

COHOMOLOGICAL SUPPORT LOCI FOR ABEL-PRYM CURVES

S. CASALAINA MARTIN - M. LAHOZ - F. VIVIANI

For an Abel-Prym curve contained in a Prym variety, we determine the cohomological support loci of its twisted ideal sheaves and the dimension of its theta-dual.

Introduction

The purpose of this paper is to study the theta-dual and the cohomological support loci for the twisted ideal sheaves of an Abel-Prym curve contained in the Prym variety associated to an étale double cover of smooth projective non-hyperelliptic curves.

Recall that given a coherent sheaf \mathcal{F} on a smooth projective variety X , the i -th cohomological support locus of \mathcal{F} is

$$V^i(\mathcal{F}) := \{ \alpha \in \text{Pic}^0(X) \mid h^i(X, \mathcal{F} \otimes \alpha) > 0 \} \subset \text{Pic}^0(X).$$

These loci have been studied in a number of contexts, and were considered for example by Green-Lazarsfeld (see [5, 6]) in order to prove a generic vanish-

Entrato in redazione 22 febbraio 2008

AMS 2000 Subject Classification: 14H40, 14K12, 14F17

Keywords: Prym varieties, Abel-Prym curves, cohomological support loci, generic vanishing, theta-dual

The first author was partially supported by NSF MSPRF grant DMS-0503228. The second author has been supported by Ministerio de Educación y Ciencia, beca de Formación del Profesorado Universitario, MTM2006-14234-C02 and 2005SGR-00557.

ing theorem for the canonical sheaf of irregular complex varieties. More precisely, they proved that if the Albanese morphism $a : X \rightarrow \text{Alb}(X) \cong \widehat{\text{Pic}}^0(X)$ has generic fiber of dimension k , then

$$\text{codim}_{\text{Pic}^0(X)} V^i(\omega_X) \geq i - k.$$

In a sequence of articles ([11, 17, 18] cf. [16]), G. Pareschi and M. Popa have studied similar questions, and have introduced the notion of a GV_k sheaf (see also [7]):

Definition. *A coherent sheaf \mathcal{F} is said to be GV_k for some $k \in \mathbb{Z}$ (which stands for generic vanishing of order k) if*

$$\text{codim}_{\text{Pic}^0(X)} V^i(\mathcal{F}) \geq i - k \text{ for all } i > 0. \tag{*}$$

Observe that we have the natural inclusions between the GV_k -sheaves:

$$(W)IT_0 = GV_{-d} \subset \cdots \subset GV_{-1} = M \subset GV_0 = GV \subset \cdots \subset GV_d = \text{Coh}(\mathcal{F})^1,$$

where the sequence becomes stationary outside the interval $[-d, d]$ for $d = \dim(X)$. With this terminology, the result of M. Green and R. Lazarsfeld says that if the Albanese morphism has generic fiber of dimension k , then ω_X is GV_k .

The above condition (*) can be expressed in terms of the Fourier-Mukai transform with respect to the kernel $\mathcal{P} = (a, \text{id})^*(\mathcal{L}) \in \mathbb{D}(X \times \hat{A})$, where \mathcal{L} is the Poincaré line bundle on $A \times \hat{A}$ (suitably normalized) and $A = \text{Alb}(X)$. G. Pareschi and M. Popa use this to study vanishing in a variety of contexts, and in particular when \mathcal{F} is an adjoint linear series or an ideal sheaf suitably twisted; this has produced many interesting applications (see [12–14, 17, 18]).

When $X = A$ is an abelian variety, the case we will be considering here, $\text{Pic}^0(A)$ is canonically isomorphic to \hat{A} , and so $V^i(\mathcal{F})$ can be considered as a subspace inside the dual abelian variety. Moreover, in the case of a principally polarized abelian variety (A, Θ) , which we will abbreviate by ppav in the sequel, we can view the cohomological support loci as subspaces of the original abelian variety using the isomorphism $\varphi_\Theta : A \xrightarrow{\sim} \hat{A}$. In this setting, Pareschi-Popa introduced the following definition of theta-dual (see [18, Def. 4.2]), which can be viewed as the cohomological support locus of a twisted ideal sheaf.

¹In the above chains of inclusions we have also indicated some other names that are used in the literature, namely: the sheaves satisfying GV_0 are also called GV -sheaves (which stands for generic vanishing sheaves), the GV_{-1} -sheaves are called M -regular sheaves (which stands for Mukai regular sheaves) and the GV_{-d} -sheaves are called $(W)IT_0$ -sheaves because they satisfy $V^i(\mathcal{F}) = \emptyset$ for all $i > 0$, or in other words they satisfies the (weak) index theorem with index 0 in Mukai’s terminology.

Definition. Given a closed subscheme X of a ppav (A, Θ) , the dual theta-dual $V(X)$ of X is the closed subset² defined by:

$$V(X) := V^0(\mathcal{I}_X(\Theta)) = \{\alpha \in \hat{A} \mid h^0(A, \mathcal{I}_X(\Theta) \otimes \alpha) \neq 0\} \subset \hat{A} \cong \text{Pic}^0(A).$$

Observe that, via the principal polarization Θ , the theta-dual $V(X)$ can be canonically identified with the locus $\{a \in A \mid X \subset \Theta_a = t_a^* \Theta\}$ of theta-translates containing X (here $t_a : A \rightarrow A$ is the translation map).

More generally, we will be interested in the cohomological support loci of the form $V^i(\mathcal{I}_X(n\Theta))$. In particular, in this paper we consider the cohomological support loci associated to the ideal sheaves of Abel-Prym curves. To fix some notation, let P be the Prym variety of dimension $g - 1$ associated to the étale double cover $\tilde{C} \rightarrow C$ of irreducible smooth projective non-hyperelliptic curves of genus $\tilde{g} = 2g - 1$ and $g \geq 3$, respectively. Let Ξ be the canonical principal polarization. Since \tilde{C} is not hyperelliptic, there is an embedding $\tilde{C} \hookrightarrow P$, unique up to translation. We denote by $\mathcal{I}_{\tilde{C}}$ the ideal sheaf of \tilde{C} inside P (see section 1 for the definition and the standard notations). Our main result is (combining Theorems 2.2, 3.1, 4.2):

Theorem A. Let \tilde{C} be a non-hyperelliptic Abel-Prym curve embedded in its Prym variety (P, Ξ) . Then we have:

