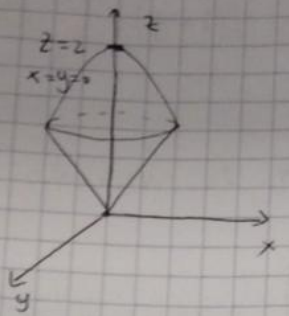

E è l'unione di un tronco di cono e un tronco di paraboloide

in coordinate cilindriche

$$\{\ 0 \le \theta < 2\pi \quad , \quad 0 \le \rho \le 1 \quad , \quad \rho \le z \le 2-\rho^2\ \}$$

1a) Calcolo della superficie del paraboloide ∂S_1

parametrizzo come

$$\rho,\theta \to \varphi(\rho,\theta) \begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = 2-\rho^2 \end{cases} \qquad 0 \le \rho \le 1 \quad 0 \le \theta < 2\pi$$

$$\varphi_\rho = \begin{pmatrix} \cos\theta \\ \sin\theta \\ -2\rho \end{pmatrix} ; \quad \varphi_\theta = \begin{pmatrix} -\rho\sin\theta \\ \rho\cos\theta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} i & j & k \\ \cos\theta & \sin\theta & -2\rho \\ -\rho\sin\theta & \rho\cos\theta & 0 \end{pmatrix} \qquad \begin{pmatrix} 2\rho^2\cos\theta \\ 2\rho^2\sin\theta \\ \rho \end{pmatrix} = \varphi_\rho \wedge \varphi_\theta$$

$$|\varphi_\rho \wedge \varphi_\theta| = \sqrt{4\rho^4+\rho^2} = \rho\sqrt{1+4\rho^2}$$

$$|\partial S_1| = \int_0^{2\pi} d\theta \int_0^1 d\rho\, |\varphi_\rho \wedge \varphi_\theta| = \cancel{\sqrt{5}\cdot 2\pi \cdot \frac{1}{2} = \sqrt{5}\pi} \quad 2\pi\int_0^1 \rho\sqrt{1+4\rho^2}\,d\rho$$

$$\partial S_1 = \frac{\pi}{4}\int_0^4 \sqrt{1+y}\; dy = \frac{\pi}{6}\left[(1+y)^{3/2}\right]_0^4 = \frac{\pi}{6}\left(5\sqrt{5}-1\right)$$

Area del cono [NOTA, la ricalcolo]

$$\rho,\theta \rightarrow \varphi(\rho,\theta) = \begin{pmatrix} \rho\cos\theta \\ \rho\sin\theta \\ \rho \end{pmatrix}$$

$$\varphi_\rho = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} \qquad \varphi_\theta = \begin{pmatrix} -\rho\sin\theta \\ \rho\cos\theta \\ 0 \end{pmatrix}$$

$$\varphi_\rho \wedge \varphi_\theta = \begin{pmatrix} -\rho\cos\theta \\ -\rho\sin\theta \\ \rho \end{pmatrix} \qquad |\varphi_\rho \wedge \varphi_\theta| = \sqrt{2}\,\rho$$

$$|\partial S_2| = \int_0^1\int_0^{2\pi} \sqrt{2}\,\rho\; d\rho\, d\theta = 2\pi\cdot\sqrt{2}\left[\frac{\rho^2}{2}\right]_0^1 = \sqrt{2}\,\pi$$

$$\frac{\pi}{6}\left(5\sqrt{5}-1\right) + \sqrt{2}\,\pi = |\partial E|$$

Sempre in coordinate cilindriche.

$$\int_E (x^2+y^2+z^2)\,dx\,dy\,dz = \int_0^1 \rho\, d\rho \int_0^{2\pi} d\theta \int_\rho^{2-\rho^2} (\rho^2+z^2)\,dz =$$

$\frac{\pi}{6}(5\sqrt{5}-1)$

$$2\pi \int \rho \, d\rho \left[\rho^2 z + \frac{z^3}{3}\right]_{z=\rho}^{z=2-\rho^2} =$$

$$2\pi \int \rho \, d\rho \left[\rho^2(2-\rho^2-\rho) + \frac{(2-\rho^2)^3}{3} - \frac{\rho^3}{3}\right]$$

$$2\pi \int \left[\rho^3(2-\rho^2-\rho) + \frac{\rho}{3}(8-\rho^6-12\rho^2+6\rho^4) - \frac{\rho^4}{3}\right] d\rho$$

$$= 2\pi \int \left[-\frac{\rho^7}{3} + \rho^5 - \frac{4}{3}\rho^4 - 2\rho^3 + \frac{8}{3}\rho\right] d\rho$$

$$= 2\pi \left(-\frac{1}{24}\rho^8 + \frac{\rho^6}{6} - \frac{4}{15}\rho^5 - \frac{1}{2}\rho^4 + \frac{4}{3}\rho^2\right]_0^1 =$$

$$2\pi\left(-\frac{1}{24} + \frac{1}{6} - \frac{4}{15} - \frac{1}{2} + \frac{4}{3}\right)$$

$$2\pi \, \frac{-5+20-32-60+160}{120} = \frac{83}{60}\pi$$

2) Parametrizzo la ~~[illegible]~~ sfera.

$0 \leq \theta \leq \pi \qquad 0 \leq \varphi \leq 2\pi$

$$S \begin{pmatrix} 2\sin\theta\cos\varphi \\ 2\sin\theta\sin\varphi \\ 2\cos\theta \end{pmatrix}$$

=

$$S_\theta = \begin{pmatrix} 2\cos\theta\cos\varphi \\ 2\cos\theta\sin\varphi \\ -2\sin\theta \end{pmatrix} \qquad S_\varphi = \begin{pmatrix} -2\sin\theta\sin\varphi \\ 2\sin\theta\cos\varphi \\ 0 \end{pmatrix}$$

$$\vec{M} = \int 2\sin\theta \begin{pmatrix} 2\sin\theta\cos\varphi \\ 2\sin\theta\sin\varphi \\ 2\cos\theta \end{pmatrix} \qquad \cdot \sin\theta \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$