$$J\varphi^{-1} = \begin{pmatrix} -2x & 1 \\ 2 & 1 \end{pmatrix} = -2(x+1) \neq 0$$

LOCALMENTE INVERTIBILE NELL'APERTO!

$$x^2 + 2x = v - u \Rightarrow$$

$$(x+1)^2 = v - u + 1 \qquad x = -1 \pm \sqrt{1 + v - u}$$

Escludo la determinazione $-$

perché otterrei $x < -1$ (ma $-1 < x < -1 + \sqrt{2}$

in T)

$$\begin{cases} x = -1 + \sqrt{1+v-u} \\ y = v - 2\left(-1 + \sqrt{1+v-u}\right) \end{cases}$$

$$J\psi = \begin{pmatrix} \frac{1}{2}\frac{-1}{\sqrt{1+v-u}} & \frac{1}{2\sqrt{1+v-u}} \\ \frac{1}{\sqrt{1+v-u}} & 1 - \frac{1}{\sqrt{1+v-u}} \end{pmatrix}$$

$$\det J\psi = -\frac{1}{2}\left(\frac{1}{\sqrt{1+v-u}} - \frac{1}{1+v-u}\right) - \frac{1}{2}\,\frac{1}{1+v-u} =$$

$$= -\frac{1}{2}\,\frac{1}{\sqrt{1+v-u}}$$

(lo sapevo già $J\psi = \left(J\psi^{-1}\right)^{-1}$!)

$$\int_0^1 du \int_0^1 dv\, \frac{1}{\sqrt{1+v-u}} \quad *$$

$$\int_0^1 du\; 2\left[\sqrt{1+v-u}\right]_{v=0}^{1} = 2\int_0^1 du\left(\sqrt{2-u} - \sqrt{1-u}\right) =$$

$$2\left(-\frac{2}{3}(2-u)^{3/2} + \frac{2}{3}(1-u)^{3/2}\right)_0^1 =$$

$$2\left(-\frac{2}{3} + \frac{2}{3}\,2^{3/2} + \frac{2}{3}\right) = \frac{4}{3}\,2^{3/2}$$

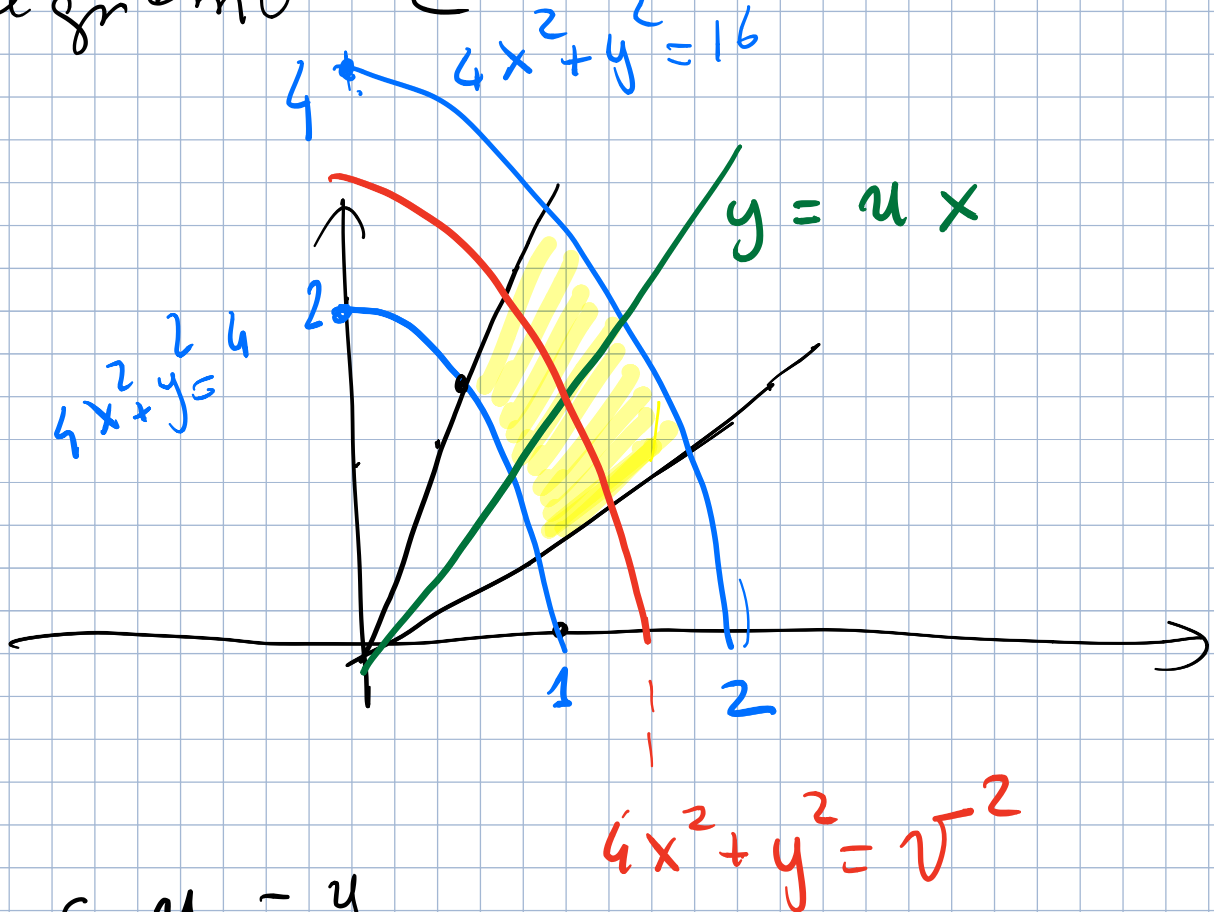
* quando $u=1$, $v=0$ il denominatore $=0$ $\Rightarrow$ integrale improprio!