1. The theta-dual of \tilde{C} has dimension

$$\dim(V(\tilde{C})) = \dim P - 3 = g - 4.$$

2. The cohomological support loci of $\mathcal{I}_{\tilde{C}}(\Xi)$ can be (non-canonically) identified with

$$\begin{cases} V^0(\mathcal{I}_{\tilde{C}}(\Xi)) = V^1(\mathcal{I}_{\tilde{C}}(\Xi)) = V(\tilde{C}), \\ V^2(\mathcal{I}_{\tilde{C}}(\Xi)) = P, \\ V^{\geq 3}(\mathcal{I}_{\tilde{C}}(\Xi)) = \emptyset. \end{cases}$$

3. The cohomological support loci of $\mathcal{I}_{\tilde{C}}(2\Xi)$ can be (non-canonically) identified with

$$\begin{cases} V^0(\mathcal{I}_{\tilde{C}}(2\Xi)) = \begin{cases} P & \text{if } g \geq 4, \\ \{0\} & \text{if } g = 3, \end{cases} \\ V^1(\mathcal{I}_{\tilde{C}}(2\Xi)) = V^2(\mathcal{I}_{\tilde{C}}(2\Xi)) = \{0\}, \\ V^{\geq 3}(\mathcal{I}_{\tilde{C}}(2\Xi)) = \emptyset. \end{cases}$$

²It is possible to put a canonical schematic structure on $V(X)$ (see [18, Def. 4.2]), which however will never play a role in this paper.

The above theorem follows by combining Theorems 2.2, 3.1, 4.2 together with the identifications that we make in formulas (1) and (2) below. As an immediate Corollary we obtain:

Corollary B. *With the notation of the above Theorem, the following hold*

- (i) $\mathcal{I}_{\tilde{C}}(\Xi)$ is not GV . More precisely, it is GV_2 but not GV_1 .
- (ii) $\mathcal{I}_{\tilde{C}}(2\Xi)$ is GV , but not IT_0 . More precisely, it is $GV_{-(g-2)}$ but not $GV_{-(g-1)}$.
- (iii) $\mathcal{I}_{\tilde{C}}(m\Xi)$ is IT_0 for every $m \geq 3$.

Statements (i) and (ii) follow by direct inspection from the theorem, while part (iii) follows from part (ii) and the general fact that if \mathcal{F} is a GV -sheaf then $\mathcal{F}(\Theta)$ is IT_0 (see [18, Lemma 3.1]).

These results should be compared to the case of an Abel-Jacobi curve. Recall that for any non-rational curve C , the Abel-Jacobi map gives a non-canonical embedding of C in its canonically polarized Jacobian $(J(C), \Theta)$. In this case, the cohomological support loci can be (non-canonically) identified with (combine [12, Thm. 4.1, Prop. 4.4] and [18, Lem. 3.3, Exa. 4.5])

$$\begin{cases} V(C) = V^0(\mathcal{I}_C(\Theta)) = V^1(\mathcal{I}_C(\Theta)) = W_{g-2}, \\ V^0(\mathcal{I}_C(n\Theta)) = J(C) \text{ for } n \geq 2, \\ V^{\geq 2}(\mathcal{I}_C(\Theta)) = V^{\geq 1}(\mathcal{I}_C(n\Theta)) = \emptyset \text{ for } n \geq 2, \end{cases}$$

where $W_{g-2} = W_{g-2}^0$ is the Brill-Noether locus of line bundles of degree $g - 2$ with non-trivial global sections. From the above description, we get that for an Abel-Jacobi curve

- (i) $\mathcal{I}_C(\Theta)$ is GV , but not IT_0 .
- (ii) $\mathcal{I}_C(m\Theta)$ is IT_0 for every $m \geq 2$.
- (iii) $\dim(V(C)) = g - 2 = \dim JC - 2$.

In [18], Pareschi-Popa have proved that the above condition (i) characterizes Abel-Jacobi curves among the non-degenerate curves inside a ppav. Moreover, they have further conjectured that the conditions (ii) and (iii) should also provide new characterizations of Abel-Jacobi curves³.

³We refer to [18] for analogous conjectures concerning the subvarieties of a ppav of minimal cohomological class.

The results in this paper show that this conjecture is not violated by Abel-Prym curves, which in a sense are the curves inside a ppav closest to the Abel-Jacobi curves.

In addition, from the results on Abel-Prym curves above, and those cited for Abel-Jacobi curves, it seems natural to ask for the relation between the following conditions on a curve X on a ppav (A, Θ) of dimension g :

- (1) $\mathcal{I}_X(e\Theta)$ is GV , but not IT_0 .
- (2) $\mathcal{I}_X((e + 1)\Theta)$ is IT_0 , but $\mathcal{I}_X(e\Theta)$ is not.
- (3) $\dim V(X) = g - e - 1$.
- (4) X is an Abel-Prym-Tyurin curve with Prym-Tyurin variety $(A, \Theta) \subset (JX, \Theta_X)$ of exponent e , that is $[X] = \frac{e[\Theta]^{g-1}}{(g-1)!}$.

Prym-Tyurin varieties of exponent 1 are precisely the Jacobians (by the Matsusaka-Ran criterion), so that the Pareschi-Popa conjecture states that the above conditions are all equivalent for $e = 1$. In the next case, it is known (see [20]) that the closure inside the moduli space \mathcal{A}_g of ppav’s of dimension g of the Prym-Tyurin varieties of exponent 2 has a unique component of maximal dimension (which is $3g$), namely the closure of the classical Prym varieties⁴. Therefore, our results in Theorem A show that “most” of the Abel-Prym-Tyurin curves of exponent 2 satisfies the conditions (1), (2) and (3).

On the other hand, Andreas Hoering has pointed out to us that condition (3) is much weaker than condition (4): any curve X on a subvariety Y with $\dim(Y) > 1$ and $\dim V(Y) = g - e - 1$ will have $\dim V(X) \geq g - e - 1$. Since Y will contain curves of arbitrarily high degree with respect to Θ , one can construct curves satisfying (3) but not (4). As a concrete example, consider a curve X lying on a W_d ($1 < d < e + 1$) inside a Jacobian or on the Fano surface inside the intermediate Jacobian of a cubic threefold. Thus we propose an alternate version of (3), which may be more closely related to the other conditions

- (3') $\dim V(X) = g - e - 1$ and X is not contained in a subvariety Y with $1 < \dim(Y) < e + 1$ and $\dim V(Y) = g - e - 1$.

Since an Abel-Prym curve of the intermediate Jacobian of a cubic threefold lies on the Fano surface F , which has class $[\Theta]^3/3!$ and $\dim V(F) = g - 3$, (4) does not imply (3'), and so we suggest the following modification of (4) as well:

⁴Among the Prym-Tyurin ppav of dimension g and exponent 2, the classical Prym varieties can also be characterized as those for which the curve X is smooth of maximal arithmetic genus, namely $2g + 1$.

