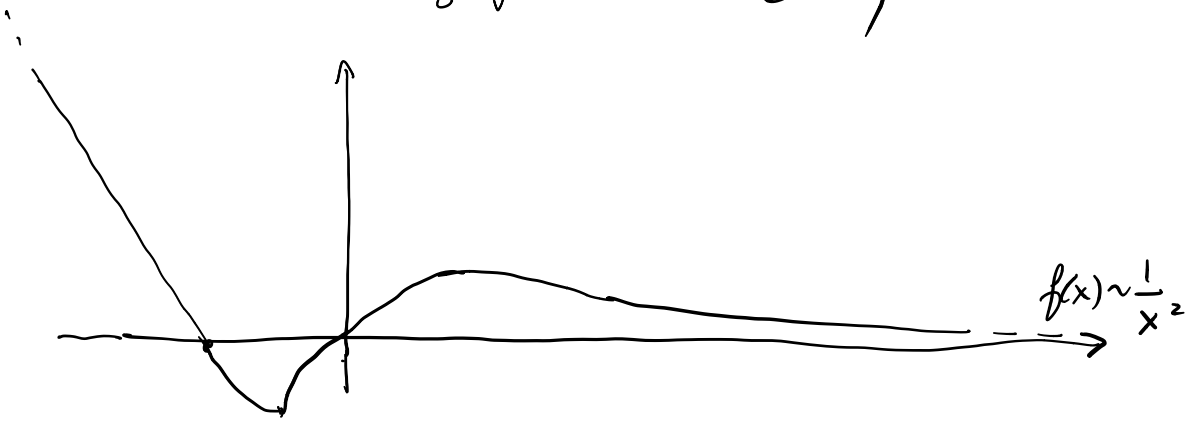
① Dato il grafico (3)



a) $\int_0^{\infty} f(x) = +\infty$ No dato che $f(x) \sim \frac{1}{x^2}$ per $x \to \infty$

b) $f'(-2) = 0$ No la derivata NON è definita in -2

infatti $\lim_{x \to -2^-} f'(x) = 0 \neq \lim_{x \to -2^+} f'(x) = +\infty$

c) $\lim_{x \to -4} \frac{f(x)}{x+4} > 0$ No il limite $= f'(-4) < 0$

$$f'(-4) \overset{def}{=} \lim_{x \to -4} \frac{f(x) - f(-4)}{x+4} = \lim_{x \to -4} \frac{f(x)}{x+4}$$

d) la preimmagine di $[0,3] = [-6,-4] \cup [0,\infty)$

② Date le due serie (4)

$$S_1 = \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+\sqrt{n}} \quad ; \quad S_2 = \sum_{n=1}^{\infty} \frac{2^n + n}{4^n + \sqrt{n}}$$

$\boxed{2)}$ S_1 e S_2 convergono $(\cos n\pi = (-1)^n!)$

S_1 converge per il criterio di Leibnitz

mentre S_2 converge dato che $\dfrac{2^n+n}{4^n+\sqrt{n}} \sim \left(\dfrac{1}{2}\right)^n$

③ $$\lim_{x\to 0} \frac{e^{x^2} - 1 - \ln(1+x^2)}{\sqrt{3x^4+1} - 1} \qquad (6)$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^4); \quad \ln(1+x^2) = x^2 - \frac{x^4}{2}$$

$$\lim_{x\to 0} \frac{e^{x^2} - 1 - \ln(1+x^2)}{\sqrt{3x^4+1} - 1} \cdot \frac{\sqrt{3x^4+1}+1}{\sqrt{3x^4+1}+1} = \frac{\left(e^{x^2} - 1 - \ln(1+x^2)\right)\left(\sqrt{3x^4+1}+1\right)}{3x^4+1-1}$$

$$\frac{\frac{x^4}{2} + \frac{x^4}{2} + o(x^4)}{3x^4} \cdot 2 = \frac{2}{3} \qquad (B)$$

④ $$\lim_{n\to\infty} \left(\frac{n^3+2n+1}{n^3-4n+2}\right)^{2n^2+n} =$$

$$\lim_{n\to\infty} \left(\frac{n^3-4n+2 + 6n - 1}{n^3-4n+2}\right)^{2n^2+n} =$$

$$\lim \left(1 + \frac{6n-1}{}\right)^{\frac{n^3-4n+2}{6n-1}\,\frac{(6n-1)(2n^2+n)}{n^3-4n+2}}$$

$n \to \infty \quad (\qquad n^3 - 4n + 2)$

$$= e^{12} \qquad (d)$$

(4)

5) Calcolare $\displaystyle\int \frac{dx}{x\sqrt{1-x^2}}$ (6)

$$y = \sqrt{1-x^2} \qquad dy = -\frac{x\,dx}{\sqrt{1-x^2}} \qquad \left(x = \sqrt{1-y^2} \right)$$

$$\int \frac{dx}{x\sqrt{1-x^2}} = -\int \frac{-x\,dx}{x^2\sqrt{1-x^2}} = -\int \frac{dy}{1-y^2} =$$

$$= \frac{1}{2}\int\left(\frac{1}{y-1} - \frac{1}{y+1}\right) dy = \frac{1}{2}\ln\left(\left|\frac{y-1}{y+1}\right|\right)\Bigg|_{y=\sqrt{1-x^2}}$$

$$\int \frac{dx}{x\sqrt{1-x^2}} = \frac{1}{2}\ln\left(\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right|\right)$$

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$$\int \frac{dx}{x\sqrt{1-x^2}} = \qquad x = \sin y \qquad dx = \cos y\,dy$$

$$dx = \sqrt{1-x^2}\,dy$$

$$\int \frac{dy}{\sin y} = \int \frac{\sin y\,dy}{1-\cos^2 y} = \int \frac{dz}{1-z^2} \qquad \text{etc...}$$

6) Studiare il grafico della funzione

$$\ln(x^2+3x+2)-1$$

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Dominio: $x^2+3x+2>0$

cioé $x>-1 \cup x<-2$

la funzione non è ne pari ne dispari

$$\lim_{x\to\pm\infty} \ln(x^2+3x+2)-1 = +\infty$$

$$\lim_{x\to -2^-} \ln(x^2+3x+2)-1 = -\infty$$

$$\lim_{x\to -1^+} \ln(x^2+3x+2)-1 = -\infty$$

$\ln(x^2+3x+2)-1 \geq 0$ se $x^2+3x+2 \geq e$

$$x_{1,2} = \frac{-3\pm\sqrt{9-4(2-e)}}{2} = \frac{-3\pm\sqrt{1+4e}}{2}$$

$f(x) \geq 0$ se $x \geq \frac{-3+\sqrt{1+4e}}{2}$ o $x \leq \frac{-3-\sqrt{1+4e}}{2}$

$$f'(x) = \frac{2x+3}{x^2+3x+2}$$

Nel dominio $x^2+3x+2>0$ quindi

$$f'(x) > 0 \quad \text{in} \quad D \cap \left\{ x > -\frac{3}{2} \right\}$$