Titles and Abstracts Poster Session, Algebraic Geometry in Roma Tre

On the occasion of Sandro Verra's 70th birthday

June 14, 2022, Aula Adalberto Libera, start at 17:00

Paweł Borówka (*Jagiellonian University in Krakow*)

Title. Non-simple abelian surfaces.

Abstract. In a joint work with Robert Auffarth (arXiv 2111.11799), we have found a characterisation for possible complementary types (exponents) of elliptic curves on a $(1, d)$-polarised abelian surfaces. Similarly to Humbert surfaces, we have shown that the locus of $(1, d)$-polarised surfaces that admit a pair of complementary elliptic curves of prescribed exponents is an irreducible surface in the moduli. The result is supplied with a few examples illustrating why the result is non-trivial.

Andrei Bud (*Humboldt University Berlin*)

Title. The Prym-Brill-Noether divisor

Abstract. Understanding the birational geometry of the moduli space $\mathcal{R}_g$ parametrizing Prym curves has been the subject of several papers, with great insight into this problem coming from the work of Farkas and Verra. Of particular importance for this study is finding divisors of small slope on the space $\overline{\mathcal{R}}_g$. Drawing parallels with the situation on $\overline{\mathcal{M}}_g$, we consider the Prym-Brill-Noether divisor and compute (some relevant coefficients of) its class. We will highlight the role of strongly Brill-Noether loci in understanding Prym-Brill-Noether loci. A consequence of our study is that the space $\mathcal{R}_{14,2}$ parametrizing 2-branched Prym curves of genus 14 is of general type.

Stefano Canino (*Politecnico di Torino*)

Title. Analytical classification of triple points Hilbert Functions of subvarieties and c.i. subvarieties of Veronese surfaces

Abstract. Which are the complete intersections lying on Veronese surfaces $V_{2,d}$? In order to answer this question, we first characterize the Hilbert functions of subvarieties on Veronese surfaces and then we use this result to find all and only complete intersections on Veronese surfaces. In particular, we show that for $d > 2$ the only reduced complete intersections of $\mathbb{P}^N$ lying on $V_{2,d}$ are finite sets of either one or two points while, for the Veronese surface $V_{2,2} \subseteq \mathbb{P}^5$, one also has plane conics and their intersections with suitable hypersurfaces. Inspired by these evidences we formulate the following conjecture: the only reduced complete intersections of $V_{n,d}, d \geq 3$ are finite sets of either one or two points while for $d = 2$ one also has plane conics and their intersections with suitable hypersurfaces. We also checked the validity of the conjecture for $V_{3,2}$.

Alessio Cela (*ETH Zurich*)

Title. Tevelev degrees

Abstract. Tevelev degrees count genus g maps of fixed complex structure in a given curve class β through n general points of a fixed smooth projective variety X. They were first introduced by A.Cela, R. Pandharipande and J.Schmitt for the case $X = \mathbb{P}^1$ via Hurwitz theory. As an application they gave a new proof of Castelnuovo's classical count of linear series on a general genus g curve. Recently, A. Buch and R. Pandharipande proposed a virtual perspective via Gromov-Witten theory and soon after C.Lian and R. Pandharipande showed that virtual and geometric Tevelev degrees agree for certain Fano varieties and large degree curve classes. The advantage is that, almost always, virtual Tevelev degrees are much better behaved than enumerative Tevelev degrees and general Gromov-Witten invariants. Complete counts are known for flag varieties and complete intersections, while the cases of Del Pezzo surfaces and Toric Varieties are work in progress.

Christian Gleissner and Federico Fallucca (*University of Bayreuth and University of Trento*)

Title. Surfaces with canonical map of degree 10, 11 and 14

Abstract. It is well known that the canonical map of a curve C of genus at least two is either an embedding or of degree two. The latter happens if and only if C is hyperelliptic.

For a smooth surface S of general type the situation is more difficult: suppose that the image of the canonical map ϕ_{K_S} is a surface, then *Beauville* observed in [1] that:

$$d := \deg(\Phi_{K_S}) \leq 9 + \frac{27 - 9q}{p_g - 2} \leq 36.$$

where $q = h^{1,0}(S)$ is the irregularity and $p_g = h^{2,0}(S)$ is the geometric genus of S. As noted first by Persson, the maximum possible degree is 36 and if $d > 27$ then $q = 0$ and $p_g = 3$.

