

$$\int \frac{dx}{x-3} = \int \frac{dy}{y} \Big|_{y=x-3} = \ln(|x-3|)$$

$$\int \frac{2x+5}{(2x+1)^2} dx = \text{prendo } y = 2x+1$$

$$dy = 2 dx$$

$$= \int \frac{y-1+5}{y^2} \frac{dy}{2} \Big|_{y=2x+1} = \frac{x = \frac{(y-1)}{2}}$$

$$= \left(\frac{1}{2} \int \frac{dy}{y} + \frac{4}{2} \int \frac{dy}{y^2} \right) \Big|_{y=2x+1} =$$

$$= \frac{1}{2} \ln(|2x+1|) - \frac{2}{2x+1} + C$$

$$\int \frac{3x+1}{x^2+1} dx = 3 \int \frac{x dx}{x^2+1} + \int \frac{dx}{x^2+1} =$$

$$\frac{3}{2} \int \frac{dy}{y} \Big|_{y=x^2+1} + \arctg(x) + C$$

N.B. i due integrali si fanno diversamente

in $3 \int \frac{x dx}{x^2+1}$ cambio variabile $y = x^2+1$

$$\text{in } \int \frac{dx}{x^2+1} = \text{arctg}(x) \quad [\text{FORMULA NOTA}]$$

$$\int \frac{dx}{x^2+4x+3} \quad ; \text{ cerco di fattorizzare}$$

il denominatore, trovo i due zeri

$$x_{1,2} = \frac{-2 \pm \sqrt{4-3}}{1} = \begin{cases} -1 \\ -3 \end{cases}$$

$$\int \frac{dx}{x^2+4x+3} = \int \frac{dx}{(x+1)(x+3)} = \int \frac{A dx}{x+1} + \int \frac{B dx}{x+3}$$

(devo trovare A, B)

$$\frac{Ax + 3A + Bx + B}{(x+1)(x+3)} = \frac{1}{(x+1)(x+3)}$$

$$\begin{cases} A + B = 0 \\ 3A + B = 1 \end{cases} \Leftrightarrow \begin{cases} A = -B \\ 3A - A = 1 \end{cases}$$

$$A = \frac{1}{2}, B = -\frac{1}{2}$$

$$\int \frac{dx}{(x+1)(x+3)} = \frac{1}{2} \left(\int \frac{dx}{x+1} - \int \frac{dx}{x+3} \right) =$$

$$\frac{1}{2} \left(\ln(|x+1|) - \ln(|x+3|) \right) = \frac{1}{2} \ln \left(\frac{|x+1|}{|x+3|} \right)$$