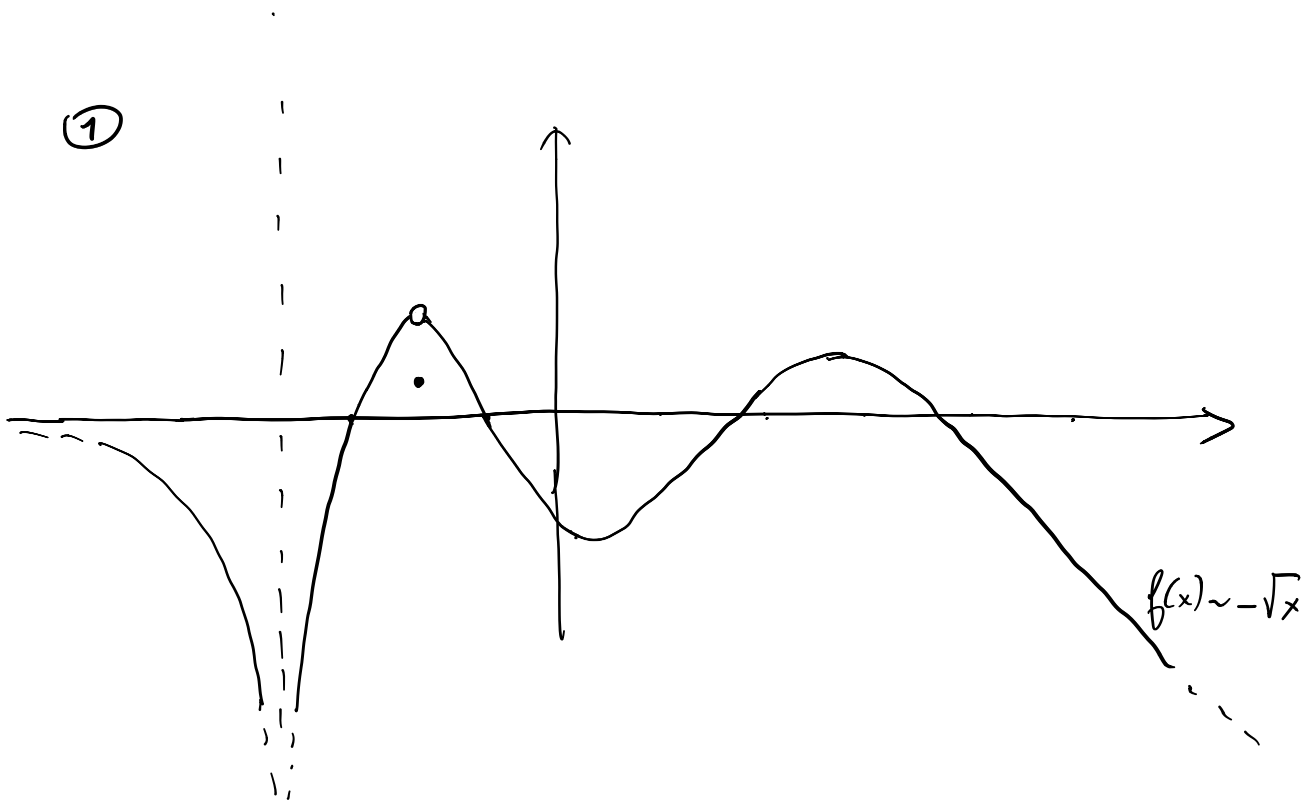
① 




SCUSATE SONO 2

Dire <u>QUALE</u> ( delle seguenti affermazioni è corretta

(a) $f([-6,-4)) = [0,3]$ NO $f([-6,-4)) = [0,3)$

(b) $\lim_{x \to -2} \frac{f(x)}{x+2} < 0$ sì $f(-2) = 0$

$$\parallel$$

$$\lim_{x \to -2} \frac{f(x) - f(-2)}{x+2} = f'(-2) < 0$$

(c) $\int_{15}^{\infty} \frac{1}{|f(x)|} < \infty$ NO $\frac{1}{|f(x)|} \sim \frac{1}{\sqrt{x}}$ e $\int_{15}^{\infty} \frac{1}{\sqrt{x}} = \infty$

