Abstracts

MINI-COURSES:

Massimiliano Berti (SISSA, Trieste)

Quasi-periodic solutions of water waves

The goal of this course is to present recent existence results of Cantor families of small amplitude, linearly stable, time quasi-periodic standing wave solutions of the water waves equations under periodic boundary conditions.

The proofs are based on several arguments: Nash-Moser implicit functions theorems, reducibility of the quasi-periodic linearized operators using pseudo-differential calculus and KAM iteration, arguments of degenerate KAM theory to verify the non-resonance conditions.

Zaher Hani (Georgia Tech, Atlanta)

Out-of-equilibrium dynamics and statistics of nonlinear dispersive waves

The focus of this mini-course is the problem of long-time dynamics of nonlinear dispersive PDE, particularly in situations characterized by instability and out-of-equilibrium behavior. This is the characteristic feature of the dynamics on compact domains (including very large ones), or more generally in settings where dispersive decay is very weak.

There are two perspectives to address this out-of-equilibrium behavior: a dynamical systems one, and a statistical mechanics one, that often goes by the name of wave turbulence theory. We shall explain the above problematic from both perspectives, and discuss some recent attempts to better understand it.

Laurent Thomann (Université de Lorraine, Nancy)

Invariant Gibbs measures for dispersive PDEs

Consider a dynamical system. We try to understand the long time behaviour of a family of trajectories, rather than the behaviour of each trajectory one by one.

An important tool is the existence of an invariant measure of probability. Here we are concerned with the dynamical system given by the flow of a Hamiltonian dispersive partial differential equation.

In this context, we can sometimes define a Gibbs measure. It turns out that such a measure is very efficient to obtain qualitative information about the long time behaviour of the solution of the PDE, and even to prove almost sure global existence results.

TALKS:

Thomas Alazard (CNRS - ENS)

Control and stabilization of the incompressible Euler equation with free surface

The incompressible Euler equation with free surface dictates the dynamics of the interface separating the air from a perfect incompressible fluid. This talk is about the controllability and the stabilization of this equation. The goal is to understand the generation and the absorption of water waves in a wave tank. These two problems are studied by two different methods: microlocal analysis for the controllability (this is a joint work with Pietro Baldi and Daniel Han-Kwan), and study of global quantities for the stabilization (multiplier method, Pohozaev identity, hamiltonian formulation, Luke's variational principle, conservation laws...).

Dario Bambusi (Università degli Studi di Milano)

On reducibility of Schrödinger equation with time dependent unbounded perturbation

Consider the Schrödinger equation

$$\mathrm{i}\dot{\psi} = -\Delta\psi + V(x)\psi + \epsilon W(x, -\mathrm{i}\nabla, \omega t)\psi \ , \quad x \in \mathbb{R}^d$$

with V a polynomial potential growing at infinity at least quadratically and W a perturbation which depends quasiperiodically on time.

I will present a method based on pseudodifferential calculus and KAM theory in order to prove reducibility under some growth assumptions on W.

The method applies to the 1-dimensional case and to some higher dimensional cases. Some cases in which the perturbation is as unbounded as the main part of the equation are included.

The results in the case $d > 1$ have been obtained in collaboration with B. Grébert.

Philippe Bolle (Université d'Avignon)

Quasi-periodic solutions of nonlinear wave equations on the torus

We present an existence result for small amplitude quasi-periodic solutions of autonomous nonlinear wave equations with a multiplicative potential on the multidimensional torus. The proof relies on a Nash-Moser procedure and uses the Hamiltonian structure of the equation. This is a joint work with M. Berti.

Aynur Bulut (University of Michigan)

Recent developments on deterministic and probabilistic well-posedness for nonlinear Schrödinger and wave equations

In this talk, we will give an overview of recent results concerning nonlinear Schrödinger and wave equations, emphasizing their treatment from both deterministic and probabilistic viewpoints. From the deterministic standpoint, a major open question is long-time existence and qualitative behavior of solutions when the nonlinearity is supercritical with respect to the conserved energy. While such questions are open even in the case of smooth compactly supported initial data with radial symmetry, we will give an overview of the partial results possible to date. Turning to the probabilistic perspective, we study initial data which is supercritical with respect to the scaling of the nonlinearity. For this class of data, the initial value problems are ill-posed: uniform continuity of the solution map cannot hold, and one is required to go beyond deterministic constructions. In particular, by considering classes of randomized initial data, one can often recover notions of well-posedness which hold except on sets of initial data occurring with low (or zero) probability.

Livia Corsi (Georgia Tech, Atlanta)

Locally integrable non-Liouville analytic geodesic flows on $\mathbb{T}^2$

A metric on $\mathbb{T}^2$ is said to be "Liouville" if in some coordinate system it has the form $ds^2 = (g_1(q_1) + g_2(q_2))(dq_1^2 + dq_2^2)$; a "folklore conjecture" states that if a metric is locally integrable then it is Liouville. I will present a counterexample to this conjecture.

