$f : E \to \mathbb{R}$

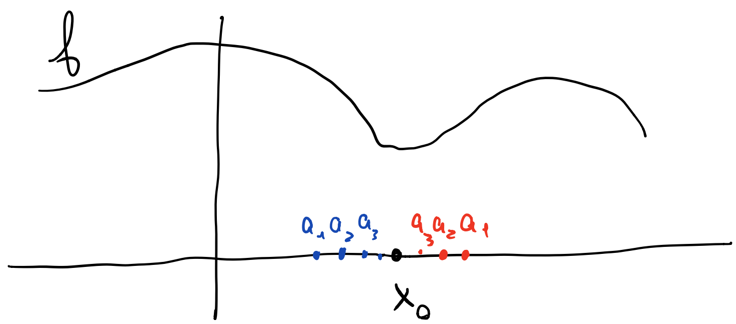
x_0 di accumulazione per E

$x_0 \in \mathbb{R}$

$$\lim_{x \to x_0^+} f(x) = \ell$$

se $\forall$ successione $a_n \in E$: $a_n \to x_0$

e $a_n > x_0$ vale $\lim_{n \to \infty} f(a_n) = \ell$



$\lim_{x \to x_0^-} f(x) = \ell$ se $\forall \{a_n\}$ $a_n \in E$

$a_n \to x_0$ e $a_n < x_0$ vale $\lim_{n \to \infty} f(a_n) = \ell$

Naturalmente se $\lim_{x \to x_0} f(x) = \ell$

allora $\Rightarrow \lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = \ell$

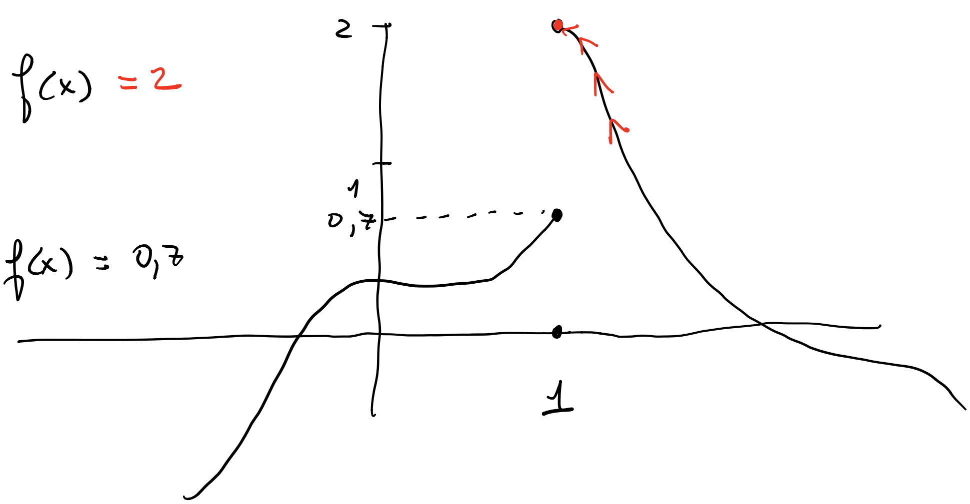
Viceversa

Se $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = \ell$

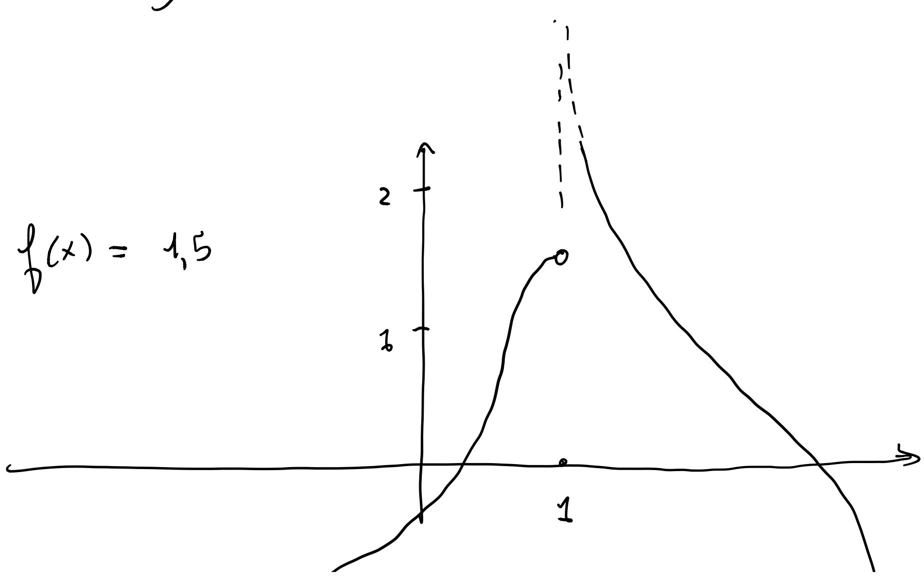
allora esiste $\lim_{x \to x_0} f(x)$ e vale ℓ

