

G. Benettin

Title: The role of Toda model in the Fermi-Pasta-Ulam problem

Abstract: In 1954 Fermi, Pasta and Ulam for the first time used a computer to investigate the ergodic behavior of a dynamical system with many degrees of freedom, interesting to study the very foundations of Statistical Mechanics. Several branches of physical and mathematical investigations started from that paper.

In 1982 Ferguson, Flaschka and McLaughlin, in an almost forgotten paper, understood that the reference integrable system for FPU, in the generic case, is the Toda model.

The aim of the talk is to revisit the FPU problem and specifically this view, in the light of some recent numerical results, partially in progress. The gap between numerical insights and mathematical results remains large, but a picture seems to be on the horizon.

W. Craig

Title: Is the solution map for a PDE a flow?

Abstract: The nonlinear Schrodinger equations and nonlinear wave equations that arise from a Lagrangian can be formulated as Hamiltonian PDEs, and are well posed locally in time in the sense of Hadamard, in a Sobolev space of sufficient spatial regularity. In particular, the solution depends continuously upon the initial data. The question is whether this dependence is smooth. Work of T. Kato (1975) shows that smoothness does not hold in general. The same considerations apply to Birkhoff normal forms transformations of PDEs. This talk will compare and contrast various cases.

V. Gelfreich (joint work with A. Vieira)

Title: Interpolating vector fields for close to identity maps and averaging

Abstract: For a smooth near-identity map, we introduce the notion of an interpolating vector field written in terms of iterates of the map. We study properties of the interpolating vector fields and explore their applications to the study of dynamics. In particular, we use them to find adiabatic invariants for symplectic near-identity maps. We also introduce the notion of a Poincaré section for a close-to-identity map. We illustrate our theory with several examples, including the Chirikov standard map and a symplectic map in dimension four, an example motivated by the theory of Arnold diffusion.

H. Ito

Title: Birkhoff normalization of non-commutatively integrable symplectic maps and its applications

Abstract: We consider the problem of analytic Birkhoff normalization for an analytic symplectic map near a resonant fixed point. We show that it is possible when the map has appropriate number of integrals according to the resonance degree. It can be applied to prove existence of generalized action-angle coordinates for a non-commutatively integrable system.

T. Kappeler

Title: On a version of the Arnold-Liouville theorem in infinite dimension: a case study

Abstract: It is well known that the focusing nonlinear Schrödinger equation (fNLS) is an integrable PDE. When considered on the circle, the periodic eigenvalues of the Zakharov-Shabat (ZS) operator, appearing in the Lax pair formulation of the fNLS equation, form an infinite set of integrals of motion. Note that unlike other integrable PDEs such as the KdV equation or the defocusing nonlinear Schrödinger equation, the fNLS equation exhibits features of hyperbolic dynamics, in particular homoclinic orbits. In form of a case study for the fNLS equation, I present a version of the Arnold-Liouville theorem in infinite dimension. This is joint work with Peter Topalov.

S. B. Kuksin

Title: KAM and mixing

Abstract: I will explain how the KAM-theory provides a powerful tool to prove the mixing for dynamical systems of finite and infinite dimension, perturbed by bounded random forces.

L. Niederman (joint work with A. Pousse and P. Robutel)

Title: Quasi periodic coorbital motions

Abstract: The motions of the satellites Janus and Epimetheus around Saturn are among the most intriguing in the solar system. These satellites exchange their orbits every four years. We give a rigorous proof of the existence of stable orbits of this kind in the three body planetary problem thanks to KAM theory.

T. Seara (joint work with M. Gidea and R. de la Llave)

Title: A general mechanism of diffusion in Hamiltonian systems

Abstract: In this talk we present a general mechanism to establish the existence of diffusing orbits in a large class of nearly integrable Hamiltonian systems. Our approach relies on successive applications of the so called 'scattering map' along homoclinic orbits to a normally hyperbolic invariant manifold. If we find pseudo-orbits of the scattering map that keep advancing in some privileged direction, we can use the recurrence property of the 'inner dynamics', restricted to the normally hyperbolic invariant manifold, to return to those pseudo-orbits. Finally, we apply topological methods to show the existence of true orbits that follow the successive applications of the two dynamics.

This method differs, in several crucial aspects, from earlier works. Unlike the well known 'two-dynamics' approach, the method we present relies on the outer dynamics alone. There are virtually no assumptions on the inner dynamics, as its invariant objects (e.g., primary and secondary tori, lower dimensional hyperbolic tori and their stable/unstable manifolds, Aubry-Mather sets) are not used at all.

The method applies to unperturbed integrable Hamiltonians of arbitrary degrees of freedom (not necessarily convex) which present some hyperbolicity and establishes diffusion in generic perturbations.

We include several applications, such as bridging large gaps in a priori unstable models in any dimension, and establishing diffusion in cases when the inner dynamics is a non-twist map.

G. Tarantello

Title: Elliptic problems in the study of Chern-Simons vortices

Abstract: We shall discuss some analytical, geometrical and topological aspects for a class of elliptic problems in connection with the description of self-dual Chern-Simons vortices.

D. Treschev

Title: Anti-integrable limit

Abstract: Anti-integrable limit is one of convenient and relatively simple methods for construction of chaotic hyperbolic invariant sets in Lagrangian, Hamiltonian and other dynamical systems.

We discuss the most natural context of the method, discrete Lagrangian systems.

Then we present examples and applications.

L. Stolovitch (joint work with Xianghong Gong)

Title: Dynamics and singularities of Cauchy-Riemann structures

Abstract: We study real analytic submanifolds of the complex space. If each tangent space contains a complex subspace the dimension of which is constant along the submanifold, then it is called a Cauchy-Riemann manifold. A point is called a CR singularity if the complex dimension drops at that point. The study of CR singularities was initiated by Bishop and then by Moser-Webster.

We study a germ of real analytic n -dimensional submanifold of \mathbb{C}^n that has a complex tangent space of maximal dimension at a CR singularity. These are higher order perturbations of a quadric.

Under some assumptions, we show its equivalence to a normal form under a local biholomorphism at the singularity. We also show that if a real submanifold is formally equivalent to its "unperturbed" quadric, it is actually holomorphically equivalent to it, if a "small divisors condition" is satisfied.

The properties of geometry of these manifolds can be read off the properties of an associated dynamical system.

This a joint work with Xianghong Gong (Madison)."

E. Valdinoci

Title: Crystal dislocation, nonlocal equations and fractional dynamical systems

Abstract: We study heteroclinic and multibump orbits for a system of equations driven by a nonlocal operator. Our motivation comes from the study of the atom dislocation function in a periodic crystal, according to the Peierls-Nabarro model. The evolution of the dislocation function can be studied by analytic techniques of fractional Laplace type. At a macroscopic scale, the dislocations have the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior potential.

Such potential turns out to be either attractive or repulsive, depending on the mutual orientation of the dislocations, and the attractive potentials generate "particle collisions" in finite time. After the collisions, the system relaxes to the equilibrium exponentially fast, and the associated steady states provide a natural setting for the study of dynamics and chaos in a fractional framework.