- (4') X is an Abel-Prym-Tyurin curve of exponent e and X is not contained in a subvariety Y with $1 < \dim(Y) = d < e + 1$ and class $[Y] = \alpha \frac{[\Theta]^{g-d}}{(g-d)!}$ with $\alpha < e$.

The paper is organized as it follows. In section 2, we review the definition of the Prym variety (P, Ξ) associated to an étale double cover $\tilde{C} \rightarrow C$ in order to fix the notation used throughout the paper. In section 3, we prove that the theta-dual of an Abel-Prym curve \tilde{C} inside P can be set-theoretically identified with the Brill-Noether locus V^2 defined in [19]. General results about these Brill-Noether ([2, 4]) give the inequality $\dim V(\tilde{C}) \geq \dim(P) - 3$. Using ideas from [10, sections 6, 7], we show that the equality holds (Theorem 2.2), which proves part (1) of the Main Theorem A. In section 4 and 5, we compute the cohomological support loci for the twisted ideal sheaves $\mathcal{I}_{\tilde{C}}(\Xi)$ (Theorem 3.1) and $\mathcal{I}_{\tilde{C}}(2\Xi)$ (Theorem 4.2), proving explicitly parts (2) and (3) of the Main Theorem A.

1. Notation and basic definitions

Throughout this paper, we work over an algebraically closed field k of characteristic different from 2. The basic results cited here are due to Mumford [9]. Let $\pi : \tilde{C} \rightarrow C$ be an étale double cover of irreducible smooth projective curves of genus \tilde{g} and g , respectively. By the Hurwitz formula, we get that $\tilde{g} = 2g - 1$. We denote by σ the involution on \tilde{C} associated to the above double cover. Consider the norm map

$$\begin{aligned} \text{Nm} : \text{Pic}(\tilde{C}) &\longrightarrow \text{Pic}(C) \\ \mathcal{O}_{\tilde{C}}(\sum_j r_j p_j) &\longmapsto \mathcal{O}_C(\sum_j r_j \pi(p_j)). \end{aligned}$$

The kernel of the norm map has two connected components

$$\ker \text{Nm} = P \cup P' \subset \text{Pic}^0(\tilde{C}),$$

where P is the component containing the identity element and is, by definition, the Prym variety associated to the étale double cover π . The above components P and P' have the following explicit description

$$\begin{aligned} P &= \left\{ \mathcal{O}_{\tilde{C}}(D - \sigma(D)) \mid D \in \text{Div}^{2N}(\tilde{C}), N \geq 0 \right\}, \\ P' &= \left\{ \mathcal{O}_{\tilde{C}}(D - \sigma(D)) \mid D \in \text{Div}^{2N+1}(\tilde{C}), N \geq 0 \right\}. \end{aligned}$$

It is often useful to consider the inverse image of the canonical line bundle of C via the norm map. This also has two connected components

$$\text{Nm}^{-1}(\omega_C) = P^+ \cup P^- \subset \text{Pic}^{2g-2}(\tilde{C}) = \text{Pic}^{\tilde{g}-1}(\tilde{C}),$$

which have the following explicit description

$$P^+ = \{L \in \text{Nm}^{-1}(\omega_C) \mid h^0(L) \equiv 0 \pmod{2}\},$$

$$P^- = \{L \in \text{Nm}^{-1}(\omega_C) \mid h^0(L) \equiv 1 \pmod{2}\}.$$

The above varieties P' , P^+ and P^- are isomorphic to the Prym variety P and, in this work, we will pass frequently from one to another.

There is a principal polarization $\Xi \in \text{NS}(P)$ induced by the principal polarization $\Theta_{\tilde{C}} \in \text{NS}(\tilde{J}\tilde{C})$. In fact, $\Theta_{\tilde{C}}|_P = 2\Xi$. One of the primary motivations for considering P^+ is the existence of a canonically defined divisor Ξ^+ whose class in the Neron-Severi group of P is Ξ :

$$\Xi^+ = \left\{L \in P^+ \subset \text{Pic}^{\tilde{g}-1}(\tilde{C}) \mid h^0(L) > 0\right\} \subset P^+.$$

On the other hand, the canonical Abel-Prym map is defined as

$$\begin{aligned} i: \tilde{C} &\longrightarrow P' \\ p &\longmapsto \sigma(p) - p. \end{aligned}$$

If \tilde{C} is hyperelliptic then the image of \tilde{C} via the Abel-Prym map is a smooth hyperelliptic curve D and the Prym variety P is isomorphic to the Jacobian $J(D)$ of D ([3, Cor. 12.5.7]). On the other hand, if C is hyperelliptic but \tilde{C} is not, then the Prym variety P is the product of two hyperelliptic Jacobians (see [10]). Therefore, since we are mostly interested in the case of an irreducible non-Jacobian ppav, we will assume throughout this paper that C is not hyperelliptic (and in particular $g \geq 3$). Note that under this hypothesis, the Abel-Prym map is an embedding ([3, Cor. 12.5.6]).

Since the Abel-Prym curve $\tilde{C} \subset P'$ and the polarization $\Xi^+ \subset P^+$ lie canonically in different spaces, the cohomological support loci for the twisted ideal sheaf $\mathcal{I}_{\tilde{C}}(n\Xi^+)$ is only defined up to a translation since we have to choose a way to translate \tilde{C} and Ξ^+ inside the Prym variety P . For this reason, we introduce the following auxiliary (canonically defined) loci

$$\tilde{V}^i(\mathcal{I}_{\tilde{C}}(n\Xi^+)) = \{E \in P^- \mid h^i(P', \mathcal{I}_{\tilde{C}}(n\Xi^+_E)) > 0\} \subset P^-, \tag{1}$$

where $\mathcal{I}_{\tilde{C}}$ is the ideal sheaf of \tilde{C} inside P' and for $E \in P^- \subset \text{Pic}^{\tilde{g}-1}(\tilde{C})$, we denote by $\Xi^+_E \subset P'$ the translate of the canonical theta divisor Ξ^+ by E^{-1} . The relation between $\tilde{V}^i(\mathcal{I}_{\tilde{C}}(n\Xi^+))$ and $V^i(\mathcal{I}_{\tilde{C}}(n\Xi))$ is easy to work out. In fact there is a choice of translate of $\tilde{C} \subset P$ and $\Xi \subset P$ so that under the isomorphism

$$\begin{aligned} \psi_{E_0}: P^- &\longrightarrow P \\ E &\longmapsto E \otimes E_0^{-1} \end{aligned}$$

induced by a line bundle $E_0 \in P^-$, we have

$$V^i(\mathcal{I}_{\tilde{C}}(n\Xi)) = \tilde{V}^i(\mathcal{I}_{\tilde{C}}(n\Xi^+))^{\otimes n} \otimes E_0^{\otimes -n} := \{(E \otimes E_0^{-1})^{\otimes n} \mid E \in \tilde{V}^i(\mathcal{I}_{\tilde{C}}(n\Xi^+))\}.$$