$\lim_{x \to 1^+} f(x) = 2$

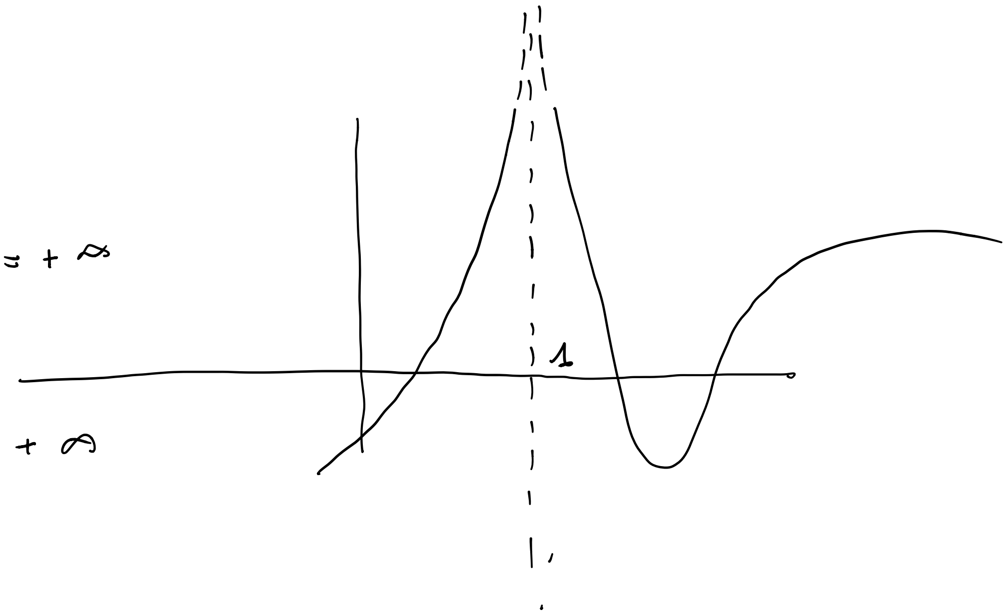
$\lim_{x \to 1^-} f(x) = 0{,}7$



$\lim_{x \to 1^-} f(x) = 1{,}5$



$$\lim_{x \to 1^-} f = +\infty$$



$$\lim_{x \to 1^+} f = +\infty$$

$$x \geq x^2$$

$$\lim_{x \to 0^+} \frac{x^2+1}{x^2-1} = \lim_{x \to 0} \frac{x^2+1}{x^2-1} = -1$$

$$\lim_{x \to 0^+} \frac{1}{x - x^2} = \frac{1}{x(1-x)} = \lim_{x \to 0^+} \frac{1}{x} \cdot \lim_{x \to 0^+} \frac{1}{1-x}$$

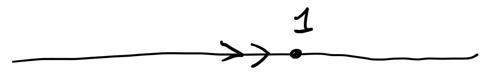
$$= \frac{1}{0^+} \cdot 1 = +\infty \cdot 1 = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x - x^2} = \frac{1}{0^-} = -\infty$$

$$x - x^2 > 0$$

$$(1-x) > 0$$

$$\lim_{x \to 1^-} \frac{1}{x - x^2} =$$



$$\lim_{x \to 1^-} \frac{1}{x(1-x)} = \lim_{x \to 1^-} \frac{1}{x} \cdot \lim_{x \to 1^-} \frac{1}{(1-x)}$$

$$= 1 \cdot \frac{1}{0^+} = +\infty$$

$$(x-1)(x+3)$$

$$x^2 + 2x - 3$$

$$\lim_{x \to 1^-} \frac{1}{x^2 - 2x + 1} = \lim_{x \to 1^-} \frac{1}{(x-1)^2} = \frac{1}{0^+} = +\infty$$

