Polinomi di Taylor. ordine 3.

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 +$$

$$\frac{1}{6} f'''(x_0)(x-x_0)^3 + R_3(x)$$

$$\left[\lim_{x \to x_0} \frac{R_3(x)}{(x-x_0)^3} = 0 \right]$$

Nel "compito" $f(x) = \ln(x^2+1)$; $x_0 = 0$

$$f'(x) = \frac{2x}{x^2+1} \ ; \ f''(x) = \frac{2}{x^2+1} - \frac{(2x)^2}{(x^2+1)^2} \ ;$$

$$f'''(x) = -\frac{2 \cdot 2x}{(x^2+1)^2} - \frac{4 \cdot 2x}{(x^2+1)^2} + 2\frac{(2x)^3}{(x^2+1)^3}$$

$$f(0) = 0 ; \ f'(0) = 0 ; \ f''(0) = 2 - 0 ; \ f'''(0) = 0$$

$$f(x) = 0 + 0 \cdot (x-0) + \frac{1}{2} \cdot 2(x-0)^2 + \frac{1}{6} 0 \cdot (x-0)^3 + R_3(x)$$

$$= x^2 + R_3(x)$$

$$\lim_{x \to 0} \frac{f(x) - x^2 + x^3}{x^3} = \lim_{x \to 0} \frac{R_3(x) + x^3}{x^3} = 1$$

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Studio di funzione

$$f(x) = \frac{x+1}{}$$

x^2+3

Dominio è $D=\mathbb{R}$.

$$\lim_{x\to\infty}\frac{x+1}{x^2+3} = \lim_{x\to\infty}\frac{x}{x^2} = 0$$

$$\lim_{x\to-\infty}\frac{x+1}{x^2+3} = 0$$

$f(x)>0$ se $x>-1$

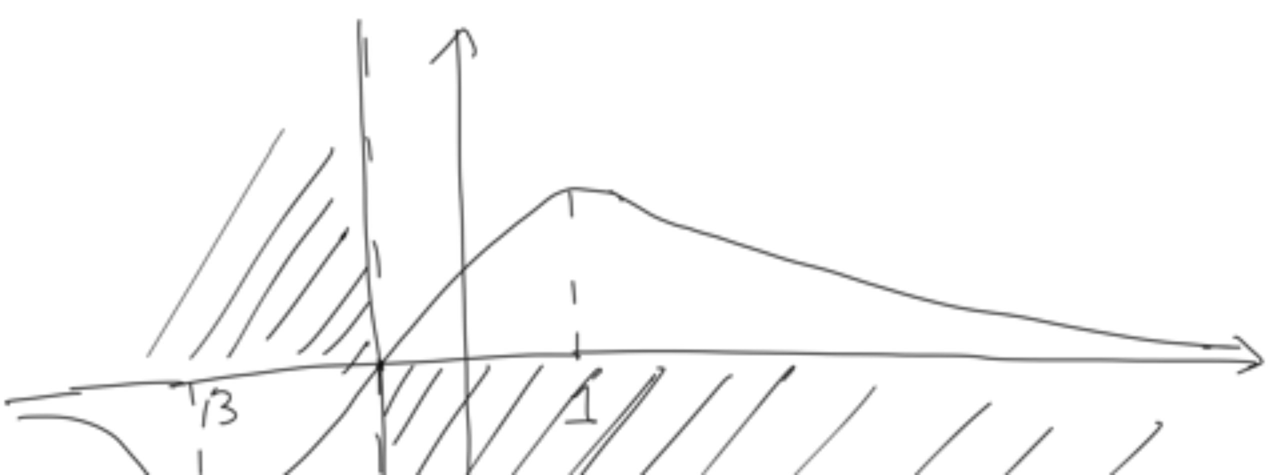
$$f'(x) = \frac{1}{x^2+3} - \frac{(x+1)2x}{(x^2+3)^2} = \frac{x^2+3-2x^2-2x}{(x^2+3)^2} =$$

$$= \frac{-x^2-2x+3}{(x^2+3)^2}$$

$f'(x)\geq 0 \iff -x^2-2x+3\geq 0$

$$x_{1,2} = \frac{1\pm\sqrt{1+3}}{-1} = \begin{cases} -3 \\ 1 \end{cases} \qquad -3\leq x\leq 1$$

Grafico qualitativo



$$\int \frac{x+1}{x^2+3}\,dx = \int \frac{x\,dx}{x^2+3} + \int \frac{dx}{x^2+3} = \frac{1}{2}\ln(|x^2+3|) + \frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right)$$

$$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{dy}{y}\Big|_{y=x^2+3} = \frac{1}{2}\ln(|x^2+3|)$$

$$\int \frac{dx}{x^2+3} = \int \frac{dx}{3\left(\frac{x^2}{3}+1\right)} = \frac{1}{3}\int \frac{dx}{\left(\frac{x^2}{3}+1\right)} =$$

$$\left(y^2 = \frac{x^2}{3} \iff y = \frac{x}{\sqrt{3}} \qquad dy = \frac{dx}{\sqrt{3}} \right)$$

$$= \frac{1}{3}\cdot\sqrt{3}\cdot\int \frac{dy}{y^2+1}\Big|_{y=\frac{x}{\sqrt{3}}} = \frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right)$$

Esercizio 0

$$\lim_{x\to-\infty} f(x) = +\infty; \quad \lim_{x\to\infty} f(x) = -2$$

$$f((3,5)) = (0,2]$$

$$f'(4) = 0 \quad ; \quad f'(1) < 0$$