considero $\mathcal{A}_r$ (campi vettoriali analitici su B_r)

$\mathcal{A}_r = \overline{\bigoplus P^{(d)}}$ chiusura rispetto alla

norma $|V|_r := \sum_{d=0}^{\infty} |V^{(d)}|_r$

con $|V^{(d)}|_r := r^d \sup_J \sum_{a : |a| = d+1} |V_{J,a}|$

$V \in \mathcal{A}_r \iff V = \sum_{d=0}^{\infty} V^{(d)}$ con $|V|_r < \infty$

(serie tot. convergenti)

(OSS. funzionerebbe anche

$\|V\|_r := \sum_J \sum_a |V_{J,a}| \, r^{|a|_1 - 1}$)

N.B. 1 se $V \in \mathcal{A}_r^{\geq d}$ (cioè $V^{(k)} = 0 \ \forall k < d$)

allora $\forall \, r' < r$ $|V|_{r_1} \leq \left(\frac{r'}{r}\right)^d |V|_r$

(in particolare $|V|_r \nearrow r$) (forse 4)

INOLTRE:

2 $|[V, W]|_{r'} \leq \frac{2r}{r - r'} |V|_r |W|_r$

3 $\sup_{x \in B_r(0)} |V_J(x)| \leq r|V|_r \quad \forall J$

$B_r(0)$ palla in ℓ_∞

Potremmo definire anche le mappe analitiche $\phi : B_r(0) \longrightarrow \mathbb{R}^n$

$\phi = (\phi_1, \dots \phi_n)$

$\phi_i = \sum \phi_{a,i} \, x^a$

con la norma $\sup_{x \in B_r} |\phi|_\infty$

$\boxed{\text{Proposizione}}$ se $0 < \rho < r$ e S un c.v. analitico tale che $\left(S = \sum_{d=0} S^{(d)} \right)$

$$|S|_{r+\rho} \leq \delta := \frac{\rho}{8e(r+\rho)}$$

Allora il flusso del c.v. S

$$\begin{cases} x_\tau = S(x) \\ x(0) = y \end{cases} \qquad y \longrightarrow \phi_S(\tau, y)$$

è ben definito fino a $\tau = 1$ inoltre

$\psi(y) := \phi_S(1, y)$ è ben definito

ed analitico con

$$\sup_{y \in B_r(0)} |\psi(y) - y|_\infty \leq (r+\rho)|S|_{r+\rho}$$

inoltre $\forall$ c.v. analitico V

$|V|_{r+\rho} < \infty$ si ha che

$e^{\mathrm{ad}S} V$ è analitico in A_r e

$$|e^{[S,\cdot]} V|_r \leq 2|V|_{r+\rho}$$

$$|e^{[S,\cdot]} V - V|_r \leq \delta^{-1} |V|_{r+\rho} |S|_{r+\rho}$$

In generale $\forall$ sequenze $(c_k)_{k\in\mathbb{N}}$ con

$|c_k| \leq \frac{1}{k!}$ si ha

$$\left| \sum_{k\geq h} c_k (\mathrm{ad}\, S)^k V \right|_r \leq 2 |V|_{r+\rho} \left(\frac{|S|_{r+\rho}}{2\delta} \right)^h$$

Dimostrazione:

$$\begin{cases} x_\tau = S(x) \\ x(0) = y \end{cases}$$

$$x_J(\tau) = y_J + \int_0^\tau S_J(x(\tau_1))\, d\tau_1$$

se $T := \inf \{ |\tau| : x(\tau) \notin B_{r+\rho}(0) \}$

vuol dire che almeno 1 componente ~~di~~ (J_0)

$x(T)$ (o $x(-T)$) è t.c.