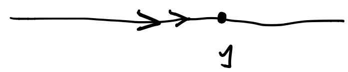
$$\lim_{x \to 1^+} \frac{1}{x^2 - 2x + 1} = \lim_{x \to 1^+} \frac{1}{(x-1)^2} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \to 1^-} \frac{10x^3 - 2x^2}{x^2 + 2x - 3} = \lim_{x \to 1^-} \frac{10x^3 - 2x^2}{(x+3)(x-1)} = \frac{8}{4 \cdot 0^-}$$

$$\shortparallel$$

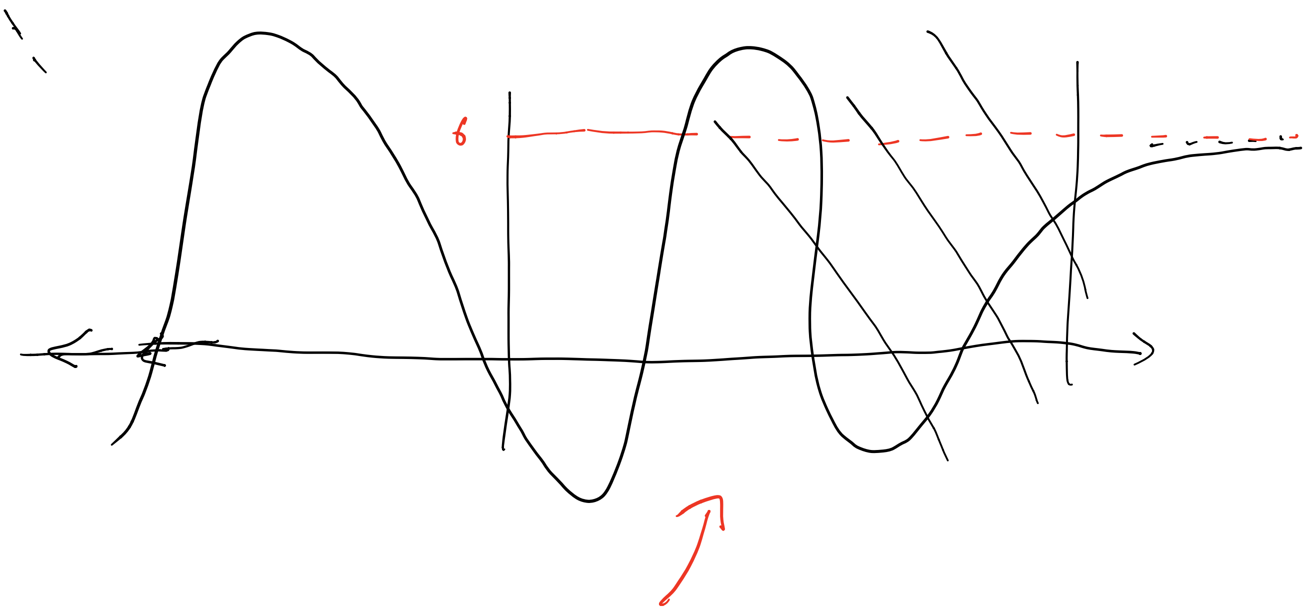
$$-\infty$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+3}}{1} = \frac{-1 \pm 2}{1} \begin{cases} 1 \\ -3 \end{cases}$$

