INTEGRAZIONE PER PARTI

$$\int f'(x)\, g(x)\, dx = -\int f(x)\, g'(x)\, dx + f(x)\, g(x)$$

Derivata della funzione composta
(cIe cambio di variabile nell'integrale)

$$\left(F(g(x))\right)' = F'(g(x)) \cdot g'(x)$$

quindi $\displaystyle\int F'(g(x)) \cdot g'(x)\, dx = F(g(x)) + \text{const}$

per esempio

$$\int \frac{2x+3}{x^2+3x+1}\, dx = \int \frac{g'(x)}{g(x)}\, dx$$

$$F(y) = \ln(|y|) \qquad F'(y) = \frac{1}{y}$$

$$\int \frac{g'(x)}{g(x)}\, dx = \int F'(g(x))\, g'(x)\, dx = \ln(|g(x)|)$$

cioè $\quad \ln(|x^2+3x+1|)$.

Si trova in varie varianti

spesso troverete

$$\int f(g(x))\, g'(x)\, dx = F(g(x))$$

se $F'(y) = f(y)$ cioè $F = \int f(y)\, dy$

cioè $$\int f(g(x))\, g'(x)\, dx = \int f(y)\, dy \Big|_{y=g(x)}$$

Esempio: $\quad \int \operatorname{tg} x \, dx =$

$$\int \frac{\sin x}{\cos x}\, dx \qquad \text{pongo } y = \cos x$$

$$\| \qquad\qquad f(y) = \frac{1}{y}$$

$$-\int \frac{(-\cos x)'}{\cos x}\, dx = -\int \frac{dy}{y} = -\ln(|y|)\Big|_{y=\cos x}$$

Esempio: $\int \frac{x\,dx}{x^4+1}$ provo $y = x^2$

$$= \frac{1}{2} \int \frac{(x^2)'\,dx}{(x^2)^2+1} = \frac{1}{2} \int \frac{dy}{y^2+1} \bigg|_{y=x^2}$$

UN PÓ più difficile

$$\int f(g(x))\,dx$$

$$\int \frac{f(g(x))}{g'(x)}\, g'(x)\,dx = F(g(x))$$

dove $F'(g(x)) = \frac{f(g(x))}{g'(x)}$

$$F(y) = \int \frac{f(y)}{g'(g^{(-1)}(y))}\,dy = \int f(y) \cdot (g^{(-1)}(y))'\,dy$$

cioé $\int f(g(x))\,dx = \int f(y)\,(g^{(-1)}(y))'\,dy \Big|_{y=g(x)}$

sempre sostituisco $g(x) = y$ e

$$dy = g'(x)\,dx \Rightarrow dx = \frac{dy}{g'(x)} = \left(g^{(-1)}(y)\right)' dy$$

Esempio

$$\int \frac{dx}{\sin x} = \int \frac{\left(1 + tg^2 \frac{x}{2}\right)}{2\, tg \frac{x}{2}}\, dx$$

provo a porre $y = tg\frac{x}{2}$

seguendo lo schema $dy = \left(tg\frac{x}{2}\right)' dx$

$$= \frac{1}{2}\left(1 + \left(tg\frac{x}{2}\right)^2\right) dx$$

$$dx = \frac{2\,dy}{1+y^2}$$

$$\int \frac{dx}{\sin x} = \int \frac{(1+y^2)}{2y} \cdot \frac{2\,dy}{1+y^2}\Bigg|_{y = tg\frac{x}{2}} =$$

Esempi $\int e^{3x+2}\,dx = \frac{1}{3}\, e^{(3x+2)}\,(3x+2)'\,dx$

$F(y) = e^{y}$; $g(x) = 3x+2$ $\qquad g^{(-1)}(y) = \frac{y-2}{3}$

$$\int e^{3x+2}\,dx = \int \frac{e^{y}}{3}\,dy = \frac{e^{y}}{3}$$

$$\int \sin(3x^2+2)\; x\,dx$$

$g(x) = 3x^2+2 \qquad dx = \frac{dy}{6x}$

sostituendo si elimina la x !

$$\int \sin(y)\; x \cdot \frac{dy}{6x} = \frac{1}{6}\int \sin(y)\,dy \Big|_{y=3x^2+2}$$

$$\int \sin(3x^2+2)\; x\,dx = -\frac{1}{6}\cos(3x^2+2)$$

$\int G(x)\,dx$ posso cambiare variabili

$\parallel$ $\qquad\qquad x = h(y)$

$$\int G(h(y))\, h'(y)\, dy \Big|_{y = h^{(-1)}(x)}$$

$$\int \frac{x^3}{x^4+6x^2+x^6}\,dx$$

provo $x = \sqrt{y}$

$(y = x^2)$

$\parallel$

$$dx = \frac{dy}{2\sqrt{y}}$$

$$\int \frac{y^{3/2}}{y^2+6y+y^3}\,\frac{dy}{2\sqrt{y}}\Big|_{y=x^2}$$

$$\int \frac{y\,dy}{y^2+6y+y^3} = \int \frac{dy}{y^2+y+6}$$