For this reason, we will also identify the cohomological support loci with the following canonical loci:

$$V^i(\mathcal{I}_{\tilde{C}}(n\Xi)) = \tilde{V}^i(\mathcal{I}_{\tilde{C}}(n\Xi^+))^{\otimes n} \subset \text{Nm}^{-1}(\omega_{\tilde{C}}^{\otimes n}) \subset \text{Pic}^{n(\tilde{g}-1)}(\tilde{C}). \quad (2)$$

For later use, we end this section with the following Lemma, which describes the restriction of the translates of the theta-divisor to the Abel-Prym curve.

Lemma 1.1. *Given any $E \in P^-$, there is an isomorphism of line bundles*

$$\mathcal{O}_{P'}(\Xi_E^+)_{|\tilde{C}} \cong E.$$

Moreover, if $E \in P^- - V(\tilde{C})$ and D is the unique divisor in $|E|$, then we have an equality of divisors

$$(\Xi_E^+)_{|\tilde{C}} = \tilde{C} \cap \Xi_E^+ = D.$$

Proof. This is standard, we include a proof for the convenience of the reader. Suppose first that $E \in P^- - V(\tilde{C})$, which, by Lemma 2.1, is equivalent to the condition $h^0(\tilde{C}, E) = 1$. Write $|E| = D = p_1 + \dots + p_{\tilde{g}-1}$, where $p_i \in \tilde{C}$. Since p_i is a fixed point of the linear series $|E|$, we have that $h^0(\tilde{C}, E \otimes \mathcal{O}_{\tilde{C}}(-p_i + \sigma p_i)) = 2$, which implies that

$$D \subset \tilde{C} \cap \Xi_E^+ = (\Xi_E^+)_{|\tilde{C}}.$$

Using that $\tilde{C} \cdot \Xi_E^+ = \tilde{g} - 1$, we get the desired second equality. Now consider the maps

$$\tilde{C} \times P^- \xrightarrow{(a, \text{id})} P' \times P^- \xrightarrow{\mu} P^+,$$

where a is the Abel-Prym map and μ is the multiplication map. Let \mathcal{P} be the Poincaré line bundle on $\tilde{C} \times P^-$, trivialized over the section $\{p\} \times P^-$ for some $p \in \tilde{C}$. Consider the line bundle on $\tilde{C} \times P^-$ given by $\mathcal{L} := (a \times \text{id})^* \mu^* \mathcal{O}_{P^+}(\Xi^+)$. We can trivialize \mathcal{L} along the given section $\{p\} \times P^-$ by tensoring with the pull back from P^- of the divisor $\Xi_{p-\sigma(p)}^+$. It is easy to check that the fibers of \mathcal{P} and \mathcal{L} over $\tilde{C} \times \{E\}$ are given by

$$\begin{cases} \mathcal{P}_{\tilde{C} \times \{E\}} = E, \\ \mathcal{L}_{\tilde{C} \times \{E\}} = \mathcal{O}_{P'}(\Xi_E^+)_{|\tilde{C}}. \end{cases}$$

By what was proved above, if $E \notin P^- - V(\tilde{C})$ then the two fibers agree. By the Seesaw theorem (e.g. [3, Lemma 11.3.4]), $\mathcal{P} \cong \mathcal{L}$ and we get the desired first equality. □

2. The theta-dual of \tilde{C}

In this section, we want to study the theta-dual of \tilde{C} in the Prym variety P ; this can be identified canonically with the set (see (2)):

$$V(\tilde{C}) := \{E \in P^- \mid h^0(P', \mathcal{I}_{\tilde{C}}(\Xi_E^+)) > 0\} \subset P^-.$$

In fact the theta-dual $V(\tilde{C})$ can be described in terms of the following standard Brill-Noether loci (see [19]):

$$V^r := \{L \in \text{Nm}^{-1}(\omega_C) \mid h^0(L) \geq r + 1, h^0(L) \equiv r + 1 \pmod{2}\},$$

where $V^r \subset P^-$ (resp. $V^r \subset P^-$) if r is even (resp. odd). We view both the theta dual and the Brill-Noether loci as sets, although they can be endowed with natural scheme structures⁵.

Lemma 2.1. *We have the set-theoretic equality*

$$V(\tilde{C}) = V^2.$$

Proof. An element $E \in P^-$ belongs to $V(\tilde{C})$ if and only if $\tilde{C} \subset \Xi_E^+$, which, by the definition of $\tilde{C} \subset P'$, is equivalent to $h^0(\tilde{C}, E \otimes \mathcal{O}_{\tilde{C}}(\sigma(p) - p)) > 0$ for every $p \in \tilde{C}$. By Mumford’s parity trick (see [10]), this happens if and only if $h^0(\tilde{C}, E) \geq 3$, that is $E \in V^2$. \square

Theorem 2.2. *For any étale double cover $\tilde{C} \rightarrow C$ as above with C not hyperelliptic of genus g , it holds that*

$$\dim(V^2) = \dim(P) - 3 = g - 4.$$

For $g = 3$, the Theorem says that $V^2 = \emptyset$. We start with the following Lemma, which is similar to [10, Lemma p. 345].

Lemma 2.3. *If $Z \subseteq V^2$ is an irreducible component, and $\dim Z \geq g - 3$, then for a general line bundle $L \in Z$, there is a line bundle M on C with $h^0(M) \geq 2$, and an effective divisor F on \tilde{C} such that $L \cong \pi^*M \otimes \mathcal{O}_{\tilde{C}}(F)$.*

Proof. Let Z and L be as in the statement. Suppose that $h^0(L) = r + 1$ for $r \geq 2$ even, so that $L \in W_{g-1}^r - W_{g-1}^{r+1}$. From the hypothesis, we get that

$$\dim T_L W_{g-1}^r \cap T_L P^- \geq g - 3 = \dim(P^-) - 2. \tag{3}$$

⁵We remark however that the V^r have been considered with different natural schematic structures (compare [19] and [4]).

