 + M_2 - 3E \sim \sum_{k=1}^{3}(A_{k1} + A_{k2} - E)$. Notice now that we can write $C - \sum_{j=1}^{i} E_j \sim B_1 + B_2 + B_3$ with $B_1 = B_2 = [\frac{d}{3}]H - \sum_{j=1}^{\delta} E_j$, and $B_3 = (d - 2[\frac{d}{3}])H - \sum_{j=1}^{i} E_j$. By hypothesis B_1 is very ample on X hence the restriction of $|B_{11} + B_{12} - E|$ on Y_i is certainly nef. But it is also big since B_1 embeds X so that E_i is a line, hence if $|B_{11} + B_{12} - E|$ were not big on Y_i, then the restriction to Y_i of the map $Y \to \mathbb{G}(1, \mathbb{P}H^0(B_1)^*)$ would have no finite fiber, hence every chord joining a point of the line $\phi_{B_1}(E_i)$ and a point of the image $\phi_{B_1}(X)$ would be contained in $\phi_{B_1}(X)$, that is $\phi_{B_1}(X)$ is a plane, a contradiction. Finally we are left to see that $B_{31} + B_{32} - E$ is nef on Y_i. Of course it can fail to be nef only on a curve contained in the indeterminacy locus of the map $Y_i \to \mathbb{G}(1, I\!PH^0(B_3)^*)$, that is a curve $Z \subset \{(x,y) \in Y_i : \phi_{B_3}(x) = \phi_{B_3}(y)\}$. By hypothesis B_3, as a divisor on X, defines an isomorphism off $E_j, j > i$, hence certainly $i < \delta$ and $Z \subset \bigcup_{j>i} U_{jj}$, but this is a contradiction since $\bigcup_{j>i} U_{jj} \cap Y_i = \emptyset$. ∎

Remark (3.11). It would be nice to have a more geometrical interpretation of the two lemmas just proved, in terms of the ideal of the nodes. By pushing down to X it can be seen that the vanishing of $H^1(Y, L_1 + L_2 - 2E)$ that we used to get the surjectivity of Φ_{K_X+C}, is in fact related to the cohomology of the normal bundle of the image of X via the linear system $K_X + C$. Therefore a good knowledge of the resolution of the ideal of this image would probably give a better result.

Proof of Theorem (1.1). If we assume (1.3) the map $\psi \circ \phi$ in diagram (1.4) is surjective by Proposition (2.2). The same is true under hypothesis (1.2) as in that case the nodes are generic [AC]. Whence the surjectivity of the Wahl map of C follows by diagram (1.4) (since p is surjective) as soon as we prove that the Gaussian map Φ_{K_X+C} is surjective. By Lemmas (3.1) and (3.10) we just need the very ampleness of $[\frac{d}{3}]H - \sum_{j=1}^{i} E_j$ on X_i. The latter follow by [AH] in case (1.2) and by [DG] in case (1.3). ∎

Remark (3.13). It is not difficult to see that, by Theorem (1.1) one gets an explicit example of a smooth irreducible curve with surjective Wahl map for all integers g such that $g \geq 149$ or $g \in [76, 81] \cup [90, 95] \cup [105, 110] \cup [114, 126] \cup [131, 143]$, thus reproving the main result of [CHM] for these genera.

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