$$\lim_{x \to 0} \frac{\operatorname{sen}(5x^2+x) \cdot \cos(3x) \cdot (1-\cos(2x^3))}{\operatorname{arctg}(x^3) \cdot \log(1-x^2) \cdot (x^2+x^3)}$$

$$= \lim_{x \to 0} \frac{(5x^2+x) \cdot 1 \cdot \frac{(2x^3)^2}{2}}{x^3 \cdot (-x^2) \cdot (x^2+x^3)} =$$

$$\lim_{x \to 0} \frac{x \cdot 2x^6}{-x^5 \cdot x^2} = -2$$

$$\lim_{x \to \infty} \frac{\ln(1+x^2)}{\sqrt{x}} = \circledast$$

prendo

dato che $\ln(y) \ll y^p$; $y = 1+x^2$, $p = \frac{1}{4}$

(se $y \to \infty$)

$$\circledast = \lim_{x \to \infty} \frac{\ln(1+x^2)}{\sqrt[4]{(1+x^2)}} \cdot \lim_{x \to \infty} \frac{\sqrt[4]{(1+x^2)}}{\sqrt{x}} =$$

$$= 0 \cdot \lim_{x \to \infty} \frac{\sqrt[4]{(1+x^2)}}{\sqrt{x}} = 1 \quad \left(\sqrt[4]{1+x^2} \sim \sqrt[4]{x^2} \right)$$

$$\lim_{x \to \infty} \frac{\ln_{\frac{1}{2}}(x)}{\sqrt{x}}$$

A con le formule del cambio di base del logaritmo $\ln_{\frac{1}{2}}(x) = \frac{\ln(x)}{\ln(\frac{1}{2})}$

$$\lim_{x \to \infty} \frac{\ln_{\frac{1}{2}}(x)}{\sqrt{x}} = \frac{1}{\ln(\frac{1}{2})} \lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}} = \frac{0}{\ln(\frac{1}{2})} = 0$$

B $y = -\ln_{\frac{1}{2}}(x)$ $\quad x = \left(\frac{1}{2}\right)^{-y} = 2^{y}$

$$\lim_{x \to \infty} -\ln_{\frac{1}{2}}(x) = +\infty$$

$$\lim_{x \to \infty} \frac{\ln_{\frac{1}{2}}(x)}{\sqrt{x}} = \lim_{y \to +\infty} \frac{-y}{2^{y/2}}$$

ma $2^{\frac{y}{2}} \gg \frac{y}{2}$ $\Rightarrow \lim_{y \to \infty} \frac{y}{2^{y/2}} = 0$

$$\lim_{x \to -\infty} x e^{x} = \lim_{x \to -\infty} \frac{x}{e^{-x}}$$

$$=$$

prendo $y = -x$ $\qquad \lim_{y \to +\infty} \frac{-y}{e^{y}} = 0$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^{x^3} = \left(\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^{x^2}\right)^{\lim_{x \to \infty} x}$$

$$= e^{\infty} = \infty$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x^3}\right)^{x^2} = \lim_{x \to \infty} \left(1 + \frac{1}{x^3}\right)^{\frac{x^3 \cdot x^2}{x^3}} =$$

$$\left(\lim_{x \to \infty} \left(1 + \frac{1}{x^3}\right)^{x^3}\right)^{\lim_{x \to \infty} \frac{x^2}{x^3}} = e^{0} = 1$$

$$\lim_{x \to 0} \frac{\sin\left(\frac{5x^3}{3x^2+x}\right)}{\left(\cos(\sqrt{x}) - 1\right)(x+x^2)} = *$$

$$\frac{5x^3}{3x^2+x} \sim \frac{5x^3}{x} = 5x^2 \quad \text{per } x \to 0$$

quindi $\sin\left(\frac{5x^3}{3x^2+x}\right) \sim \frac{5x^3}{3x^2+x} \sim 5x^2$ per $x \to 0$

$$\cos(\sqrt{x}) - 1 \sim \frac{(\sqrt{x})^2}{2} = \frac{x}{2} \quad \text{per } x \to 0$$

$$x + x^2 \sim x \quad \text{per } x \to 0$$

$$* = \lim \frac{5x^2}{\frac{x}{2} \cdot x} = 10$$

ATTENZIONE: SEMPRE VERIFICARE CHE L'ARGOMENTO tende a zero.

Es. $\sin\left(\frac{x^4+x}{x^3}\right) \not\sim \frac{x^4+x}{x^3}$ (NON è equivalente)

infatti $\frac{x^4+x}{x^3} \sim \frac{x}{x^3} = \frac{1}{x^2}$ e $\frac{1}{x^2} \to \infty$ se $x \to 0$

La FORMULA VALE SOLO SE L'ARGOMENTO tende a ZERO.

$$\lim_{x \to \infty} \frac{x^2 \sin x + x}{x^3 + x^2} = 0$$

teorema dei CARABINIERI.