The Zariski tangent space to W_{g-1}^r at L is given by the orthogonal complement to the image of the Petri map (e.g. [1, Prop. 4.2]):

$$H^0(\tilde{C}, L) \otimes H^0(\tilde{C}, \sigma^*(L)) \rightarrow H^0(\tilde{C}, \omega_{\tilde{C}}),$$

where we have used that $\omega_{\tilde{C}} = \pi^*(\omega_C) = L \otimes \sigma^*(L)$. On the other hand, the tangent space to the Prym is by definition $T_L P^- = H^0(\tilde{C}, \omega_{\tilde{C}})^-$, the (-1) -eigenspace of $H^0(\tilde{C}, \omega_{\tilde{C}})$ relative to the involution σ . Therefore, it is easy to see that the intersection of the Zariski tangent spaces $T_L W_{g-1}^r \cap T_L P^-$ is given as the orthogonal complement to the image of the map

$$v_0 : \wedge^2 H^0(\tilde{C}, L) \rightarrow H^0(\tilde{C}, \omega_{\tilde{C}})^-$$

defined by $v_0(s_i \wedge s_j) = s_i \sigma^* s_j - s_j \sigma^* s_i$.

The inequality (3) is equivalent to $\text{codim}(\ker v_0) \leq 2$. On the other hand, the decomposable forms in $\wedge^2 H^0(\tilde{C}, L)$ form a subvariety of dimension $2r - 1 \geq 3$, and so there is a decomposable vector $s_i \wedge s_j$ in $\ker v_0$. This means that $s_i \sigma^* s_j - s_j \sigma^* s_i = 0$, or in other words that $\frac{s_j}{s_i}$ defines a rational function h in C . We conclude by taking $M = \mathcal{O}_C((h)_0)$ and F the be the maximal common divisor between $(s_i)_0$ and $(s_j)_0$. □

Proof of Theorem 2.2. The dimension of $V(\tilde{C}) = V^2$ is at least $g - 4$ by the theorem of Bertram ([2], see also [4]). Suppose, by contradiction, that there is an irreducible component $Z \subseteq V^2$ such that $\dim Z = m \geq g - 3$. Then, by applying the preceding Lemma 2.3 for the general element $L \in Z$,

$$L \cong \pi^* M \otimes \mathcal{O}_{\tilde{C}}(B)$$

where M is an invertible sheaf on C such that $h^0(M) \geq 2$, and B is an effective divisor on \tilde{C} such that $Nm(B) \in |K_C \otimes M^{\otimes -2}|$. The family of such pairs (M, B) is a finite cover of the set of pairs $\{M, F\}$ where:

- M is an invertible sheaf on C of degree $d \geq 2$ such that $h^0(M) \geq 2$,
- F is an effective divisor on C of degree $2g - 2 - 2d \geq 0$, such that $F \in |K_C \otimes M^{\otimes -2}|$.

By Marten’s theorem applied to the non-hyperelliptic curve C (see [1, Pag. 192]), the dimension of the above family of line bundles M is bounded above by

$$\dim(W_d^1) < d - 2. \tag{4}$$

Fixing a line bundle M as above, the dimension of possible F satisfying the second condition is bounded by Clifford’s theorem,

$$h^0(K_C \otimes M^{\otimes -2}) - 1 \leq g - 1 - d, \tag{5}$$

By putting together the inequalities (4) and (5), we get that the dimension m of our family of pairs $\{M, F\}$ is bounded above by $m < d + 2 + g - 1 - d = g - 3$, contradicting our hypothesis. \square

3. Computing the $V^i(\mathcal{I}_{\tilde{C}}(\Xi))$

In this section we compute the cohomological support loci for the ideal sheaf $\mathcal{I}_{\tilde{C}}(\Xi)$, which can be identified with the auxiliary canonical loci

$$\tilde{V}^i(\mathcal{I}_{\tilde{C}}(\Xi^+)) \subset P^-$$

defined in (1).

Theorem 3.1. *The cohomological support loci for $\mathcal{I}_{\tilde{C}}(\Xi)$ are*

$$\begin{cases} V^0(\mathcal{I}_{\tilde{C}}(\Xi)) = V^1(\mathcal{I}_{\tilde{C}}(\Xi)) = V(\tilde{C}), \\ V^2(\mathcal{I}_{\tilde{C}}(\Xi)) = P^-, \\ V^{\geq 3}(\mathcal{I}_{\tilde{C}}(\Xi)) = \emptyset. \end{cases}$$

Proof. The equality $V^0(\mathcal{I}_{\tilde{C}}(\Xi)) = V(\tilde{C})$ is just the definition of the theta-dual of \tilde{C} . Consider the exact sequence defining the ideal sheaf $\mathcal{I}_{\tilde{C}}$ twisted by the divisor Ξ_E^+ , for $E \in P^-$:

$$0 \rightarrow \mathcal{I}_{\tilde{C}}(\Xi_E^+) \rightarrow \mathcal{O}_{P'}(\Xi_E^+) \rightarrow \mathcal{O}_{\tilde{C}}(\Xi_E^+) \rightarrow 0.$$

By taking cohomology and using the vanishing $H^j(P', \mathcal{O}_{P'}(\Xi_E^+)) = 0$ for $j > 0$, we get the emptiness of $\tilde{V}^i(\mathcal{I}_{\tilde{C}}(\Xi^+))$ for $i \geq 3$ and the two exact sequences

$$\begin{aligned} 0 \rightarrow H^0(\mathcal{I}_{\tilde{C}}(\Xi_E^+)) \rightarrow H^0(\mathcal{O}_{P'}(\Xi_E^+)) \xrightarrow{\psi_E} H^0(\mathcal{O}_{\tilde{C}}(\Xi_E^+)) \rightarrow H^1(\mathcal{I}_{\tilde{C}}(\Xi_E^+)) \rightarrow 0, \\ 0 \rightarrow H^1(\mathcal{O}_{\tilde{C}}(\Xi_E^+)) \rightarrow H^2(\mathcal{I}_{\tilde{C}}(\Xi_E^+)) \rightarrow 0. \end{aligned}$$

Using the second exact sequence and the Lemma 1.1, we get that

$$\tilde{V}^2(\mathcal{I}_{\tilde{C}}(\Xi^+)) = \{E \mid h^1(\tilde{C}, E) > 0\}.$$

Since E has degree $\tilde{g} - 1$, by Riemann-Roch we have that $h^1(\tilde{C}, E) = h^0(\tilde{C}, E)$, which is greater than 0 for all $E \in P^-$ by the definition of P^- .