A question posed by M. Lopes and R. Pardini in [2] is if for each $d \leq 36$ there exists an algebraic surface S such that the degree of its canonical map is equal to d. At the moment there are only examples in literature of surfaces with canonical map of degree

$$d = 2, \ldots, 9, 12, 16, 20, 24, 27, 32, 36.$$

Our aim is to present surfaces with canonical maps of degree $d = 10$, 11 and 14. According to our knowledge they are the first examples for these values of d. They can be described using Pardini's theory of branched abelian covers [3], which is one of the standard techniques in this subject, cf. [2]. However, we decided to present them in an elementary way using plane curves and basic algebraic geometry. Our construction is completely self-contained, basically reference free and easily accessible: we construct our surfaces S as quotients of a product of two Fermat septics

$$F = \{x_0^7 + x_1^7 + x_2^7 = 0\} \subset \mathbb{P}^2$$

modulo certain free and diagonal actions of the group $\mathbb{Z}_7^2$. The surfaces are regular with geometric genus $p_g(S) = 3$. We write the canonical system of each S explicitly in terms of $\mathbb{Z}_7^2$-invariant holomorphic two-forms on the product $F \times F$. It turns out that $|K_S|$ is not base-point free, i.e. the canonical map $\Phi_{K_S} \colon S \dashrightarrow \mathbb{P}^2$ is just a rational map. To compute its degree $d = \deg(\Phi_{K_S})$, we resolve the indeterminacy by a sequence of blowups and compute the degree of the resulting holomorphic map via elementary intersection theory.

References

[1] A. Beauville, L'application canonique pour les surfaces de type général, Inv. Math., 1979, 121–140

[2] M.M. Lopes, R. Pardini, On the degree of the canonical map of a surface of general type, https://arxiv.org/abs/2103.01912, 2021

[3] R. Pardini. Abelian covers of algebraic varieties. J. Reine Angew. Math., **417**, (1991), 191–213.

Elsa Maneval (*ETH Zurich*)

Title. Donaldson-Thomas theory using finite and local fields

Abstract. This poster is about computations of Donaldson-Thomas invariants using finite fields and non-archimedean local fields (e.g. p-adic fields $\mathbb{Q}_p$). DT invariants are defined on moduli spaces of sheaves over a smooth projective variety of low dimension (up to 4). These enumerative invariants are a sensible ways of countings curves in surfaces or 3-folds. Although the theory was developed at first using integrals over a virtual class, the development of a motivic Donaldson-Thomas theory leads recently to compare such invariants with some well-chosen p-adic integrals. There are two main situations depending on whether there are strictly semi-stable sheaves in the moduli or not. In cases where the moduli is smooth, it is also natural to use finite fields countings. These different contexts leads us to study different examples. Woonam Lim and Franscesca Carocci are guiding me for this Master's level project.

Miguel Moreira (*ETH Zurich*)

Title. Virasoro constraints for moduli of sheaves on curves

Abstract. Virasoro constraints for Gromov-Witten invariants have a rich history tied to the very beginning of the subject. Recently, Virasoro constraints for moduli spaces of stable pairs on 3-folds were found using the Gromov-Witten/Pandharipande-Thomas correspondence. This discovery led to a new study of such constraints for descendents of integrals in different moduli of sheaves. In joint work with A. Bojko and W. Lim, we propose a general conjecture and fit the Virasoro operators in the vertex algebra Joyce recently introduced to study wall-crossing. We then use this framework to show compatibility between the constraints and wall-crossing. As an application, we prove that Virasoro holds for moduli of stable sheaves on curves.

Federico Moretti (*Humboldt University Berlin*)

Title. Brill-Noether theory and Green's conjecture for general curves on simple abelian surfaces

Abstract. Brill-Noether theory has been deeply investigated for curves lying on K3 surfaces. Lazarsfeld proved that a smooth curve in the primitive linear system of a K3 surface is Brill-Noether general, giving a proof of Brill-Noether-Petri conjecture. Green- Lazarsfeld proved that the Clifford index of a smooth curve on a K3 is constant varying the

curve in its linear system and Ciliberto-Pareschi characterized when also the gonality is constant. Voisin and later Aprodu-Farkas used the techninques developed to study Brill-Noether theory to prove Green's conjecture for such curves. We mirror some of the work done for K3 surfaces to continue the study of Brill-Noether theory for curves on abelian surfaces, recently started by (among others) Knutsen, Lelli-Chiesa and Mongardi. We focus on the dimension of $W^1_d|L|$ for non primitive linear system L, computing the gonality of the curves and discussing implications on Green's conjecture.

Nicola Picoco (*Università di Bari*)

Title. Cayley–Bacharach property and applications

Abstract.

A celebrated theorem by Cayley and Bacharach asserts that if $\Gamma = \{P_1, \dots, P_{de}\} \subset \mathbb{P}^2$ is the collection of intersection points between two plane curves of degree d and e respectively, then any curve of degree $d + e - 3$ passing through all but one points of Γ passes through the last point. More generally, we say that a finite set of points $\Gamma \subset \mathbb{P}^n$ satisfies the Cayley-Bacharach condition with respect to the complete linear system of hypersurfaces of degree k, or more briefly that Γ is $CB(k)$, if whenever a hypersurface X of degree k passes through all but one points of Γ, X passes through the last point as well. We study the properties of points satisfying the Cayley-Bacharach condition and their applications.