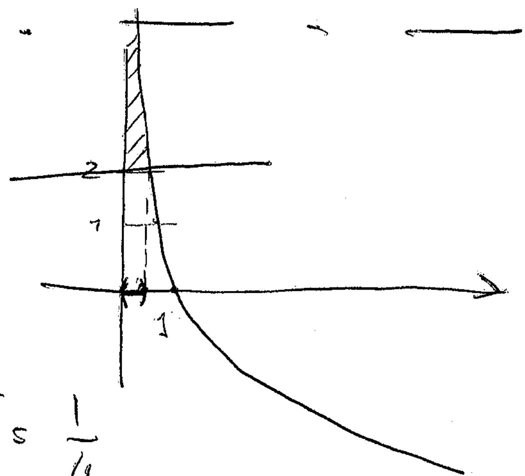
dato che $-1 \leq \sin x \leq 1$ segue che

$$\frac{-x^2+x}{x^3+x^2} \leq \frac{x^2 \sin x + x}{x^3+x^2} \leq \frac{x^2+x}{x^3+x^2}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

$$\ln_{\frac{1}{2}} (x^2 + 3x - 1) \leq 2$$



RISOLVO PRIMA

$$\ln_{\frac{1}{2}} (z) \leq 2 \iff 0 < z \leq \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

equivale a $\quad 0 < x^2 + 3x - 1 \leq \frac{1}{4}$

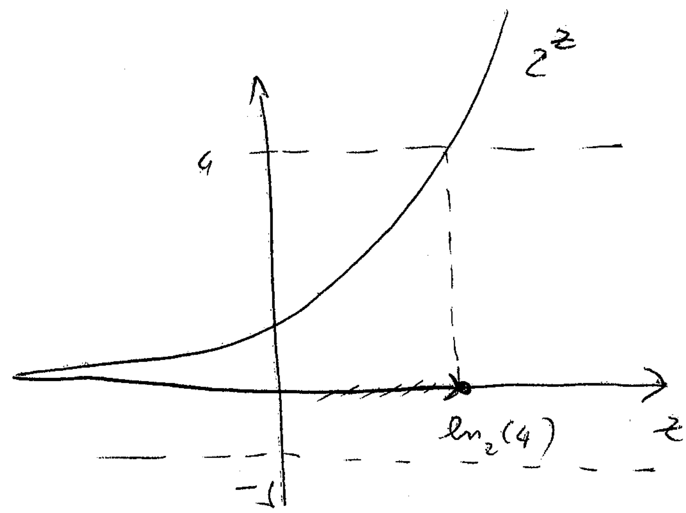
$$\Rightarrow \begin{cases} x^2 + 3x - 1 > 0 & \Rightarrow \quad x > \frac{-3+\sqrt{9+4}}{2} \; ; \; x < \frac{-3-\sqrt{13}}{2} \\ x^2 + 3x - \frac{5}{4} \leq 0 & \frac{-3 \pm \sqrt{9+5}}{2} \leq x \leq \frac{-3+\sqrt{14}}{2} \end{cases}$$

RISULTATO: $\frac{-3-\sqrt{14}}{2} \leq x < \frac{-3-\sqrt{13}}{2} \quad \cup \quad \frac{-3+\sqrt{13}}{2} < x \leq \frac{-3+\sqrt{14}}{2}$

$$-1 < 2^{\frac{x^2+1}{x-2}} < 4$$

La prima DISEQUAZIONE è SEMPRE SODDISFATTA!

$$-1 < 2^z < 4$$



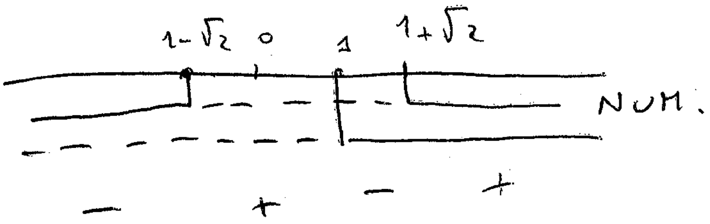
$z < \ln_2(4) = 2$

$$\frac{x^2+1}{x-1} < 2$$

$$\frac{x^2+1-2(x-1)}{x-1} < 0 \qquad \frac{x^2-2x-1}{x-1} < 0$$

x^2-2x-1 è positivo all'esterno dei suoi zeri

$x^2-2x-1 > 0 \Rightarrow x < 1-\sqrt{2}$; $x > 1+\sqrt{2}$



SOLUZIONE : $x < 1-\sqrt{2} \;\cup\; 1 < x < 1+\sqrt{2}$