Consider now the first above exact sequence. Since $H^0(P', \mathcal{O}_{P'}(\Xi_E^+)) = 1$ and $\tilde{V}^1(\mathcal{I}_{\tilde{C}}(\Xi^+))$ consists of the elements E such that the map ψ_E is not surjective, we get using again Lemmas 1.1 and 2.1

$$\begin{aligned} \tilde{V}^1(\mathcal{I}_{\tilde{C}}(\Xi^+)) &= \{E \mid h^1(\tilde{C}, \mathcal{O}_{\tilde{C}}(\Xi_E^+)) > 1\} \cup \{E \mid h^0(P', \mathcal{I}_{\tilde{C}}(\Xi_E^+)) = 1\} = \\ &= \tilde{V}^2 \cup V(\tilde{C}) = V(\tilde{C}). \end{aligned}$$

\square

4. Computing the $V^i(\mathcal{I}_C(2\Xi))$

In this section we determine the cohomological support loci of the ideal sheaf $\mathcal{I}_{\tilde{C}}(2\Xi^+)$. In the proof of the next Theorem, we will need to know that the set $S(\tilde{C})$ of theta-characteristics of \tilde{C} has a point in P^- . It is not much more work to show the stronger lemma below.

Lemma 4.1. *A theta-characteristic $L \in S(\tilde{C})$ belongs to $\text{Nm}^{-1}(\omega_C) = P^+ \cup P^-$ if and only if it is the pull-back of a theta-characteristic M on C . Moreover, it holds that*

$$|S(\tilde{C}) \cap P^-| = |S(\tilde{C}) \cap P^+| = 2^{2g-1}.$$

Proof. If M is a theta-characteristic on C , then $\pi^*(M) \in S(\tilde{C}) \cap \text{Nm}^{-1}(\omega_C)$ as

$$\begin{cases} \text{Nm}(\pi^*(M)) = M^{\otimes 2} = \omega_C, \\ \pi^*(M)^{\otimes 2} = \pi^*(M^{\otimes 2}) = \pi^*(\omega_C) = \omega_{\tilde{C}}. \end{cases}$$

Conversely, if $L^{\otimes 2} = \omega_{\tilde{C}}$ and $\text{Nm}(L) = \omega_C$, then

$$L \otimes L = \omega_{\tilde{C}} = \pi^*(\omega_C) = \pi^*(\text{Nm}(L)) = L \otimes \sigma^*(L),$$

which implies that $L = \pi^*(M)$ for some $M \in \text{Pic}(C)$. By applying the Norm map, we obtain $\omega_C = \text{Nm}(L) = \text{Nm}(\pi^*(M)) = M^{\otimes 2}$.

Moreover, if we denote by η_0 the line bundle of order two on C satisfying $\pi_*(\mathcal{O}_{\tilde{C}}) = \mathcal{O}_C \oplus \eta_0$, then the pull-back $\pi^*(M)$ of a theta-characteristic M on C belongs to P^- if and only if $h^0(C, M) \not\equiv h^0(C, M \otimes \eta_0) \pmod{2}$ (and to P^+ otherwise), as follows from the formula $H^0(\tilde{C}, \pi^*(M)) = H^0(C, M) \oplus H^0(C, M \otimes \eta_0)$.

Now, fix a theta-characteristic M_0 of C . Then all the theta-characteristics of C are of the form $M_0 \otimes \eta$ for a unique $\eta \in J_2(C)$, where $J_2(C)$ is the group of the 2^{2g} line bundles of C whose square is trivial. Consider the following map

$$\begin{aligned} q_0 : J_2(C) &\longrightarrow \mathbb{Z}/2\mathbb{Z} \\ \eta &\longmapsto h^0(M_0 \otimes \eta \otimes \eta_0) - h^0(M_0 \otimes \eta) \pmod{2} \end{aligned}$$

The Riemann-Mumford relation (see [9, p. 182]) yields

$$q_0(\eta) = q_0(\eta_0) + \text{In } e_2(\eta, \eta_0),$$

where $e_2 : J_2(C) \times J(C) \rightarrow \{\pm 1\}$ is the Riemann skew-symmetric bilinear form (recall that $\text{char}(k) \neq 2$) and In is defined by $\text{In}(+1) = 0$ and $\text{In}(-1) = 1$. From the nondegeneracy of the form e_2 and the fact that $\eta_0 \neq \mathcal{O}_C$, it follows that the function

$$\begin{aligned} J_2(C) &\longrightarrow \{\pm 1\} \\ \eta &\longmapsto e_2(\eta_0, \eta) \end{aligned}$$

takes half the times the value $+1$ and half the value -1 . This concludes our proof. □

To complete the main theorem announced in the introduction we have the following:

Theorem 4.2. *For a fixed $E_0 \in P^-$, the cohomological support loci for the ideal sheaf $\mathcal{I}_{\tilde{C}}(2\Xi^+)$ are isomorphic to*

$$\begin{cases} V^0(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = \begin{cases} (P^-)^{\otimes 2} & \text{if } g \geq 4, \\ \{\omega_{\tilde{C}}\} & \text{if } g = 3, \end{cases} \\ V^1(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = V^2(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = \{\omega_{\tilde{C}}\}, \\ V^{\geq 3}(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = \emptyset. \end{cases}$$

Proof. We first compute the auxiliary canonical loci $\tilde{V}^i(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \subset P^-$ introduced in (1). To this end, consider the exact sequence defining the ideal sheaf $\mathcal{I}_{\tilde{C}}$ twisted by the divisor $2 \cdot \Xi_E^+$ with $E \in P^-$:

$$0 \rightarrow \mathcal{I}_{\tilde{C}}(2 \cdot \Xi_E^+) \rightarrow \mathcal{O}_{P'}(2 \cdot \Xi_E^+) \rightarrow \mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+) \rightarrow 0.$$

By taking cohomology and using the vanishing $H^j(P', \mathcal{O}_{P'}(2 \cdot \Xi_E^+)) = 0$ for $j > 0$, we get the emptiness of $\tilde{V}^i(\mathcal{I}_{\tilde{C}}(2\Xi^+))$ for $i \geq 3$ and the two exact sequences

$$H^0(\mathcal{I}_{\tilde{C}}(2 \cdot \Xi_E^+)) \hookrightarrow H^0(\mathcal{O}_{P'}(2 \cdot \Xi_E^+)) \xrightarrow{\phi_E} H^0(\mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+)) \rightarrow H^1(\mathcal{I}_{\tilde{C}}(2 \cdot \Xi_E^+)),$$

$$0 \rightarrow H^1(\mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+)) \rightarrow H^2(\mathcal{I}_{\tilde{C}}(2 \cdot \Xi_E^+)) \rightarrow 0.$$

