Some exercises on Linear systems

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Exercise 1. Study the solutions of the system

$$\dot{x} = Ax\,, \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 1 & 3 & 2 \end{pmatrix}\,, \quad x(t) \in \mathbb{R}^3$$

Exercise 2. Find the exponential of the following matrices

$$\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}\,, \quad \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\,, \quad \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}\,, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,,$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}\,, \quad \begin{pmatrix} \lambda & 0 & 0 \\ 1 & \lambda & 0 \\ 0 & 1 & \lambda \end{pmatrix}\,, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\,,$$

Give a general formula for the exponent of a Jordan block.
Compute the logarithm of the invertible matrices.

Exercise 3. Study the solutions of the system

$$\dot{x} = Ax + b(t)\,, \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 1 & 3 & 2 \end{pmatrix}\,, \quad b = \begin{pmatrix} \cos(t) \\ t \\ 1 \end{pmatrix}\,, \quad x(t) \in \mathbb{R}^3$$

Exercise 4. Reduce to constant coefficients the complex system

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & e^{it} \\ e^{-it} & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

find a fundamental solution and solve the associated non-homogeneous problem

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & e^{it} \\ e^{-it} & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ e^{-2it} \end{pmatrix}$$