12.23. Calcolare:

1. $\int_E dxdy$ 2. $\int_E xdxdy$ 3. $\int_E \frac{dxdy}{xy}$

dove E è l'insieme delimitato dalle rette $y = 2x$ e $2y = x$ e dalle ellissi $4x^2 + y^2 = 4$ e $4x^2 + y^2 = 16$ e contenuto nel primo quadrante:

Disegnamo E



$$\psi^{-1} = \begin{cases} u = \dfrac{y}{x} \\ v = \sqrt{4x^2+y^2} \end{cases}$$

ovvero ψ ($J\psi^{-1} = \begin{pmatrix} -\dfrac{y}{x^2} & \dfrac{1}{x} \\ \dfrac{4x}{\sqrt{4x^2+y^2}} & \dfrac{y}{\sqrt{4x^2+y^2}} \end{pmatrix}$

$\det J\psi^{-1} = -\sqrt{4x^2+y^2}$

$$\begin{cases} x = \dfrac{v}{\sqrt{4+u^2}} \\ \\ y = \dfrac{uv}{\sqrt{4+u^2}} \end{cases}$$

$$\det J\psi = -\frac{1}{v}$$

① $$A(E) = \int_{\frac{1}{2}}^{2} du \int_{1}^{2} \frac{1}{v}\, dv =$$

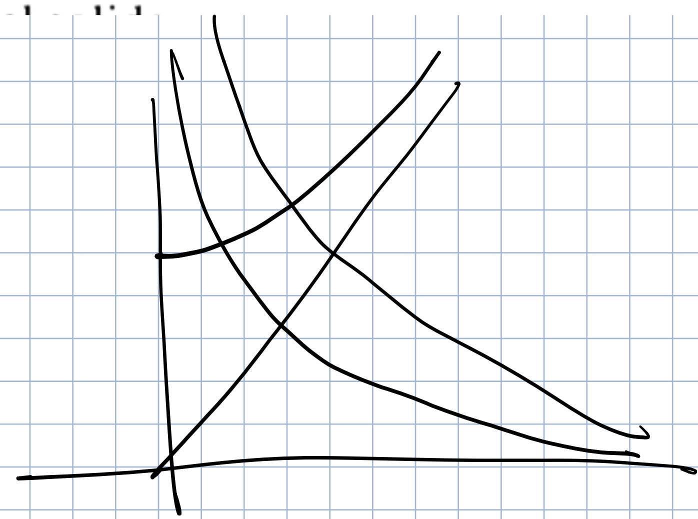
$$= \int_{\frac{1}{2}}^{2} du \left[\ln v\right]_1^2 = \left(2-\frac{1}{2}\right)\ln 2$$

② $$\int_{\frac{1}{2}}^{2} du \int_{1}^{2} \frac{1}{4+u^2}\, dv = \ldots$$

12.26 Calcolare l'integrale

$$\star = \int_D (y^2 - x^2)^{xy}(x^2 + y^2)\,dxdy$$

con $D = \{(x, y) : x > 0, y > 0, 0 < a \leqslant xy \leqslant b, 0 < y^2 - x^2 < 1\}$.



$$\Psi^{-1}: \begin{cases} u = xy \\ v = y^2 - x^2 \end{cases} \qquad J\Psi^{-1} = \begin{pmatrix} y & x \\ -2x & 2y \end{pmatrix} =$$

$$\det J\Psi^{-1} = 2(x^2 + y^2)$$

$$\star = \int_a^b du \int_0^1 dv \left((y^2 - x^2)^{xy} (x^2 + y^2) \right)_{\substack{x = x(u,v) \\ y = y(u,v)}} |\det J\Psi(u,v)|$$

Ma dato che

$$\det(J\varphi) = \frac{1}{\det(J\varphi^{-1})}\Bigg|_{\substack{x = x(u,v) \\ y = y(u,v)}}$$

$$A = \int_a^b du \int_0^1 \frac{v^u}{2}\, dv =$$

$$\frac{1}{2}\int_a^b \frac{1}{u+1}\left[v^{u+1}\right]_0^1 du =$$

$$\frac{1}{2}\int_a^b \frac{1}{u+1}\, du = \frac{1}{2}\ln(b) - \ln(a)$$