By Lemma 1.1, we have that $\mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+) = E^{\otimes 2}$ and therefore the second exact sequence implies that

$$\tilde{V}^2(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = \{E^{\otimes 2} \mid h^1(\tilde{C}, E^{\otimes 2}) > 0\} = S(\tilde{C}) \cap P^-.$$

Moreover, from the Proposition 4.3 below and the first exact sequence, we have that

$$\tilde{V}^1(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \subset S(\tilde{C}) \cap P^-.$$

In order to determine $\tilde{V}^0(\mathcal{I}_{\tilde{C}}(2\Xi^+))$, we consider the first above exact sequence. If $g \geq 4$, then we have the inequality

$$h^0(P', \mathcal{O}_{P'}(2 \cdot \Xi_E^+)) = 2^{g-1} > 2g - 1 = \tilde{g} \geq h^0(\tilde{C}, \mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+)),$$

from which we conclude that

$$\tilde{V}^0(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = P^- \text{ if } g \geq 4.$$

On the other hand, if $g = 3$ and E is not a theta characteristic, then the map ϕ_E is a surjection (as we will prove in the Proposition 4.3) between two spaces of the same dimension and therefore an isomorphism. This implies that

$$\tilde{V}^0(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \subset S(\tilde{C}) \cap P^- \text{ if } g = 3.$$

Observe that $S(\tilde{C}) \cap P^- \neq \emptyset$ according to Lemma 4.1. Therefore, using our canonical identification (2), we get for the cohomological support loci

$$\begin{cases} V^0(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \begin{cases} = (P^-)^{\otimes 2} & \text{if } g \geq 4, \\ \subset \{\omega_{\tilde{C}}\} & \text{if } g = 3, \end{cases} \\ V^1(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \subset \{\omega_{\tilde{C}}\}, \\ V^2(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = \{\omega_{\tilde{C}}\}, \\ V^{\geq 3}(\mathcal{I}_{\tilde{C}}(2\Xi^+)) = \emptyset. \end{cases}$$

We deduce that the ideal $\mathcal{I}_{\tilde{C}}(2\Xi^+)$ is GV-sheaf in the sense of [17, Def. 3.1] and hence, by [17, Prop. 3.13], we get that

$$\{\omega_{\tilde{C}}\} = V^2(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \subset V^1(\mathcal{I}_{\tilde{C}}(2\Xi^+)) \subset V^0(\mathcal{I}_{\tilde{C}}(2\Xi^+)),$$

which gives the desired conclusion. □

The next Proposition is due to Beauville; a proof is given in [8, Lemma 2.4] for the case of genus four curves. We give a proof here for the sake of the completeness.

Proposition 4.3. *If $E \in P^-$ is not a theta-characteristic, then the restriction map*

$$\phi_E : H^0(P', \mathcal{O}_{P'}(2 \cdot \Xi_E^+)) \rightarrow H^0(\tilde{C}, \mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+))$$

is surjective.

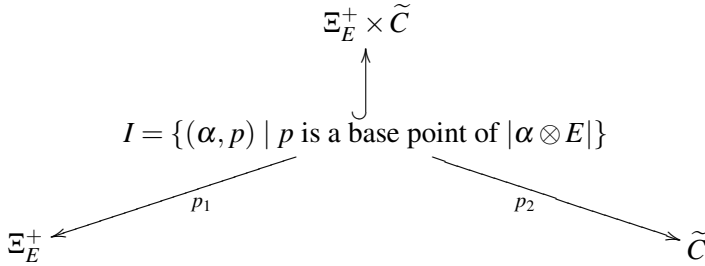
Proof. We start by proving that a general $\alpha \in \Xi_E^+ \subset P'$ satisfies

1. $(-1)^*(\alpha) = \alpha^{-1} \notin \Xi_E^+$,
2. $|E \otimes \alpha|$ is a pencil without base points.

The first assertion follows from the fact that Ξ_E^+ is not symmetric. Indeed, Ξ_E^+ is symmetric if and only if E is a theta-characteristic (see [3, Thm. 11.2.4]), which we have excluded by hypothesis.

The fact that the complete linear series $|E \otimes \alpha|$ is a pencil for a general $\alpha \in \Xi_E^+$ follows from the fact that a generic element $L \in \Xi^+$ has $h^0(L) = 2$.

Now we want to see that the linear system $|E \otimes \alpha|$ has no base points for a general $\alpha \in \Xi_E^+$. Consider the incidence variety



For every point $q \in \tilde{C}$, the following injection

$$\begin{array}{ccc}
 p_2^{-1}(q) & \longrightarrow & V^2 \\
 \alpha & \longmapsto & \alpha \otimes E \otimes \mathcal{O}(-q + \sigma q)
 \end{array}$$

is well-defined since q is a fixed point of $|E \otimes \alpha|$. Therefore, by Theorem 2.2, the fibers of p_2 have dimension at most $g - 4$ and hence I has dimension at most $g - 3$. Since the dimension of Ξ_E^+ is $g - 2$, this implies that the first projection is not dominant and hence the conclusion.

Now we want to find elements in $H^0(P', \mathcal{O}_{P'}(2 \cdot \Xi_E^+))$ that form a basis when restricted to $H^0(\tilde{C}, \mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+))$. From Lemma 1.1 and the fact that E is not a theta characteristic, we get that $h^0(\tilde{C}, \mathcal{O}_{\tilde{C}}(2 \cdot \Xi_E^+)) = h^0(\tilde{C}, E^{\otimes 2}) = \tilde{g} - 1$. We begin by choosing an $\alpha \in \Xi_E^+ \subset P^-$ satisfying the two conditions above. In particular, from the condition (2), we can choose an effective divisor $E_\alpha = \sum_{1 \leq i \leq \tilde{g}-1} x_i \in |E \otimes \alpha|$ such that all the points x_i are distinct.

We define the following effective divisors in P^-

$$E_{\alpha,j} := E_\alpha - x_j + \sigma x_j = \sigma x_j + \sum_{i \neq j} x_i \quad \text{for } j = 1, \dots, \tilde{g} - 1.$$

By condition (2), we get that $h^0(P', \mathcal{O}_{P'}(E_{\alpha,j})) = 1$ and therefore, by Lemma 1.1, $(\Xi_{E_{\alpha,j}}^+)_{|\tilde{C}} = E_{\alpha,j}$.

