FOGLIO DI ESERCIZI 4: DIMENSIONE E BASI DI SOTTOSPAZI DI $\mathbb{R}^n$

ESERCIZIO 1

Per ciascun sottospazio $U \leq \mathbb{R}^4$ dato come insieme di soluzioni dei seguenti sistemi lineare omogenei, si determini la dimensione di U e si trovi una base di U.

(A)

$$\{X_1 + X_2 - X_3 + 2X_4 = 0.$$

(B)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ X_1 - X_2 - X_4 = 0. \end{cases}$$

(C)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ X_1 - X_2 - X_4 = 0, \\ -X_1 + X_2 - X_3 = 0. \end{cases}$$

(D)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ X_1 - X_2 - X_4 = 0, \\ -X_1 + X_2 - X_3 = 0, \\ X_2 - X_4 = 0. \end{cases}$$

(E)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ X_1 - X_2 - X_4 = 0, \\ -X_1 + X_2 - X_3 = 0, \\ X_2 - X_4 = 0, \\ 2X_1 - 3X_4 = 0. \end{cases}$$

(F)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ 2X_1 + 2X_2 - 2X_3 + 4X_4 = 0. \end{cases}$$

(G)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ X_1 - X_2 - X_4 = 0, \\ 2X_1 + 2X_2 - X_3 + X_4 = 0. \end{cases}$$

(H)

$$\begin{cases} X_1 + X_2 - X_3 + 2X_4 = 0, \\ X_1 - X_2 - X_4 = 0, \\ -X_1 + X_2 - X_3 = 0, \\ X_1 + X_2 - 2X_3 + X_4 = 0. \end{cases}$$

ESERCIZIO 2

Per ciascuno dei seguenti sottospazi W di $\mathbb{R}^4$ dati in forma parametrica, si determini la dimensione di W e si determini una base di W.

(A)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix} \right\rangle.$$

(B)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\rangle.$$

(C)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle.$$

(D)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix} \right\rangle.$$

(E)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ -2 \end{pmatrix} \right\rangle.$$

(F)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -2 \\ -4 \end{pmatrix} \right\rangle.$$

(G)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \\ -1 \end{pmatrix} \right\rangle.$$

(H)

$$\left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 0 \\ -1 \end{pmatrix} \right\rangle.$$

ESERCIZIO 3

Da ciascuno dei seguenti sistemi di vettori di $\mathbb{R}^4$, si estragga una base del loro span lineare.

(A)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

(B)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 0 \\ 2 \end{pmatrix} \right\}.$$

(C)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

(D)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

(E)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

(F)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 2 \end{pmatrix} \right\}.$$

(G)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 2 \end{pmatrix} \right\}.$$

(H)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

(I)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\}.$$