$x_{J_0}(T) = r + \rho$ quindi $(|y_{J_0}| \leq r$

$$r+\rho \leq r + T \sup_{x \in B_{r+\rho}} |S_{J_0}(x)|$$

$$r+\rho \leq r + (r+\rho) T \, |S|_{r+\rho} \Big)$$

$$\rho \leq \frac{T \rho}{8e} \quad \Rightarrow \quad T \geq 8e > 1$$

quindi $\forall \; y \in B_r(0)$

$\phi(1,y)$ è ben definito e inoltre

$$|\phi(1,y) - y|_\infty \leq \Big| \int_0^1 S(\phi(\tau,y)) \, d\tau \Big|$$

$$|\phi(1,y) - y|_\infty \leq (r+\rho) \, |S|_{r+\rho}$$

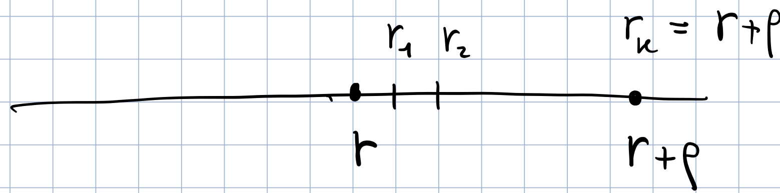
$$V = \sum V^{(d)} \qquad \text{con} \quad \sum |V^{(d)}|_{r+\rho} < \infty$$

$$e^{[S,\cdot]} V = \sum_{k=0}^{\infty} \frac{(\mathrm{ad}\, S)^k}{k!} V$$

$$\left| e^{[S,\cdot]} V \right|_r \le \sum_{k=0}^{\infty} \frac{|(\mathrm{ad}\, S)^k V|_r}{k!}$$

$r_1 > r$

Rem $$|[S,V]|_r \le 4 \frac{r_1}{r_1 - r} |S|_{r_1} |V|_{r_1}$$



(so controllare S, V in norme $r+\rho$)

$$r_i = r + i\frac{\rho}{k} \qquad (r_0 = r)$$

$$|(\mathrm{ad}\, S)^k V|_r = |[S, (\mathrm{ad}\, S)^{k-1} V]|_r$$

$$\le 4 \frac{r_1}{r_1 - r_0} |S|_{r_1} |(\mathrm{ad}\, S)^{k-1} V|_{r_1}$$

$$\le 4 \frac{(r+\rho)}{\rho} k \, |S|_{r+\rho} |(\mathrm{ad}\, S)^{k-1} V|_{r_1}$$

$$= 4\left(\frac{r+\rho}{\rho}\right) k\, |S|_{r+\rho} \left| [S, (\mathrm{ad}\, S)^{k-2} V] \right|_{r_1}$$

$$\leq 4\left(\frac{r+\rho}{\rho}\right) k\, |S|_{r+\rho} \frac{4 r_2}{r_2 - r_1} |S|_{r_2} \left| (\mathrm{ad}\, S)^{k-2} V \right|_{r_2}$$

$$\leq \left(4\left(\frac{r+\rho}{\rho}\right) k\, |S|_{r+\rho} \right)^2 \left| (\mathrm{ad}\, S)^{k-2} V \right|_{r_2}$$

$$\vdots$$

$$\leq \left(4\left(\frac{r+\rho}{\rho}\right) k\, |S|_{r+\rho} \right)^k |V|_{r+\rho}$$

quindi

$$\sum_{k=0}^{\infty} \frac{\left| (\mathrm{ad}\, S)^k V \right|_r}{k!} \leq$$

$$\sum_{k=0}^{\infty} \frac{k^k}{k!} \left(4\frac{(r+\rho)}{\rho} |S|_{r+\rho} \right)^k |V|_{r+\rho} = *$$

$$\left(k! \geq k^k e^{-k} \qquad \forall k \right)$$

$$* \leq \sum \left(4e \, \frac{r+\rho}{\rho} \, |S|_{r+\rho} \right)^{k}$$

$\swarrow$

se questo è $< \frac{1}{2}$ ho convergenza

totale + tutte le stime