Consider next the line bundle $E \otimes \alpha^{-1} \otimes \mathcal{O}_{\tilde{C}}(x_j - \sigma x_j) \in P^-$. Since $h^0(\tilde{C}, E \otimes \alpha^{-1}) = 0$ by condition (1), using Mumford's parity trick we deduce that

$$h^0(\tilde{C}, E \otimes \alpha^{-1} \otimes \mathcal{O}_{\tilde{C}}(x_j - \sigma x_j)) = 1.$$

Define $E'_{\alpha,j}$ to be the unique effective divisor of $|E \otimes \alpha^{-1} \otimes \mathcal{O}_{\tilde{C}}(x_j - \sigma x_j)|$ By Lemma 1.1, we get that $(\Xi_{E'_{\alpha,j}}^+)_{|\tilde{C}} = E'_{\alpha,j}$.

Summing up, we have constructed $\tilde{g} - 1$ couples of divisors $(E_{\alpha,j}, E'_{\alpha,j})$ satisfying

$$\begin{cases} \mathcal{O}_{P'}(\Xi_{E_{\alpha,j}}^+ + \Xi_{E'_{\alpha,j}}^+) \cong \mathcal{O}_{P'}(2 \cdot \Xi_E^+), \\ (\Xi_{E_{\alpha,j}}^+ + \Xi_{E'_{\alpha,j}}^+) |_{\tilde{C}} = E_{\alpha,j} + E'_{\alpha,j}. \end{cases}$$

It remains to show that the $\tilde{g} - 1$ divisors $E_{\alpha,j} + E'_{\alpha,j}$ corresponds to independent sections on $H^0(\tilde{C}, 2 \cdot E)$. This will follow from the next

Claim: $x_j \notin E_{\alpha,j} + E'_{\alpha,j}$ and $x_j \in E_{\alpha,k} + E'_{\alpha,k}$ for every $k \neq j$.

By the definition of the $E_{\alpha,j}$ and using that σ has no fixed points, we get that $x_j \notin E_{\alpha,j}$ while $x_j \in E_{\alpha,k}$ for every $k \neq j$. Finally, observe that $\mathcal{O}_{\tilde{C}}(E'_{\alpha,j}) \otimes \mathcal{O}_{\tilde{C}}(\sigma x_j - x_j) = E \otimes \alpha^{-1}$ which by condition (1) has no sections. This can happen only if $E'_{\alpha,j} - x_j$ is not effective, or in other words $x_j \notin E'_{\alpha,j}$. □

Acknowledgements

We are grateful to prof. A. Ragusa for organizing an excellent summer school “Pragmatic 2007” held at the University of Catania, where the three authors began their joint work on this subject. We also thank prof. G. Pareschi and prof. M. Popa who, during that summer school, suggested to us an interesting research problem from which this work originated, and have since then followed the progress of this work, providing useful suggestions.

REFERENCES

- [1] E. Arbarello - M. Cornalba - P. A. Griffiths - J. Harris, *Geometry of algebraic curves*, Vol. I. Grundlehren der Mathematischen Wissenschaften 267. Springer-Verlag, New York, 1985.
- [2] A. Bertram, *An existence theorem for Prym special divisors*, Invent. Math. 90 (1987), 669-671.
- [3] C. Birkenhake - S. Lange, *Complex abelian varieties*, Second edition. Grundlehren der Mathematischen Wissenschaften, 302. Springer-Verlag, Berlin, 2004.
- [4] C. De Concini - P. Pragacz, *On the class of Brill-Noether loci for Prym varieties*, Math. Ann. 302 (1995), 687-697.

- [5] M. Green - R. Lazarsfeld, *Deformation theory, generic vanishing theorems, and some conjectures of Enriques, Catanese and Beauville*, Invent. Math. 90 (1987), 389-407.
- [6] M. Green - R. Lazarsfeld, *Higher obstructions to deforming cohomology groups of line bundles*, J. Amer. Math. Soc. 1 (1991), 87-103.
- [7] C. Hacon, *A derived category approach to generic vanishing*, J. Reine Angew. Math. 575 (2004), 173-187.
- [8] E. Izadi - D. van Straten, *The intermediate Jacobians of the theta divisors of four-dimensional principally polarized abelian varieties*, J. Alg. Geom. 4 (1995), 557-590.
- [9] D. Mumford, *Theta characteristics of an algebraic curve*, Ann. Sci. École Norm. Sup. 4 (1971), 181-192.
- [10] D. Mumford, *Prym varieties. I*, Contributions to analysis (a collection of papers dedicated to Lipman Bers), pp. 325-350. Academic Press, New York, 1974.
- [11] G. Pareschi, *Generic vanishing, gaussian maps, and Fourier-Mukai transform*, arXiv:math/0310026.
- [12] G. Pareschi - M. Popa, *Regularity on abelian varieties I*, J. Amer. Math. Soc. 16 (2003), 285-302.
- [13] G. Pareschi - M. Popa, *Regularity on abelian varieties II: basic results on linear series and defining equations*, J. Algebraic Geom. 13 (2004), 167-193.
- [14] G. Pareschi - M. Popa: *Regularity on abelian varieties III: relationship with Generic Vanishing and applications*, arXiv:0802.1021.
- [15] G. Pareschi - M. Popa, *Castelnuovo theory and the geometric Schottky problem*, to appear in J. Reine Angew. Math. (available at arXiv:math/0407370).
- [16] G. Pareschi - M. Popa, *M-regularity and the Fourier-Mukai transform*, to appear in Pure and Applied Math. Quarterly, issue in honor of F. Bogomolov (available at arXiv:math/0512645).
- [17] G. Pareschi - M. Popa, *GV-sheaves, Fourier-Mukai transform, and Generic Vanishing*, arXiv:math/0608127.
- [18] G. Pareschi - M. Popa, *Generic vanishing and minimal cohomology classes on abelian varieties*, Math. Annalen 340 (2008), 209-222.
- [19] G. E. Welters, *A theorem of Gieseker-Petri type for Prym varieties*, Ann. Sci. École Norm. Sup. 18 (1985), 671-683.
- [20] G. E. Welters, *Curves of twice the minimal class on principally polarized abelian varieties*, Nederl. Akad. Wetensch. Indag. Math. 49 (1987), 87-109.

SEBASTIAN CASALAINA MARTIN
Harvard University,
Department of Mathematics,
One Oxford Street,
Cambridge, MA 02138 (USA).
e-mail: casa@math.harvard.edu

MARTÍ LAHOZ
Universitat Politècnica de Catalunya,
Departament de Matemàtica Aplicada I,
Av. Diagonal, 647
08028 Barcelona (Spain).
e-mail: marti.lahoz@upc.edu

FILIPPO VIVIANI
Humboldt Universität zu Berlin,
Institut für Mathematik,
Rudower Chausse 25,
10099 Berlin (Germany).
e-mail: viviani@math.hu-berlin.de