In particular, we improve a result by Lopez and Pirola and we show that, if $k \geq 1$ and $\Gamma = \{P_1, \dots, P_d\} \subset \mathbb{P}^n$ is a set of distinct points $CB(k)$, with $d \leq h(k - h + 3) - 1$ and $3 \leq h \leq 5$, then Γ lies on a curve of degree $h - 1$. Then we apply this result to the study of linear series on curves on smooth surfaces in $\mathbb{P}^3$.

Moreover, in collaboration with F. Bastianelli, we study a property of Cayley-Bacharach type on Grassmannians, and we apply our results to compute the covering gonality of the threefold and fourfold symmetric product of a smooth curve C, which is the least gonality of an irreducible curve passing through the general point of $C^{(k)}$, with $k = 3, 4$.

Debaditya Raychaudhury (*Fields Institute*)

Title. Positivity of zero-regular bundles, continuous regularity, and generic vanishing

Abstract. We study the positivity of 0-regular bundles on polarized smooth projective varieties for divisible polarizations. To do so, we use a notion of partial regularity, and we further combine it with the so-called covering trick to prove generic vanishing theorems for continuously 1-regular torsion-free coherent sheaves. This continuous variant of

Castelnuovo-Mumford regularity was introduced by Mustopa, and he raised the question of whether a continuously 1-regular such sheaf is GV. Here we answer the question in the affirmative for a class of polarizations on varieties whose Albanese maps are either surjective, or are finite onto their images.

Saverio Andrea Secci (*Università di Torino*)

Title. Fano 4-folds having a prime divisor of Picard number 1

Abstract. We discuss a classification result for smooth, complex Fano 4-folds of Picard number 3, having a prime divisor of Picard number 1. They form 28 families. We compute the main numerical invariants, determine the base locus of the anticanonical divisor, and study their deformations to give an upper bound of the dimension of the base of the Kuranishi family of a general member.

Stefano Serpente (*Università di Roma Tre*)

Title. Graph Complexes and Moduli of Hasset-stable curves

Abstract. We consider the alternative compactifications of the moduli stack of smooth curves introduced by Hassett. We generalize the graph complexes used to study the top weight cohomology of this moduli space in the classical Deligne-Mumford compactification to this case. We show also that for fixed g and n, there are particular filtrations of these graph complexes.

Anatoli Shatsila (*Jagiellonian University in Krakow*)

Title. Geometry of Elliptic Normal Curves of Degree 6

Abstract. In our work we focus on the geometry of elliptic normal curves of degree 6 embedded in $\mathbb{P}^5$.

We determine the space of quadric hypersurfaces through an elliptic normal curve of degree 6 and find the explicit equations of generators of $\mathrm{I}(\mathrm{Sec}(C_6))$.

We study the images C_p and C_{pq} of a sextic C_6 under the projection from a general point $P \in \mathbb{P}^5$ and a general line $\overline{PQ} \subset \mathbb{P}^5$. In particular, we show that C_p is k-normal for all $k \geq 2$ and $I(C_p)$ is generated by three homogeneous polynomials of degree 2 and two

homogeneous polynomials of degree 3. We then show that C_{pq} is k-normal for all $k \geq 3$ and $I(C_{pq})$ is generated by two homogeneous polynomials of degree 3 and three homogeneous polynomials of degree 4.

Mohammad Zaman Fashami (*Pedagogical University of Cracow*)

Title. Asymptotic invariants of schemes come from symbolic powers and unexpected hypersurfaces

Abstract. The Waldschmidt constant $\widehat{\alpha}(I)$ of a radical ideal I in the coordinate ring of $\mathbb{P}^N$ measures the degree of a hypersurface passing through the set defined by I in $\mathbb{P}^N$. Let I be homogeneous ideal in the ring $R = K[x_0, \ldots, x_N]$. For a positive integer m, let $I^{(m)}$ be the m^{th} symbolic power of I defined as $I^{(m)} = R \cap \bigcap_{P \in \mathrm{Ass}(I)} I^m R_P$, where the intersection is taken in the ring of fractions of R and $\mathrm{Ass}(I)$ is the set of associated primes of I. Then $\widehat{\alpha}(I) = \lim_{m \to \infty} \frac{\alpha(I^{(m)})}{m}$.

In this paper we quote and review some know fact about Waldschmidt constant, then in first part we deal with the Waldschmidt constant of general lines in $\mathbb{P}^N$. In the second part we compute Waldschmidt constant for some special configuration of points and conection of them to unexpected hypersurfaces.

Angelina Zheng (*University of Pavia*)

Title. Stable cohomology of the moduli space of trigonal curves

Abstract. The rational cohomology of the moduli space T_g of trigonal curves of genus g has been computed by Looijenga for $g = 3$, by Tommasi for $g = 4$ and by myself for $g = 5$. In this poster I would like to present the rational cohomology of T_g for higher genera. Specifically, we prove that the rational cohomology $H^i(T_g)$ is independent of g for $g > 4i + 3$ and that it coincides with the tautological ring in this range. This result is achieved by studying the embedding of trigonal curves in Hirzebruch surfaces and using Gorinov-Vassiliev's method.