PARTE 2. SOLUZIONI

1. $$\int x\left(\ln(x)\right)^2 dx = \frac{x^2}{2}\left(\ln(x)\right)^2 -$$

$$\int \frac{x^2}{2} \cdot \frac{2\ln(x)}{x}\, dx = \frac{x^2}{2}\left(\ln(x)\right)^2 - \int x\ln(x)\, dx$$

$$= \frac{x^2}{2}\left(\ln(x)\right)^2 - \left(\frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\, dx\right) =$$

$$= \left\{\frac{x^2}{2}\left(\ln(x)\right)^2 - \frac{x^2}{2}\ln(x) - \frac{x^4}{4} + c\right\}_{c\in\mathbb{R}}$$

2. $$\int \frac{dx}{x^2+2x+1+4} = \frac{1}{4}\int \frac{dx}{\left(\frac{x+1}{2}\right)^2+1} =$$

$$= \frac{1}{2}\int \frac{dy}{y^2+1}\Bigg|_{y=\frac{x+1}{2}} = \left\{\frac{1}{2}\operatorname{arctg}\left(\frac{x+1}{2}\right) + c\right\}_{c\in\mathbb{R}}$$

3. $$\int \frac{\cos(x)+\sin(x)}{1+\cos(x)}\, dx =$$

$$\int \frac{1+\cos x}{[illegible]}\, dx + \int \frac{\sin(x)\, dx}{[illegible]} - \int \frac{dx}{1+\cos x} =$$

$$\int \frac{\ldots}{1+\cos x} \quad \int \frac{\ldots}{1+\cos(x)} \quad \int \frac{\ldots}{1+\cos x}$$

$$= x - \int \frac{dv}{1+v}\Bigg|_{v=\cos x} - \int \frac{dx}{1+\cos x} =$$

$$= x - \ln(1+\cos(x)) - 2\int \frac{dt}{(1+t^2)\left(1+\frac{1-t^2}{1+t^2}\right)}\Bigg|_{t=\operatorname{tg}\left(\frac{x}{2}\right)} =$$

$$= x - \ln(1+\cos(x)) - \int dt\Big|_{t=\operatorname{tg}\left(\frac{x}{2}\right)} =$$

$$= \left\{x - \ln(1+\cos x) - \operatorname{tg}\left(\frac{x}{2}\right) + c\right\}_{c\in\mathbb{R}}$$

4. $x^4 + x^2 + 1 = (x^3 + x^2 + 2x + 2)(x-1) + 3$

$$\int \frac{x^4+x^2+1}{x-1} = \left\{\frac{x^4}{4} + \frac{x^3}{3} + x^2 + 2x + 3\ln|x-1| + c\right\}_c$$

5. $* = \lim\limits_{x\to 0} \dfrac{\tan(x) - \operatorname{sen}(x)}{x - \operatorname{sen} x} = \lim\limits_{x\to 0} \dfrac{f(x)}{g(x)}$

① $f(x), g(x) \longrightarrow 0$ per $x \to 0$

② $g'(x) = 1 - \cos(x) > 0$ per $x \in \mathring{I}(0)$

Applico teorema del l'Hopital

$$* = \lim_{x\to 0} \frac{\frac{1}{\cos^2 x} - \cos x}{1 - \cos(x)} =$$

$$1 - \cos^3(x)$$

$$\lim_{x\to 0} \frac{1}{\cos^2(x)} \cdot \lim_{x \to 0} \frac{1-\cos\ldots}{1-\cos(x)} =$$

$$= \lim_{x\to 0} \frac{(1-\cos x)\,(1+\cos^2(x)+\cos x)}{(1-\cos x)} = 3$$

6. $f(x) := x^2 - \ln(1+x) = 0$

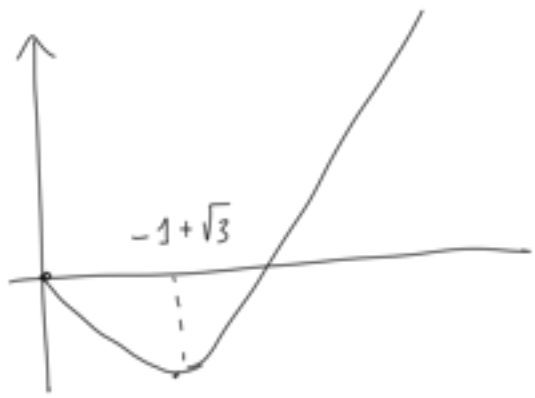
Noto che $f(0) = 0$ e

$$\lim_{x\to\infty} f(x) = \infty$$

calcolo $f'(x) = 2x - \frac{1}{1+x} = \frac{2x^2+2x-1}{1+x}$

$f'(x) = 0 \Rightarrow x = -1 \pm \sqrt{1+2}$

(N.B. la funzione è definita per $x > -1$)



(N.B. $-1+\sqrt{3} > 0$!)

7. $f(x) = x\,(x^2-4)^{-\frac{1}{3}}$

$$f'(x) = 1 \cdot (x^2-4)^{-\frac{1}{3}} + x \cdot \left[-\frac{1}{3}(x^2-4)^{-\frac{1}{3}-1} \cdot (2x)\right]$$

$$= (x^2-4)^{-\frac{1}{3}} - \frac{2}{3}x^2(x^2-4)^{-\frac{4}{3}}$$