(d) $\lim_{x \to 8^-} \frac{f(x)}{x-8} = -\infty$ SI $f(8) > 0$

$$\frac{f(8)}{0^-} = -\infty$$

(2) $\lim_{n \to \infty} \left( \frac{3n^2+4}{3n^2+n+1} \right)^n = \lim_{n \to \infty} \left( 1 + \frac{4-n-1}{3n^2+n+1} \right)^n$

$$= \lim_{n \to \infty} \left( 1 + \frac{3-n}{3n^2+n+1} \right)^{\frac{3n^2+n+1}{3-n} \cdot \frac{n(3-n)}{3n^2+n+1}}$$

$$= e^{-\frac{1}{3}}$$

$\boxed{e}$ nessuna delle precedenti

(3) $\int_0^\infty \frac{dx}{4+x^2} \overset{\text{def}}{=} \lim_{t \to \infty} \int_0^t \frac{dx}{4+x^2}$

Calcolo la primitiva

$$\int \frac{dx}{4+x^2} = \frac{1}{4} \int \frac{dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{2} \int \frac{dy}{1+y^2} \Big|_{y=\frac{x}{2}} = \frac{1}{2} \operatorname{arctg} \frac{x}{2}$$

$$\int_0^\infty \frac{dx}{4+x^2} = \frac{1}{2}\lim_{t\to\infty}\left(\operatorname{arctg} t - \operatorname{arctg}(0)\right)$$

$$= \frac{1}{2}\operatorname{arctg}(\infty) = \frac{\pi}{4}$$

4

$$\lim_{x\to 0} \frac{\ln(1+x-x^2) - x + \frac{3x^2}{2}}{x^3}$$

$\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$ quindi

$$\ln(1+x-x^2) = (x-x^2) - \frac{1}{2}(x-x^2)^2 + \frac{(x-x^2)^3}{3} + o(x^3)$$

$$= (x-x^2) - \frac{1}{2}(x^2-2x^3) + \frac{x^3}{3} + o(x^3)$$

e infine

$$\lim_{x\to 0} \frac{\ln(1+x-x^2) - x + \frac{3x^2}{2}}{x^3} =$$

$$\lim_{x\to 0} \frac{\frac{4}{3}x^3 + o(x^3)}{x^3} = \frac{4}{3} \quad (c)$$

(5) $$\int \frac{dx}{\sqrt{x^2-2x+2}} = \quad \text{( Eulero )}$$

$$x^2+y^2-2xy = x^2-2x+2$$

$$y = \sqrt{x^2-2x+2} + x \; ; \qquad y^2-2 = 2x(y-1)$$

$$dx = \frac{1}{2}\left(1+\frac{1}{(y-1)^2}\right)dy \qquad x = \frac{y+1}{2} - \frac{1}{2(y-1)}$$

$$\sqrt{x^2-2x+2} = y-x = \frac{y-1}{2} + \frac{1}{2(y-1)} = \frac{y-1}{2}\left(1+\frac{1}{(y-1)^2}\right)$$

$$\int \frac{dx}{\sqrt{x^2-2x+2}} = \int \frac{\left(1+\cancel{\frac{1}{(y-1)^2}}\right)dy}{(y+1)\left(1+\cancel{\frac{1}{(y-1)^2}}\right)} = \int \frac{dy}{y-1}$$

$$= \ln\left(\left|\sqrt{x^2-2x+2}+x-1\right|\right)$$

Oppure

$$\int \frac{dx}{\sqrt{x^2-2x+2}} = \int \frac{dx}{\sqrt{(x-1)^2+1}} = \int \frac{dt}{\sqrt{t^2+1}}\Bigg|_{t=x-1}$$

$$\int \frac{dt}{\sqrt{t^2+1}} \overset{\text{Noto}}{=} \operatorname{arcsinh}(t)$$

(oppure faccio eulero da qui!)

⑥ Studiare la funzione

$$f(x) = \sqrt{x} \ln x$$

Definita per $x > 0$

Positiva per $\ln x > 0$ cioè $x > 1$

$$\lim_{x \to 0^+} \sqrt{x} \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} \overset{\text{Hospital}}{=}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2} x^{-\frac{3}{2}}} = -2 \lim_{x \to 0^+} x^{\frac{3}{2}-1} = 0^-$$

$$\lim_{x \to \infty} \sqrt{x} \ln x = +\infty$$

$$f'(x) = \frac{1}{2} \frac{1}{\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \left( \frac{\ln x}{2} + 1 \right)$$

$$f'(x) \geq 0 \qquad \ln x \geq -2 \qquad \Rightarrow \qquad x \geq e^{-2}$$

$$f''(x) = -\frac{1}{2}\frac{1}{\sqrt{x^3}}\left(\frac{\ln x}{2}+1\right) + \frac{1}{\sqrt{x}}\,\frac{1}{2x}$$

$$= -\frac{\ln x}{2\sqrt{x^3}}$$