Precisely I will show that there exists an analytic, non-separable, mechanical Hamiltonian $H = H(p, q)$ which is integrable on an open subset $\mathcal{U}$ of the energy surface $\{H = 1/2\}$. Moreover in $\{H = 1/2\} \setminus \mathcal{U}$ the flow of H has horseshoes and hence chaotic behavior, which in turn means that there is no analytic first integral (independent of H) on the whole energy surface.

This is a work in progress with V. Kaloshin.

Walter Craig (McMaster University, Hamilton)

Birkhoff normal form for nonlinear wave equations

Theorems on global existence of solutions of nonlinear wave equations in $\mathbb{R}^n$ depend upon a competition between the time decay of solutions and the degree of the nonlinearity. Decay estimates are more effective when inessential nonlinear terms are able to be removed through a well-chosen transformation. In this talk, we construct Birkhoff normal forms transformations for the class of wave equations which are Hamiltonian PDEs and null forms, giving a new proof via canonical transformations of the global existence theorems for null form wave equations of S. Klainerman and J. Shatah in space dimensions $n \geq 3$. The critical case $n = 2$ is also under consideration. These results are work-in-progress with A. French and C.-R. Yang.

Jean-Marc Delort (Université Paris 13)

Almost global solutions for the capillarity wave equation with small periodic data

We prove that the capillarity waves equation in one dimension and finite depth has solutions over time intervals of length $c_N \epsilon^{-N}$ for any N, if the Cauchy data are of small size ϵ and space periodic, and if the gravity, or the surface tension, is taken outside a subset of zero measure. The proof relies on normal forms and on the use of the reversibility of the equation. This is joint work with Massimiliano Berti.

Patrick Gérard (Université Paris-Sud)

Quasiperiodic solutions of the cubic Szego equation

We classify all the quasiperiodic solutions of the cubic Szego equation in the natural Dirichlet energy space. In particular, we prove that there are many more quasiperiodic solutions than the rational solutions, and that one can choose them as singular as possible.

The proof relies on the representation formula of solutions recently established in collaboration with Sandrine Grellier.

Benoît Grébert (Université de Nantes)

On reducibility of quantum harmonic oscillator on $\mathbb{R}^d$ with quasiperiodic in time potential

We prove that a linear d-dimensional Schrödinger equation on $\mathbb{R}^d$ with harmonic potential $|x|^2$ and small t-quasiperiodic potential

$$i\partial_t u - \Delta u + |x|^2 u + \varepsilon V(t\omega, x)u = 0, \quad x \in \mathbb{R}^d$$

reduces to an autonomous system for most values of the frequency vector $\omega \in \mathbb{R}^n$. As a consequence any solution of such a linear PDE is almost periodic in time and remains bounded in all Sobolev norms. (joint work with E. Paturel)

Marcel Guardia (Universitat Politècnica de Catalunya, Barcelona)

Growth of Sobolev norms for the analytic non-linear Schrödinger equation

Consider the completely resonant non-linear Schrödinger equation on the two dimensional torus with any analytic gauge invariant nonlinearity. Fix s>1. We show the existence of solutions of this equation which achieve arbitrarily large growth of H^s Sobolev norms. We also give estimates for the time required to attain this growth. This is a joint work with Emanuele Haus and Michela Procesi.

Vadim Kaloshin (University of Maryland)

Stochastic Arnold diffusion of deterministic systems

During the talk we describe a class of nearly integrable deterministic systems, of Arnold's example type, where we prove stochastic diffusive behavior. Namely, we show that distributions given by deterministic evolution of certain random initial conditions weakly converge to a diffusion process. This result is conceptually different from known mathematical results, where existence of "diffusing orbits" is shown. This work is based on joint papers with O. Castejon, M. Guardia, J. Zhang, and K. Zhang.

Thomas Kappeler (Universität Zürich)

Canonical coordinates with tame estimates for integrable PDEs

In a case study for integrable PDEs, we construct canonical coordinates for the defocusing NLS equation on the circle, taylored to the needs in perturbation theory. The coordinates are defined in neighbourhoods of families of finite dimensional invariant tori and satisfy together with their derivatives tame estimates. In these coordinates, the dNLS Hamiltonian is in normal form up to order three. This is joint work with Riccardo Montalto.

Riccardo Montalto (Universität Zürich)

On the exact controllability for quasi-linear Hamiltonian NLS equation

In this talk I will show how to prove the exact controllability for quasi-linear Hamiltonian NLS equations. The proof is based on a refined version of the Nash-moser Theorem proved by P. Baldi and E. Haus. The controllability of the linearized problem is proved by the Hilbert Uniqueness method (HUM method) using a reduction to constant coefficients up to order 0 of the linearized NLS equation. This is a joint work with P. Baldi and E. Haus.