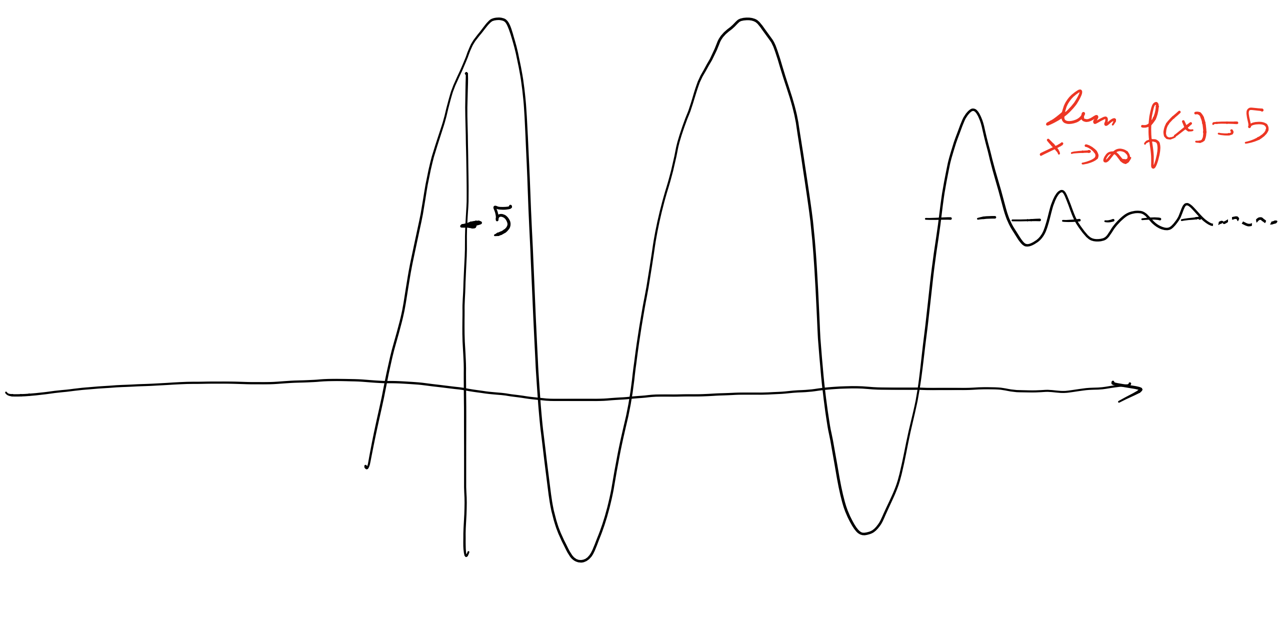


$$= \lim_{x \to 1^-} \frac{10x^3 - 2x^2}{x+3} \cdot \lim_{x \to 1^-} \frac{1}{x-1}$$

$$\frac{8}{4} \cdot \frac{1}{0^-}$$



$$\lim_{x \to \infty} f(x) = 6^-$$



$$f(x) \sim g(x) \qquad \text{per} \quad x \to x_0$$

se $$\lim_{x \to x_0} \frac{f(x)}{g(x)} = 1$$

$$\sin x \sim x \qquad \text{per} \quad \underline{\underline{x \to 0}}$$

$$\ln(1+x) \sim x \qquad \text{per} \quad x \to 0$$

$$tg(x) \sim x \qquad \text{per} \quad x \to 0$$

$$x^4 - 3x^2 + 2 \sim x^4 \qquad \text{per} \quad x \to \infty$$

$$x^4 - 3x^2 + \boxed{2} \sim 2 \qquad \text{per} \quad x \to 0$$

$$x^4 - 3x^2 \sim -3x^2 \qquad \text{per} \quad x \to 0$$

$$\lim_{x \to 0} \frac{x^4 - 3x^2}{-3x^2} = \lim_{x \to 0} -\frac{x^2}{3} + 1 = 1 \quad \checkmark$$

$$\lim_{x \to 0} \frac{\ln(1 + \sin x)}{x} = \lim_{x \to 0} \frac{\ln(1+x)}{x} =$$

$\sin x \sim x$

$\ln(1+x) \sim x$

$$\lim_{x \to 0} \frac{x}{x} = \underline{1}$$

[$\ln(1+y) \sim y$ se $y \to 0$

$y = \sin x$ $\quad \sin x \to 0$ se $x \to 0$

$\ln(1 + g(x)) \sim g(x)$ $\quad g(x) \to 0$ per $x \to 0$

$$\lim_{x \to x_0} f(g(x)) = \lim_{y \to y_0} f(y)$$

dove $y_0 = \lim_{x \to x_0} g(x)$

$\ln(1+\operatorname{sen} x) \sim \operatorname{sen} x$ per $x \to 0$

$\operatorname{sen} x \sim x$ per $x \to 0$

quindi $\ln(1+\operatorname{sen} x) \sim \operatorname{sen} x \sim x$

$\ln(\cos x)$ per $x \to 0$

$\ln(1+y) \sim y$ per $y \to 0$

$y = \cos x - 1$ $\qquad y \to 0$

$$\ln(\cos x) = \ln(1 + \overbrace{\cos x - 1}^{y}) \sim \cos x - 1$$

$\ln(\cos x) \sim \cos x - 1 \sim -\frac{x^2}{2}$ per $x \to 0$

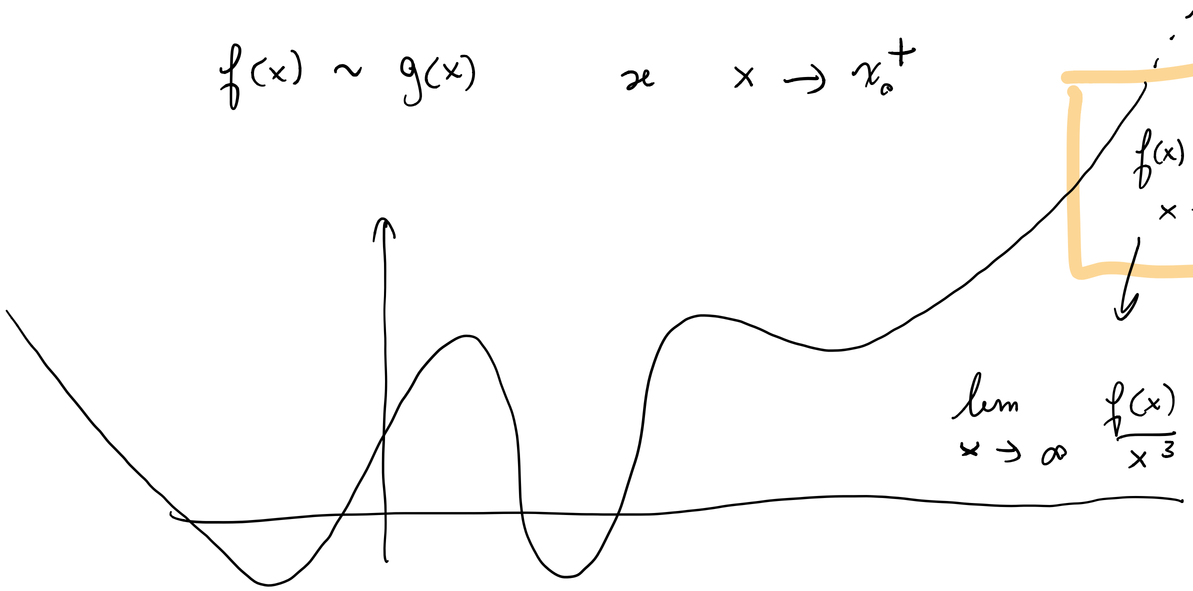
$$\boxed{\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}} \iff 1-\cos x \sim \frac{x^2}{2}$$

$$\lim_{x \to 0} \frac{1-\cos x}{\frac{x^2}{2}} = 2 \lim_{x \to 0} \frac{1-\cos x}{x^2} = 2 \cdot \frac{1}{2} = 1$$

$0 < f(x) \ll g(x)$ se $x \to x_0$

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$$

$f(x) \sim g(x)$ se $x \to x_0^+$

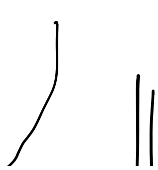


$\boxed{f(x) \sim x^3 \quad x \to \infty}$

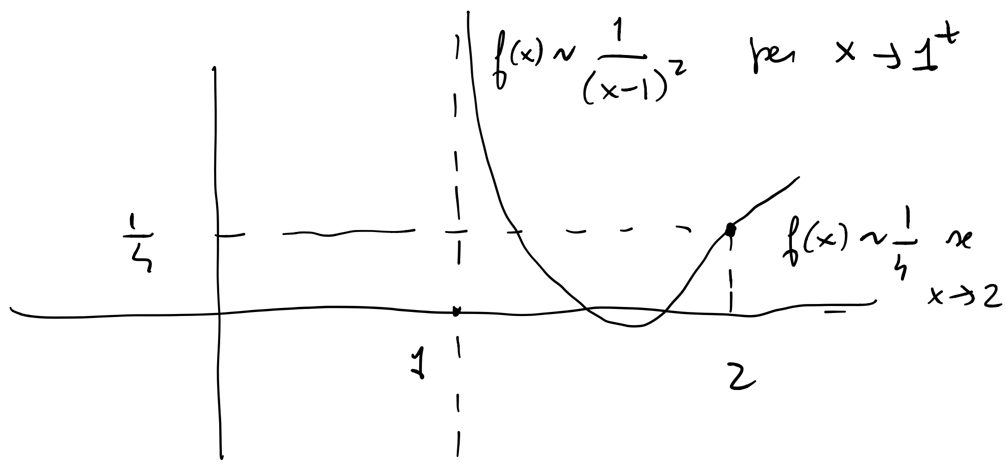
$$\lim_{x \to \infty} \frac{f(x)}{x^3} = 1$$

$$\lim_{x \to +\infty} \frac{f(x)}{x^2+1} = \lim_{x \to \infty} \frac{x^3}{x^2+1} = \lim_{x \to \infty} \frac{x^3}{x^2}$$

$$= \lim_{x \to \infty} x = +\infty$$



$$\lim_{x\to\infty} \frac{f(x)}{x^3}\cdot\frac{x^3}{x^2+1} = \boxed{\lim_{x\to\infty}\frac{f(x)}{x^3}}\cdot \lim_{x\to\infty}\frac{x^3}{x^2+1} = +\infty$$



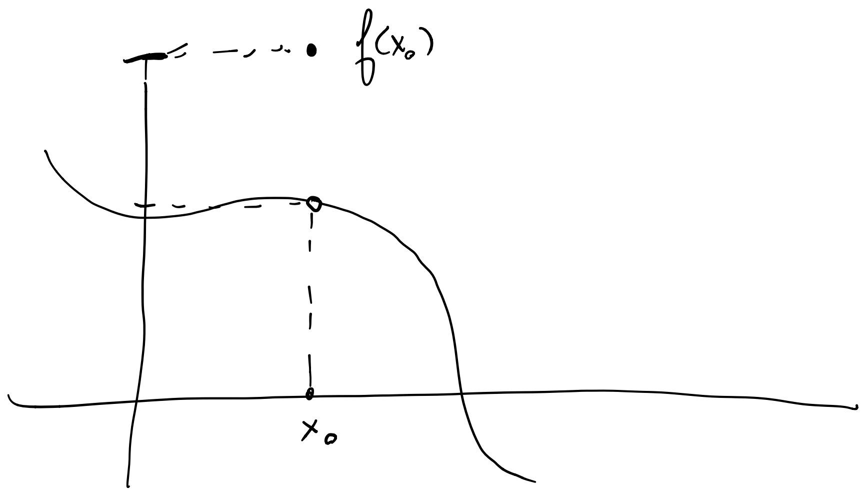
$$\lim_{x\to 1^+} (x-1)^3 f(x) = \lim_{x\to 1^+} (x-1)^3\cdot\frac{1}{(x-1)^2} =$$

$$\lim_{x\to 1^+} (x-1) = 0^+$$

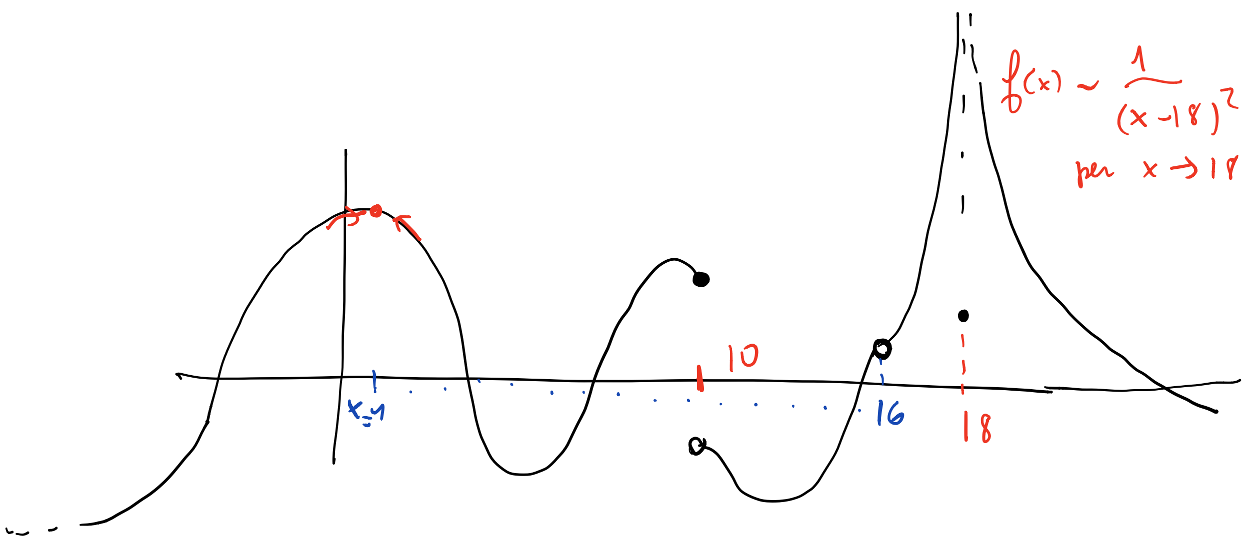
Definizione:

data $f: D \to \mathbb{R}$ e dato $x_0 \in D$, pto di acc.

f è continua in x_0 se $\lim_{x\to x_0} f(x) = f(x_0)$



f(x₀)
x₀



x=4
10
16
18
f(x) ~ 1/(x-18)²
per x → 18